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## Simulation of the thermal behavior of machine tools for efficient machine development and online correction of the Tool Center Point (TCP)-displacement

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### Abstract

Machine tool models play an essential role in the development process and during the whole lifetime of machines. On the one hand, they enable the developer to predict the thermo-elastic behavior of a new machine without the necessity for a prototype. Therefore, time and money can be spared in the development process. On the other hand, model predictive control can be applied to reduce the manufacturing error, induced by the displacement of the tool center point (TCP).

This paper provides a brief overview of the methodology used and developed in the Collaborative Research Center/Transregio 96 (CRC/TR 96) to simulate the thermal behavior of complete machine tools. As an illustration, the newly developed approach is used to simulate the behavior of a simple test model. The different approaches are compared and a short guideline, which method can be used for a given task, will be given.

Thermal Behavior, Simulation, Model Order Reduction (MOR), Finite Element Method (FEM)

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### 1 Introduction

Saving resources is of increasing importance for sustainable production processes. To account for this issue, it is essential to reduce the energy consumption of modern machine tools by smart cooling strategies, further enhanced construction techniques etc., to compensate for an unwanted tool center point (TCP) displacement, before it actually appears. Furthermore, the simulation of the thermal behavior allows model predictive control to reduce the error induced by thermo-elastic deformations, which yield an unwanted tool center point (TCP) displacement.

This paper describes an approach to simulate the thermal behavior of complete machine tools, which is developed and used by projects in the CRC/TR 96.

First, the general methodology will be introduced and the steps to generate a finite element model to calculate the thermal field are described. After that, it is shown, how model order reduction (MOR) can be used to speed up the simulation process. To illustrate the strengths and weaknesses of the different approaches, the thermal field of a simple test model is calculated. These results are compared. To conclude, a short summary of the different approaches will be given and the next development steps are outlined.

### 2 Modelling of machine tools

The methodology of modelling the machine tool including geometric simplification, the choice of boundary conditions and choosing the right mathematical models, plays a significant role in the calculation of the thermal behavior.

#### 2.1 General Considerations

To model the thermo-elastic behavior of machine tools, two different kinds of coupling have to be taken into account:

- the physical coupling between the heat equation and the equations of elasticity,
- the geometrical coupling between different components that can also move relative to each other (contact problem).

These coupling types must be treated with different mathematical approaches and have their own challenges. For example, the geometrical coupling of the relatively moved parts leads to discontinuous sources in space and time. These sources can be modelled with functions on the part interface, which depend on different temperature fields.

#### 2.2 CAD-Model

The starting point of all simulations is a CAD-Model of the investigated system. The model must contain all the geometric details which are relevant for the simulation. Small features of the geometric model like fillets, bevels or drillings should only be kept in the model, if the behavior at those items is of interest. A simplified geometry can reduce the effort for the next modelling steps significantly. For the given examples, the CAD-Software SolidWorks® was used.

#### 2.3 FE-Model

After generating a sufficiently detailed geometric model of the system, a finite element (FE) model must be created.

Therefore the CAD-Model is imported into the FE-Tool and the simulation domain is discretized into finite elements. The material parameters like specific heat capacity or thermal conductivity, are set for each individual material of all parts of the model.

The boundary conditions take a special role in the modeling of the machine behavior. To calculate the thermal field as accurate as possible, the process heat, the cooling system, the environment interaction, the heat fluxes from the axis drives and the friction losses of the bearings and linear guides have to be taken into account. Modelling of those boundary conditions plays a significant role in the CRC/TR 96. Models for e. g. the dissipative heat of the cutting process or friction losses in linear guides are incorporated into the model. For the presented methodology and workflow, ANSYS Mechanical as an industry-standard computer aided engineering (CAE)-Tool is used to generate the FE-Model.

The equation that describes the thermal field is

$$c\rho \frac{\partial T(t)}{\partial t} - \vec{\nabla} \cdot (\lambda \vec{\nabla} T(t)) = q(t). \quad (1)$$

and is discretized by using the FE-method.

For simplicity, constant, isotropic, material properties are assumed in this presentation. The material coefficients are the specific heat  $c$ , the density  $\rho$  and the thermal conductivity  $\lambda$ .  $T$  is the temperature at the given material point,  $q$  can be any heat source and the variable  $t$  signifies the time.

