Special Interest Group Meeting on Thermal Issues
Laboratory for Machine Tools and Production Engineering (WZL) of
RWTH Aachen, Germany, February 2020
www.euspen.eu

Applied thermo-elastic model reduction

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Abstract
Errors induced by thermal deformations, i.e. undesired deformations and/or relative displacement differences due to temperature fluctuations, are becoming one of the more significant contributions to the overall accuracy budget during the design of high precision equipment. In order to optimize the system design for performance, transient thermo-mechanical analyses are performed. However, these analyses can become computationally very expensive and thus time intensive. In order to accelerate this process, while retaining accurate models, model reduction techniques are used in practice. This paper presents a short overview of commonly used model reduction techniques for thermo-elastic systems and gives an insight on the advantages and the ease with which they can be applied in the actual development of high precision equipment. As an example an actual developed high precision stage is used, which has nanometre performance requirements.

Keywords: Thermal-elastic model, model order reduction, modal analysis, Krylov subspace method, design optimization.

1. Introduction

When accounting for thermal induced deformation errors, during the concept and/or design stage of the development of a high precision machine, transient simulations of thermal-mechanical models are needed. When thermal models become quite complex and when multiple variations of thermal loads need to be investigated, these simulations can become very computational expensive. To accelerate this process model reduction techniques are used.

The following section gives a very short overview of the mathematical framework together with commonly used reduced bases.

2. Model reduction framework and reduced bases

Given a full order model (FOM) Σ of a MIMO LTI system with N states, p inputs and m outputs:

$$\boldsymbol{\Sigma}: \begin{cases} \boldsymbol{E}\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{u}(\boldsymbol{t}) \\ \boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{C}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D}\boldsymbol{u}(\boldsymbol{t}) \end{cases} \quad (1)$$

Model reduction assumes that state trajectories, $\boldsymbol{x}(\boldsymbol{t})$ are contained in lower dimensional subspaces of size $\boldsymbol{k} \ll \boldsymbol{N}$ and can be written as $\boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{V}\boldsymbol{q}(\boldsymbol{t})$, with $\boldsymbol{V} \in \mathbb{R}^{\boldsymbol{N}\times\boldsymbol{k}}$, the reduced basis vector matrix and $\boldsymbol{q}(\boldsymbol{t}) \in \mathbb{R}^{\boldsymbol{k}}$, the reduced state vector. By applying the standard Galerkin projection [1, 2] the full order model $\boldsymbol{\Sigma}$ can be projected to a reduced order model (ROM) $\boldsymbol{\Sigma_k}$:

$$\boldsymbol{\Sigma_k}: \begin{cases} \widehat{\boldsymbol{E}}\dot{\boldsymbol{q}}(\boldsymbol{t}) = \widehat{\boldsymbol{A}}\boldsymbol{q}(\boldsymbol{t}) + \widehat{\boldsymbol{B}}\boldsymbol{u}(\boldsymbol{t}) \\ \boldsymbol{y}(\boldsymbol{t}) = \widehat{\boldsymbol{C}}\boldsymbol{q}(\boldsymbol{t}) + \boldsymbol{D}\boldsymbol{u}(\boldsymbol{t}) \end{cases} \quad (2)$$

With $\widehat{\boldsymbol{E}} = \boldsymbol{V^T}\boldsymbol{E}\boldsymbol{V}, \widehat{\boldsymbol{A}} = \boldsymbol{V^T}\boldsymbol{A}\boldsymbol{V} \in \mathbb{R}^{\boldsymbol{k}\times\boldsymbol{k}}, \widehat{\boldsymbol{B}} = \boldsymbol{V^T}\boldsymbol{B} \in \mathbb{R}^{\boldsymbol{k}\times\boldsymbol{p}}$ and $\widehat{\boldsymbol{C}} = \boldsymbol{C}\boldsymbol{V} \in \mathbb{R}^{\boldsymbol{m}\times\boldsymbol{k}}$.

Most commonly used reduced basis vectors are the *eigenvectors* (modal method), *Krylov basis vectors* (moment matching method) of the system Σ or the basis vectors calculated from a proper orthogonal decomposition (POD) method. The POD method can be applied when the actual inputs of the system are well known and/or state-trajectory data is already available, see [1, 2].
Methods for solving the eigenvalue problem are standard available in e.g. Matlab, ANSYS and Comsol. However for the Krylov basis vectors or POD methods one has to implement the commonly available algorithms, see [2] for more details. In this paper only the application of eigenvectors and the Krylov basis are considered. The use of POD basis vectors will be a topic for another paper.

