Special Interest Group Meeting on Thermal Issues
Laboratory for Machine Tools and Production Engineering (WZL) of
RWTH Aachen, Germany, February 2020
www.euspen.eu

Thermal Modal Analysis including Static correction: an efficient tool to model and design thermal compensation systems

Nicolas JOBERT[1] , Muriel THOMASSET[1], Gilles CAUCHON[1], David DENNETIERE[1]

[1]*Synchrotron SOLEIL, l'Orme des Merisiers, Gif-sur-Yvette, 91192 Saint-Aubin, France*

nicolas.jobert@synchrotron-soleil.fr

Abstract

Designing a thermal compensation system always involves dealing with conflicting requirements, such as minimizing the number of sensors and transducers, cable routing and control power while ensuring maximal disturbances rejection. To assist in the trade-off process, it is essential to have a means to quickly investigate the merits of various configurations. Starting from the dynamics of the thermal systems, the residual position error for any control loop can be quickly evaluated provided that such a model is given in an efficient and compact manner. State-space models provide a convenient way to capture the system dynamics, and allow for the exploration of dozens of control configurations with no appreciable computational burden. However, conventional finite element packages do not have built-in capacities to efficiently generate such compact models, and it is extremely inefficient to work with the original detailed models.

In this paper, it is shown that using thermal modal analysis, thermal mechanical state-space models can be efficiently built. In particular, it is shown that adding one extra state per actuator allows for capturing the full static response of the system. While the impact on the thermal response is generally limited, the mechanical response fidelity is very significantly improved, particularly in bending dominated problems. The method is validated against the full, detailed thermal mechanical response, and it shows that the reduced models are both efficient and conservative.

Model reduction, thermal modal analysis, state-space, thermal effects compensation

1. Scope and motivation

The idea of using modal analysis to reduce the complexity of thermal transient analyses is nothing new (see [1] or [2] for example). However, in contrast with structural dynamics where it is *de facto* the standard approach, for thermal effects modal superposition techniques have never seemed to have received a broad acceptance. One reason for this might be that convergence is comparatively slow, i.e. generally a large number of thermal modes are needed to obtain acceptable agreement with the original model.

Firstly, it must be acknowledged that this limitation is a fact, and arises particularly in situations where point-like heat loads are applied. However, as noted in [3], while the temperature distribution can be underestimated in the vicinity of the load, this does not mean that the quantity of interest (generally, positional or angular deviation) is severely affected.

Secondly, when higher accuracy is required, then the modal basis can be simply augmented using the so-called residual vector approach. A nice discussion of the method as applied in the field of structural dynamics, where it originated, can be found in [4]. To the best of the knowledge of the authors, the method has never been applied to thermal response estimation. In the following sections we outline the corresponding procedure and show its benefits when applied to ultra-precision components.

2.1. Thermal response using residual modal vector method

Mathematically, including a residual vector this simply amounts to evaluating the offset between the static response vector of the system obtained firstly using the original model, and secondly using a modal basis, as follows:

$$T_{exact} = K^{-1} P$$, and

$$T_{reduced} = \sum_{i=1}^{n} {}^{t}\Phi_i P . \Phi_i / \lambda_i$$

Where *K* is the conductivity matrix, *P* is the nodal load vector (thermal power fed into the system on a node basis) and (λ_i , Φ_i) are the eigenvalues and eigenvectors of the thermal system, i.e solutions to the following matrix equation.

$$\lambda C + K = 0$$

Physically, for each mode the corresponding eigenvalue is equal to the inverse of the corresponding time constant, that is, the modes with the lowest eigenvalues correspond to solutions to the free thermal response of the system with the longest decay time. The residual vector *R* is obtained by :

$$R = T_{exact} - T_{reduced}$$

The process can be repeated with each load case, i.e. the response to the j^th^ load vector will need to account for the j^th^ residual vector to be exact in the static domain.

2.2. State-space thermal-mechanical model using residual modal vector

Thanks to linearity the mechanical response $\Phi_{i,s}$ to all of the n_m thermal modes $\Phi_{i,t}$ can be evaluated and superimposed to obtain the complete thermal mechanical response. By the same procedure, the responses to the n_a residual vectors are estimated.