Applying the FE method and using equation (1) leads to the following system equation to calculate the discrete temperature vector  $\mathbf{T}$  at the FE nodes

$$M\dot{\mathbf{T}}(t) + D\mathbf{T}(t) = \mathbf{q}(t) \quad (2)$$

of one continuous part, with the matrices  $M$  (capacity matrix),  $D$  (conductivity matrix) and the time dependent load vector  $\mathbf{q}$ .

To model thermal coupling by the exchange of heat between multiple parts, different approaches [8][3][11] can be incorporated into the vector  $\mathbf{q}$ .

The calculation of the transient behavior of machines modeled by the FE-Method can be very time consuming, because a fine discretization, resulting in lots of degrees of freedom (DOF), is needed for a good approximation of the real field. To use model predictive control systems for real time error correction and for parameter and design studies to optimize the machine design during the development, the simulations must be carried out faster, while maintaining the accuracy of the FE-Model.

Model order reduction (MOR) becomes crucial to reduce the simulation time for those models. To build those reduced-order models, by the class of methods addressed in the CRC/TR96, it is necessary to transform the FE-Model into a so called input-output-model (IO-Model).

#### 2.4 IO-Model

All the data necessary to generate IO-Models are extracted from ANSYS Mechanical, written into an open source, machine-readable description format and processed in specialized software tools and algorithms, developed as part of the research in CRC/ TR96, afterwards.

The formerly described FE-Model is transformed into a model in generalized state space form

$$\begin{aligned} E\dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t) \\ \mathbf{y}(t) &= C\mathbf{x}(t). \end{aligned} \quad (3)$$

The system (3) represents a linear time-invariant (LTI) system with the state  $\mathbf{x} \in \mathbb{R}^n$ , the input  $\mathbf{u} \in \mathbb{R}^p$  and the output  $\mathbf{y} \in \mathbb{R}^m$ . The matrices  $E \in \mathbb{R}^{n \times n}$ ,  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times p}$  and  $C \in \mathbb{R}^{m \times n}$  characterize the system. Note that for the here

considered thermal simulation of machine tools and their subsystems, the vectors  $\mathbf{x}$  and  $\mathbf{y}$  represent the nodal temperatures  $\mathbf{T}$  and the model output temperatures in points and faces of interest, e.g. at sensor positions and the TCP.

To generate the IO-form of the model from the data extracted from the ANSYS-FE-Model, the open source FE-Toolbox DUNE [6] is used. DUNE provides a modularized software framework, which allows for different mesh structures, different basis functions, including discontinuous functions, several mesh operations, in particular mesh intersections and an interface to sophisticated linear solver libraries like SuiteSparse [7] or PETSc [2] in an exchangeable manner.

For the generation of the IO-system, we implemented the computation of the coefficient matrices in a general way, such that different coupling strategies are easier to extend to newly developed coupling strategies.

#### 2.5 Reduced IO-Model

After the IO-generation, there is the possibility to reduce the high-order IO-Model (3) to a low-dimensional surrogate model that approximates the dynamical behavior of the original model by means of MOR. Several projection-based techniques [1], including balanced truncation, moment matching and the iterative rational Krylov algorithm (IRKA) are implemented in a CRC/TR 96-tailored MOR-tool in MATLAB®.

During this reduction process, projection matrices  $V$  and  $W \in \mathbb{R}^{n \times r}$  are computed and the reduced system of order  $r \ll n$  is obtained by

$$\begin{aligned} W^T E V \dot{\mathbf{x}}_r(t) &= W^T A V \mathbf{x}_r(t) + W^T B \mathbf{u}(t) \\ \mathbf{y}_r(t) &= C V \mathbf{x}_r(t). \end{aligned} \quad (4)$$

The vectors  $\mathbf{x}_r$  and  $\mathbf{y}_r$  denote the state vector and the output vector of the reduced model.

The MOR tool, further, incorporates treatment of inhomogeneous initial conditions, following [4] and allows for the preservation of physical parameters in the surrogate models via adapted MOR-techniques [5].