3. Thermo-mechanical case

As a case, an actual developed high precision positioning stage is considered. This stage, with a total mass of 6.2 kg, is part of a larger machine and its movement is limited to one direction only. The other degrees of freedom are constrained by means of 3 bearings and 2 guidance rails. The actuation, along the X direction, is performed by an ironless linear motor. See figures 1 and 3 for a more detailed overview.

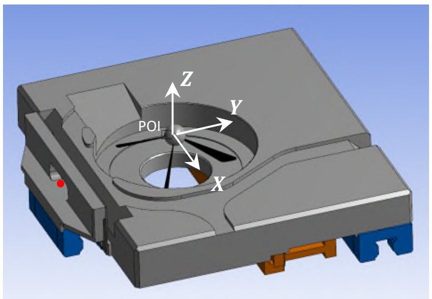

Figure 1: Top view model high precision positioning stage. Red dot indicates encoder location. Sphere at (0,0,0) of coordinate system is POI.

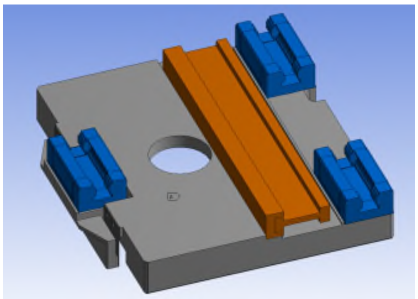

Figure 2: Bottom view model high precision positioning stage. Bearings are indicated in blue and the actuator is indicated in orange.

It is assumed that the stage is in ambient conditions at a nominal environment temperature of 22 $°C$. Faces in contact with air have appropriate convection boundary conditions, ranging from 2.5 – 5 W/m^2/K, depending on the face orientation. Additionally linearized radiation is taken into account, in the order of 0.6 – 1.5 W/m^2/K, as temperature offsets, due to heat loads are expected to be in the sub Kelvin range.

For this example two inputs are considered, i.e. the air temperature at the top surface (+Z side) of the stage and heat generation in the linear motor. However it is not uncommon to have a larger amount of inputs, i.e. $\mathcal{O}(10)$, can easily be incorporated in the framework. As for temperature outputs, the nodes on the outer faces of the large carrier part (grey coloured part in Figure 1 and 2) are used.

To assess the resulting thermal induced deformations, the displacement of a point-of-interest (POI) is considered in conjunction with constraints at the three bearings and in the encoder degree of freedom, i.e. along the X-axis. The POI can be seen in Figure 1, coinciding with the coordinate system and is rigidly connected to the stage. In this example it is assumed that the POI is thermally isolated from the stage.

4. Implementation and model reduction details

The finite element (FE) implementation, i.e. meshing, boundary conditions and contacts, is done using ANSYS and the model reduction steps are performed in an ANSYS/Matlab developed toolbox. The toolbox combines the efficient ANSYS solvers with the flexibility of Matlab.

The first input is heat generation in the linear motor and results from a 300 mm scanning setpoint, i.e. the stage scans multiple times back and forth within a fixed time of 7.5 s. The maximum achieved velocity is 0.5 m/s and has maximum acceleration levels of 22 m/s^2. This setpoint results in a maximum dissipating power of 225 W. In figure 3 the first two seconds of the heat generated in the linear actuator is shown.

As for the second input, the topside air temperature is stepped to 32$°C$ at the beginning of the simulation.

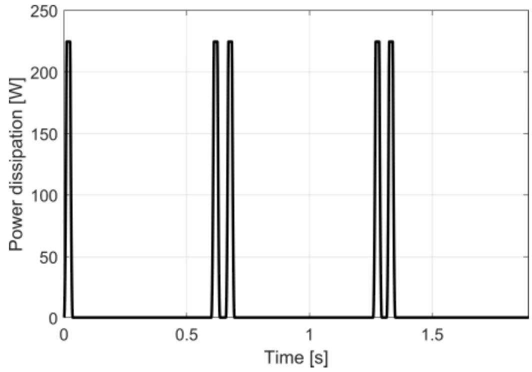

Figure 3: First two seconds of the actuator internal heat generation.