This is sufficient to conveniently build a state-space model for the thermal-mechanical response, i.e.:

$$x' = Ax + Bu \text{ , and}$$
$$y = Cx + Du$$

Where:

- x is the state vector, i.e containing the n_m modal amplitudes
- y is the output vector, containing responses at the n_s sensors
- u is the input vector, containing the thermal power delivered by each of the n_a actuators

By definition, A is a square diagonal matrix containing the eigenvalues, B is a $n_m x n_m$ matrix containing the load vectors. C is a $n_s x n_m$ matrix containing the modal amplitudes at each of the sensor locations.

Under normal circumstances, D would be zero. When adding residual vectors, each of their contributions will appear as a *feedthrough*, i.e. an additional response whose contribution linearly (and instantly) follows the excitation, hence in the D matrix. Mathematically, D will be a $n_s x n_a$, each column of which will correspond to the j-th residual vector. See for example [5] for a discussion of the residual vector as a feedthrough for structural dynamics.

3. Practical application

The methodology outlined previously is applied to a geometrically simple optical component for which flatness requirements are tight, while thermal effects are of crucial importance. This is the typical configuration for primary mirrors used in Synchrotron Light Source facility such as SOLEIL. In this example, the heat deposited can exceed 10^3 W/cm^2, while the local slope error should be kept within 1 µrad, down to 0.2 µrad in the near future. In order to control the temperature, the mirror is cooled via water or liquid nitrogen circulating into a copper heat exchanger, tightly on each of the mirrors side. The mirror itself is fitted with regularly spaced holes in order to allow bolting of each half of the heat exchanger to the mirror. Such a mirror is shown on Figure 1.

Figure 1. primary mirror for high-energy X-ray beamline (SOLEIL Synchrotron)

While both heat load and heat sinks are aimed at being stationary, there are inevitable residual fluctuations. The heat load deposited on the optical surface will change over time, in intensity, and in space because of the photon beam jitter, or whenever because of required changes in undulator gap. Conversely, the cooling water will neither keep its temperature constant to better than 1 °C nor will the flow rate remain stable within less than 10 to 20 %. This will induce some modification of the heat circulating in the mirror, then of the temperature distribution and ultimately the mechanical distortions of the optical surface.

In order to evaluate the optical surface distortions due to thermal transients, a thermal mechanical finite element model has been developed (using ANSYS rev 19.1), as shown on figure 2. It consists of both the mirror and the cooler, and is meshed using 20-nodes brick elements. In order to simplify the interpretation, we assume a uniform thermal conductance at the mirror/cooler interface of 50 000 W/m²/K, as would be obtained using a thermal contact enhancement foil (indium) as is typically employed in such a situation.

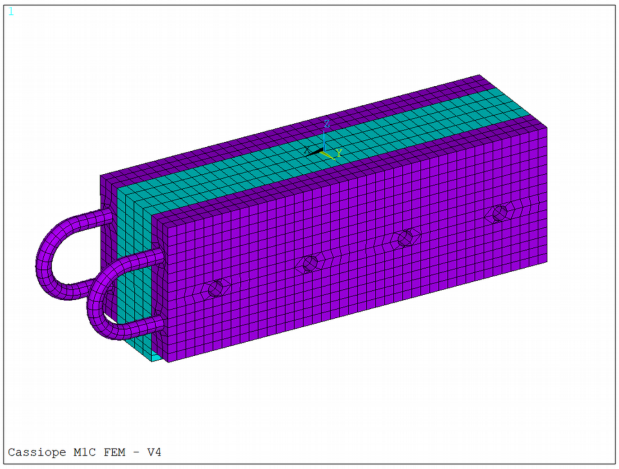

Figure 2. Finite Element model - mirror and cooler

The heat load is provided by the photon beam. It is assumed to be centered on the optical surface, with a maximum flux of 2500 W/cm^2. The beam profile is assumed to be gaussian, with a full-width-at-half-maximum equal to half the optical surface dimensions (160x25 mm^2), hence a total input power of about 400 W.