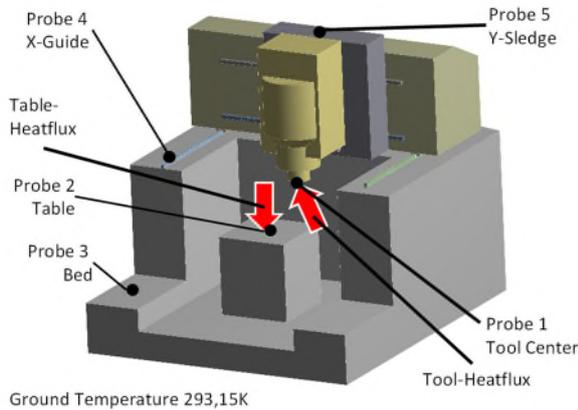
#### 2.6 Time Integration Step

The different machine tool models are used to calculate the transient temperature field.

To compute the FE-solution, ANSYS and DUNE are used. The IO- and the reduced IO-Models are integrated with the second-order Rosenbrock-2 method [9]. This particular method belongs to the subclass of W-methods, which means we can use a fixed Jacobian over some time steps without the loss of too much accuracy. Furthermore, our experiments showed, that the method also remained stable with the explicit coupling terms.

### 3 Example Machine Results

The CAD model along with the used boundary conditions and probe positions to illustrate the workflow and to compare the results is shown in Figure 1.



**Figure 1.** Example Machine and Boundary Conditions

A simplified machine tool, consisting of a machine bed and three sledges is simulated. All parts consist of the same, isotropic material with constant thermal properties. The connection between those components is realized by linear guides, consisting of rails and guide-wagons.

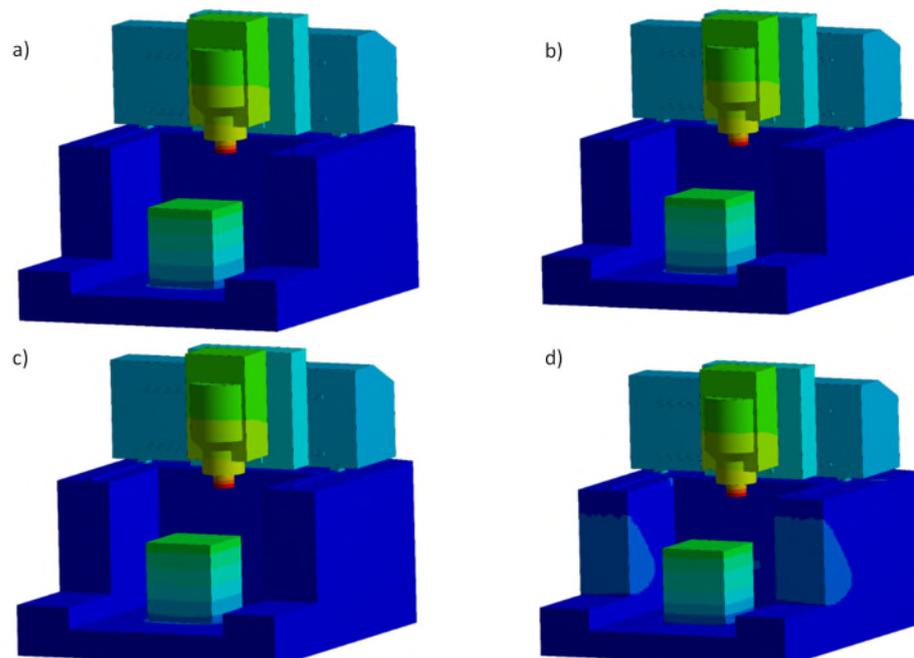
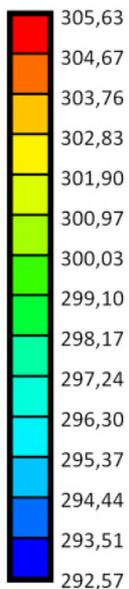
A constant temperature at the ground (293,15K) and two heat flux boundary conditions are applied. No relative movement of the parts is modeled, which means that all parts are fixed.

The model outputs for the reduced model are defined as the average temperatures of the faces that are subject to heat flux boundary conditions.

The coupling between the parts is realized by a simple output coupling using the averaged temperatures on the surfaces in contact.

The described model is used to calculate the thermal field with all four approaches. The reduced model was generated using Balanced Truncation. Figure 2 shows the results of the

Temperature / K



**Figure 2.** Thermal field of the four different simulation technologies a) ANSYS, b) DUNE, c) IO-Solution and d) Reduced IO-Solution

calculation after a transient simulation of 80 000s (500s steps) real time.