The defined inputs will be used for transient simulations of three systems, i.e. the FOM, the modal ROM and the Krylov ROM. Initially, a thermal simulation will be performed, which calculates temperature fields with a time-step of 1 ms. Next the thermally induced deformations, $\vec{d}(t)$, are calculated, by solving the following set of linear equations for each time step:

$$K_s\vec{d}(t) = K_{th}T_{FOM}(t) \tag{3}$$

with K_s the FE stiffness matrix, K_{th} the thermo-elastic coupling matrix and $T_{FOM}(t)$ the calculated FOM temperature field at time t. To be able to calculate the deformations for the ROMs, it is only necessary to calculate the so-called thermo-elastic basis vectors, V_d, which is found by solving a similar set of equations as (3), i.e.

$$K_sV_d = K_{th}V_{ROM} \tag{4}$$

with V_{ROM} the reduced, *modal* or *Krylov,* basis vectors. Note that by changing the output matric $\hat{C}$ of (2) to $\hat{C} = CV_d$, the reduced system Σ_k can calculate the deformations for any *other* given input and time. Giving a large flexibility in input variation/sensitivity analysis.

The size of the thermal ROMs is chosen by using a rule of thumb value of $k = 30$ per input for the Krylov method and $k = 120$ per input for the modal method. The reason the modal method requires a factor four more states, is slower error convergence for thermal systems than for the modal method than for Krylov based methods , see [1, p. 284]. Additionally, the modal method requires the use of residual vectors in order to compensate for relatively large DC errors. The effect of taking this correction into account will be shown in the next section.

Although the modal method requires a significantly larger order, the modal method is still a preferred method, due to the efficient calculation of the eigenvalue problem in FE packages. Furthermore, there is an intuitive physical interpretation of mode shapes and an extensive knowledge, toolboxes and experimental techniques are available originating from structural dynamic related problems.

5. Results and conclusions

First the impact on computational performance is considered and afterwards the accuracy of the ROMs are analysed.

Implementing the models with the inputs as described above, results in transient simulation timings given in Table 1. It can be seen that by applying model reduction, a computational performance increase for transient simulations of a factor $7\ (modal) - 117 (Krylov)$ can be achieved. Also the timings for generating the ROMs are significant less when compared to performing one FOM simulation. Indicating that there is no significant impact on overall calculation timings when recalculating the ROM.

When considering the calculations of the deformations, the time for evaluating a data-set with 50 time-steps of the FOM or 50 basis vectors is approximately $28\ min$. Note that after generating the thermo-elastic basis vectors, V_d, every alternative input can be simulated efficiently. For the FOM, however, this would imply that the thermal transient and deformation calculations need to be redone completely, which will take approximately $50\ min$ per new loadcase.

Table 1. Overall timings for one transient thermal simulation, performed on an i7-7700HQ mobile workstation with 32 Gb RAM

Timings [s]	Full model N ≈ 240k	Modal k=240	Modal k=240 + residual vector correction	Krylov k=60
Basis vector calculation		148	148	21
Correction term		-	2.4	-
Projection + state space		0.2	0.5	0.7
Thermal Simulation	1290	181	187.4	11

To evaluate the accuracy of the ROMs the following spatial RMS error measure over time is used:

$$\varepsilon_{rms}(t) = \frac{\sqrt{\sum_{i=1}^{N_{nodes}} [T_{FOM}(i,t) - T_{ROM}(i,t)]^2}}{\sqrt{\sum_{i=1}^{N_{nodes}} T_{FOM}^2(i,t)}} \qquad (5)$$

with N_{nodes} the number of FE nodes, T_{FOM} and T_{ROM} the temperature of the FOM and respectively the ROM. Figure 4 shows $\varepsilon_{rms}(t)$ for the 3 different ROMS, i.e. modal, modal + residual vectors correction terms and the Krylov model. Figure 5 shows the relative spatial temperature error at the final simulation time ($t = 7.5\ s$).

It can be concluded that the Krylov method can very accurately estimate the temperature fields over space as well as time, i.e. $\varepsilon_{rms} < 0.04$ within 0.3 s. For this case it out performs the modal method. However the modal method is sufficiently accurate at the spots where there is a significant temperature offset from $22°C$. See e.g. the hot-spot at location $(x, y, z,) = (0, -0.15, 0.13)\ m$ in the top-left of figure 5. What is also interesting to note is that the addition of the residual correction vectors, significantly improves the accuracy of the modal method and that it should therefore be included when the actual inputs are known.