This system is cooled by circulating water, and again we simplify things by assuming that the flow rate is large enough to maintain a fluid bulk temperature at 21°C, and the fluid convection (film) coefficient is equal to 8000 W/m²/K .

Under those hypotheses, the temperature distribution is shown in figure 3, and the out-of-plane motion (with respect to the optical surface) in figure 4. Although the temperature elevation is about 23 K, and the resulting distortion is about 5 µm, this is already enough to cause some loss of performance for the overall system, since the corresponding slope is of the order of 100 µrad.

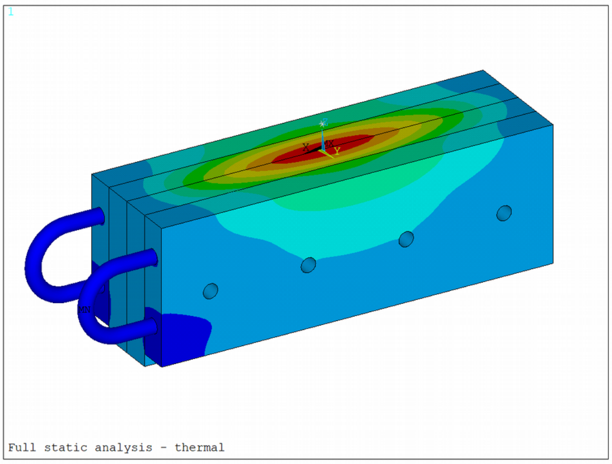

Figure 3. Temperature for nominal beam power (Tmax=42.8 °C)

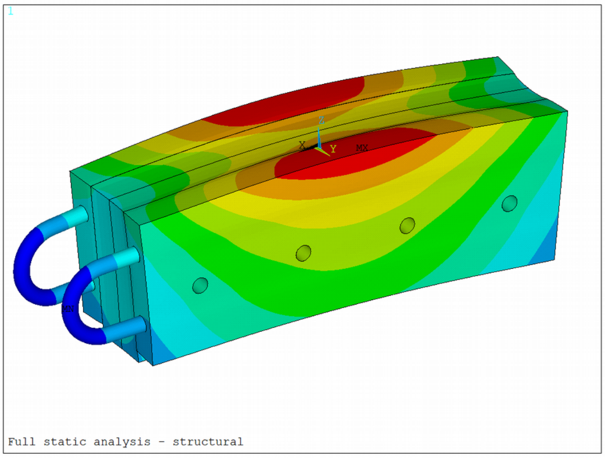

Figure 4. Distortion for nominal beam power dZmax=5.1 µm)

Equipped with those reference values, we can estimate the convergence rate of the modal superposition method.

First of all, we begin by estimating the modes, see [6] for generalities about the ANSYS matrix manipulation language (APDL Math), [7] for details specific to thermal modal analysis practical implementation and [8] for thermal harmonic analysis.

Table 1 - Thermal Modes

Mode		Description
#	τ [s]	
1	12.0	Heat dumped into water
2	8.1	Heat traveling longitudinally (order 1)
3	4.3	Heat traveling longitudinally (order 2)
4	3.0	Heat travelling vertically (order 1)

Obviously, modes 1 and 4 involve differential expansion of the upper and lower part of the system (bi metal effect), and should be a major contributor to both the temperature and structural response. Other modes might contribute to temperature distribution, but minimally to the distortions of the optical surface.