Therefor, the reduced state vector  $x_r$  is expanded to obtain an approximate solution  $\hat{x}$  for the full model, using the relation  $\hat{x}(t) = Vx_r(t)$ . Note that this approach should be used with care. MOR aims at finding a good approximation of the output  $y$ , but not necessarily the whole high-dimensional state  $x$ . Hence, the outputs of the IO-Model should be selected with regard to the desired application of the reduced model.

The solutions of ANSYS, Dune and the IO-Model are nearly identical. As expected, the expanded solution of the reduced model shows differences in areas away from the defined model outputs. To illustrate this statement, Table 1 shows the calculated temperatures at the points marked in Figure 1. The ANSYS solution was used as the benchmark for the calculation of the absolute errors.

**Table 1** Probe temperatures and errors in Kelvin

	ANSYS	DUNE	IO	Red-IO
Probe 1	298,729	298,733	298,733	298,582
$\Delta$ Probe 1	-	4,00E-03	4,00E-03	1,47E-01
Probe 2	305,621	305,623	305,623	305,623
$\Delta$ Probe 2	-	2,00E-03	2,00E-03	2,00E-03
Probe 3	293,162	293,162	293,162	293,419
$\Delta$ Probe 3	-	0,00E+00	0,00E+00	2,57E-01
Probe 4	293,324	293,32	293,32	293,334
$\Delta$ Probe 4	-	4,00E-03	4,00E-03	1,00E-02
Probe 5	295,661	295,659	295,659	295,657
$\Delta$ Probe 5	-	2,00E-03	2,00E-03	4,00E-03

The biggest observed error between the full and reduced model for those representative points is approximately 0,15 K.

The model orders used to calculate those solutions are listed in Table 2. By further increasing the order of the reduced model, a better approximation at the cost of higher simulation times is possible.

**Table 2:** Comparison of model size and simulation times in s

	Tool/ Model	Complete	Model- setup	Integration
Rough – DOFs (Full 45260, Reduced 174)	ANSYS	53,4	7,10	46,30
	DUNE	52,16	31,03	21,13
	IO	27,03	9,15	18,88
	Red-IO	5,91	5,84	0,07
Fine – DOFs (Full 167121, Reduced 183)	ANSYS	193,66	16,94	176,72
	DUNE-FE	224,56	115,23	109,33
	IO	130,09	35,24	94,85
	Red-IO	21,94	21,84	0,1

A comparison of the simulation times for the different approaches shows a remarkable reduction of the computation time by using MOR (Table 2). By reducing the model order of the rough mesh to 0,38 % of the full model, the simulation time to calculate the thermal field can be reduced by 78.14 % with respect to the full IO-Model. The integration step of the reduced model is 270 times faster.

For those Timings, ANSYS 19.2 and DUNE in version 2.4.1 were used. The integration of the Models uses SuiteSparse (5.6.0 - UMFPAK) with the BLAS-Libraries OpenBLAS (0.3.7 for DUNE) and the intel MKL (2019.4.245) together with MTL in svn revision 9592 for the IO-Models. All computations were carried out on a Windows-PC with Intel Core i5-6500 processor.

#### 4 Summary and Outlook

In this paper, a new workflow to calculate the transient behavior of machine tools was introduced. It consists of the usual modeling of a finite element model with subsequent steps to generate a state space (IO-) model and a machine-readable description of the finite element model.

The main advantage in the open format is the exchange with other software tools enabling their features to the standard engineering workflow. Further extensions include problem-tailored time integration methods, which make use of the different time scales between the heat conduction and the relative moving components.

The IO-Model can easily be replaced by a much smaller surrogate model, generated by MOR-techniques, within the same workflow.

The potential for the usage of MOR for different use cases was shown by comparing simulation times and accuracy of the results against the results obtained by using the state of the art FE-tool ANSYS. Significant amounts of computational effort can be spared.

Concerning the ongoing advancement of the modeling techniques and the intended extension to the consideration of

uncertain and nonlinear effects in the model, further methods for MOR will be included. Furthermore, the presented workflow will be extended to calculate the deformation of machine tools. This will enable the user to investigate the whole thermo-elastic functional chain of machine tools during machine development and usage.

The shown workflow is wrapped into an ANSYS Mechanical ACT Extension and is used by multiple subprojects of the CRC/TR 96.

#### Acknowledgement

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