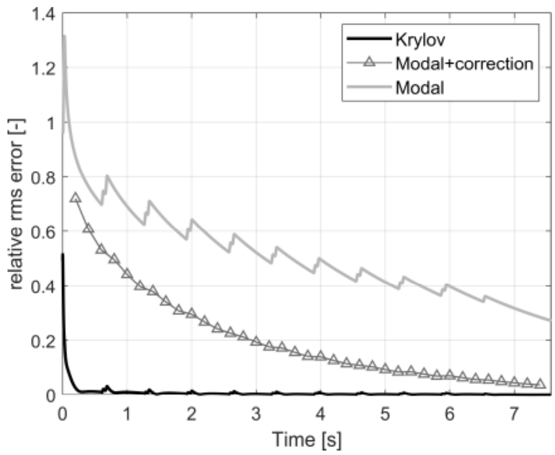

Figure 4. $\varepsilon_{rms}(t)$, relative RMS temperature error over time.

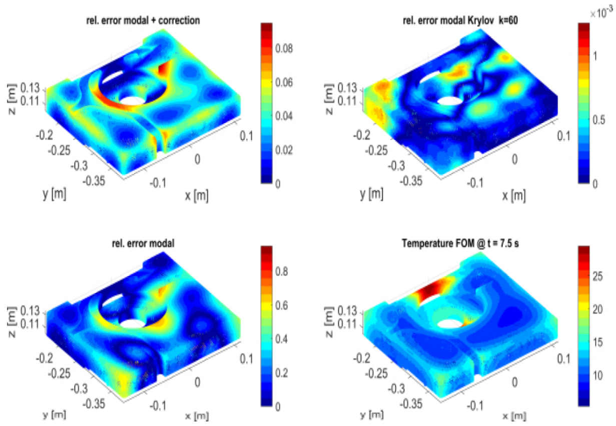

Figure 5. Relative temperature errors, compared to the FOM temperature field at t =7.5 s.

Finally the accuracy with regard to the POI deformations is shown in figure 6. In this figure the 2-norm of the calculated deformation vector over time is compared. These deformation calculations only take the first 50 basis vectors into account in order to limit the overall computation time. From figure 6 it can be seen that both reduction methods accurately (within 1 nm absolute difference) describe the POI thermally induced drift.

Note that for this case only a smaller subset of the reduced basis vectors is sufficient to accurately estimate the POI deformations and indicates that not all basis vectors contribute equally to the final deformation.

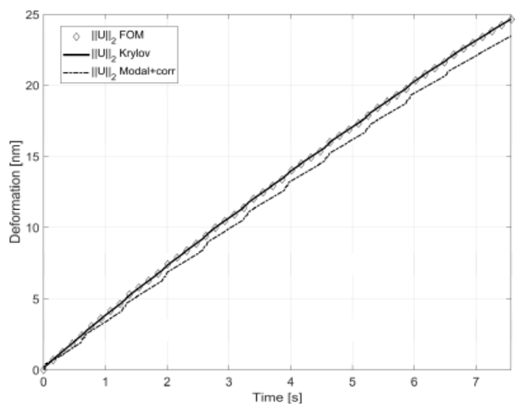

Figure 6. Norm of the POI deformations in [nm].

Current practical limitations of model reduction techniques are a lack of a-priori knowledge on what the accuracy will be, which basis vectors should be taken into account and which size of the reduced order should be chosen given a certain tolerance on allowed errors.

Taken these practical limitations into account, it can still be stated that using model reduction techniques in practice, the development high precision systems can be effectively and accurately be accelerated. Especially when used early on in the concept / design phase.

As a final note, one of the main benefits of using ROMs is the huge flexibility in applying input variations and different initial conditions. This flexibility is very helpful in determining critical sensitivities of the system and selecting the most demanding inputs for future analysis.

6. Future work

This paper only presented a short overview what can be achieved with model reduction techniques. Future work will focus more on the application of advanced model reduction related topics. These topics will go into more depth regarding automated reduced order selection, optimal selection of basis vectors for thermo-elastic deformations calculations and optimal sensor/actuator placement together with thermal error correction models, see e.g. [3].

References

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