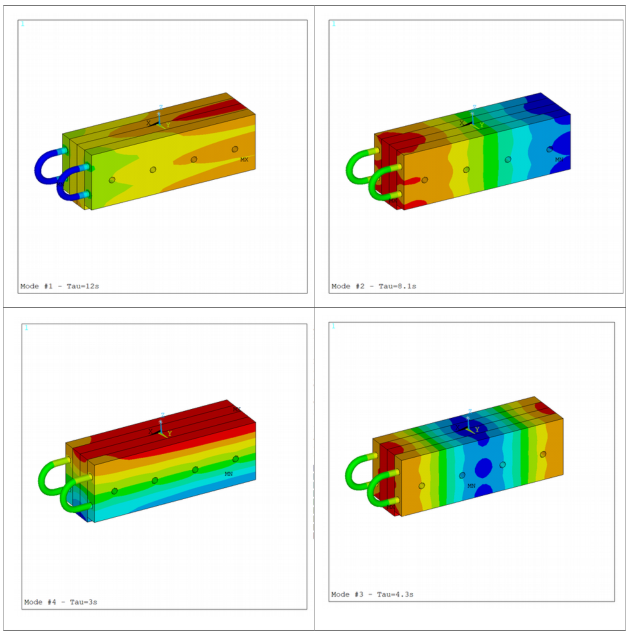

Figure 5. Temperature distribution for modes 1 to 4.

The thermal time constants distribution is given in figure 6. Since we are aiming at building a model that would have a useful bandwidth extending up to 1 Hz, we need to include modes with time constants shorter than approximately 0.5 to 1 s. From the thermal model (half model using the XZ plane of symmetry), we see that there are less than 10 modes with thermal time constants longer than 2 s, but more than 100 modes when setting the limit at 0.5 s. Obviously, the modal method is extremely efficient for slow dynamics, but there is a cliff-edge effect in the required number of modes as soon as one tries to extend the bandwidth further.

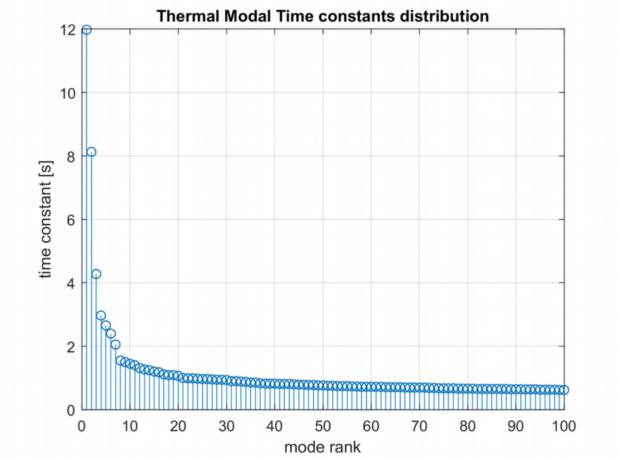

Figure 6. Thermal time constants distribution

In our case we might want to track the accuracy of temperature and structural responses obtained by including an increasing number of modes. Again, the static response is straightforward to obtain. The states (thermal modes) are obtained as $x = -A^{-1}Bu$ and hence the static response reads (including the residual vector):

$$T_{static} = Cx_{static} + Du = (-CA^{-1}B + D)u$$

Applying the procedure to the peak local temperature (on the optical surface), it appears that the convergence rate is terribly slow. The relative error exceeds 30% even for 100 modes included, and shows no sign of decrease (see Figure 7). On the contrary, the bump magnitude can be estimated within 1% by using as few as 20 modes.

In this context, it is clear that adding a single residual vector would definitely help to overcome convergence problems.

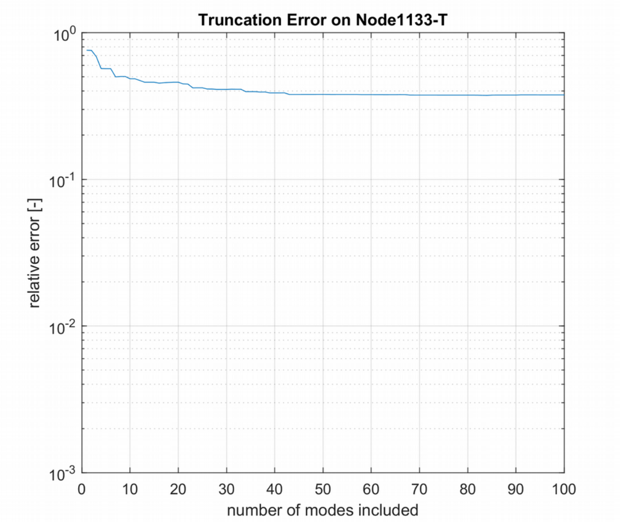

Figure 7. Relative error on local temperature

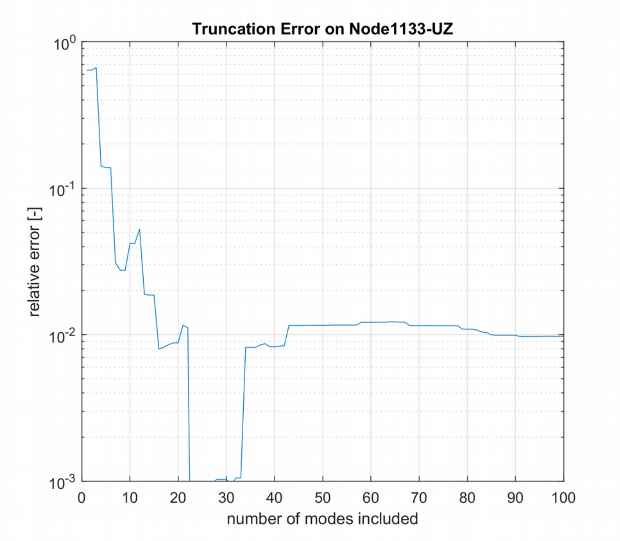

Figure 8. Relative error on bump amplitude

Residual vector effect on transfer functions

Since the quantity of interest is the optical surface distortion, from this point on we will focus on the thermal bump. The frequency response estimated *in the absence of residual vector* is given in figure 9 below. The input is expressed as a fraction of total thermal power, and the output is the thermal bump (in mm). We can check that for 10 and 100 modes, the static response is close to the reference value of 5.1µm.

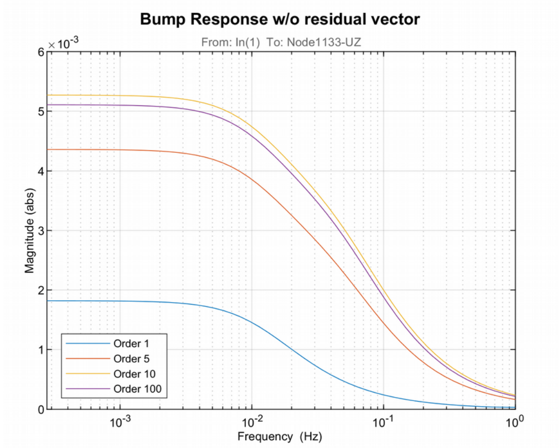

Figure 9. Transfer function without residual vector : magnitude

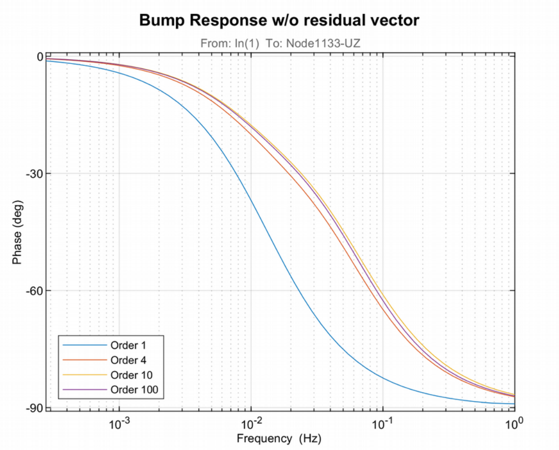

Figure 10. Transfer function without residual vector : phase

The benefit of including a residual vector is shown in figures 11 and 12, for magnitude and phase. Clearly, as far as magnitude is concerned, the model validity is greatly improved, and the usable bandwidth extends largely above 0.1Hz. In terms of phase, however, it is clear that it cannot be reliably estimated for frequencies above 0.5 Hz. Inconsistent results are obtained, clearly showing limitations of the model (and to begin with, space discretization is probably insufficient: for silicon, thermal diffusivity is about 90 mm^2/s, and a mesh with an element size 6 mm might not be fine enough).

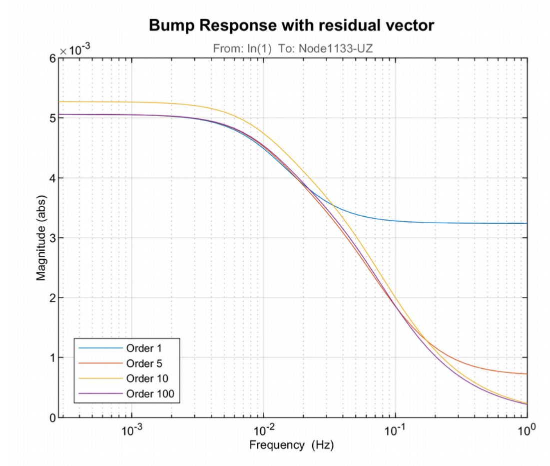

Figure 11. Transfer function with residual vector : magnitude

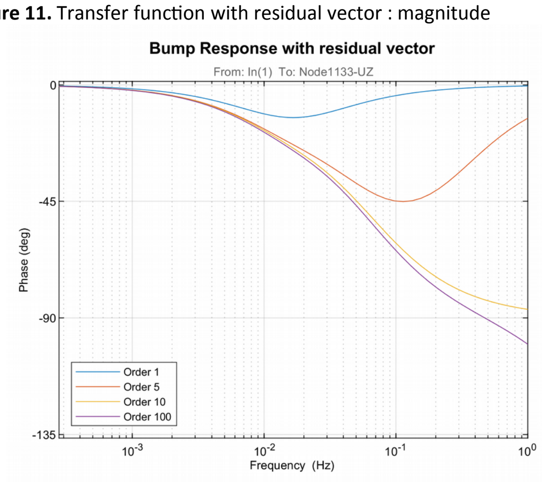

Figure 12. Transfer function with residual vector : phase

4. Conclusion

In this paper, we have shown that thermal mechanical state-space models can be efficiently built by using the modal method. In particular, it is shown that adding one extra state per actuator allows for capturing the full static response of the system. While the impact on the thermal response is generally limited, the mechanical response fidelity is very significantly improved, particularly in bending dominated problems, or if local results are to be obtained. In particular, it has been shown that convergence can be obtained at drastically different rates, depending on the quantity of interest: for the same number of modes included in the analysis, the relative error could vary by as much as two order of magnitudes.

Since it is not feasible to obtain accurate results with confidence by solely relying on engineering judgment, it is recommended to systematically add a residual vector, thus largely improving the robustness of the analysis. It has been shown, however, that while the system response in the low frequency range is clearly improved, the asymptotic (high-frequency) response is biased, and in particular, starting from a limit frequency the phase is clearly corrupted (i.e. it begins to increase), a clear warning sign that the model should not be used above that particular frequency. To summarize, the residual vector method only dramatically enhances the fidelity of model in the low frequency range, but does not extend its validity in the higher frequency range.

This work could be improved by modifying the correction: the residual vector could be modified into a residual mode, with a specific time constant chosen so as to minimize the deviation from the exact response above the model cut-off frequency.

References

[1] Palmieri and Rathjen 1978 - CAVE3 - A General Transient Heat Transfer Computer Code Utilizing Eigenvectors and Eigenvectors – NASA Contractor Report 145290

[2] Ruijl, Compensation model based on thermal mode shape. In Euspen SIG Seminar2006, Eindhoven, The Netherlands

[3] Morishima, 2016, Novel thermal error reduction techniques in temperature domain

[4] Roy and Girard, 2005, Impact of Residual Modes in Structural Dynamics, European Conference on Spacecraft Structures, Materials and Mechanical Testing.

[5] Preumont, 2017, Vibration Control of Active Structures 3rd edition

[6] ANSYS Inc, Mechanical APDL, ANSYS Parametric Design Language Guide

[7] Jobert, 2017, Working Wonders with APDL Math: Thermal Modal Analysis, PADT Blog «The Focus»

[8] Jobert, 2019, Working with APDL Math Illustrated : Thermal Harmonic Analysis, ANSYS White Paper