Brunel University London

Investigation on a NURBS based approach to modelling and analysis of freeform surfaces with the application to design and ultraprecision machining of vari-focal lenses

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Presentation Outline

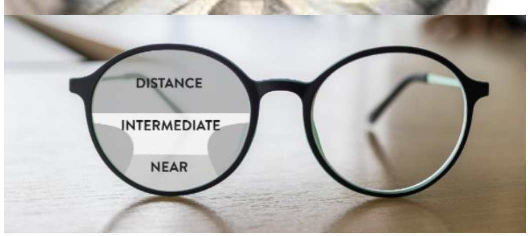
- Research background
- NURBS based approach to freeform surface modelling and construction
- Integrated digital assessment and its implementation
- Ultraprecision machining trials
- Concluding remarks

Research background

- Freeform surfaced devices and applications

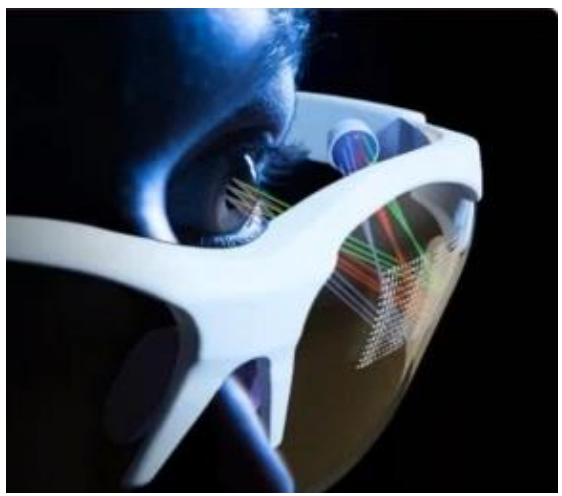
Medical and Ophthalmics

- Assistive Technologies
- Ophthalmic and Microscopy



Mobile Displays

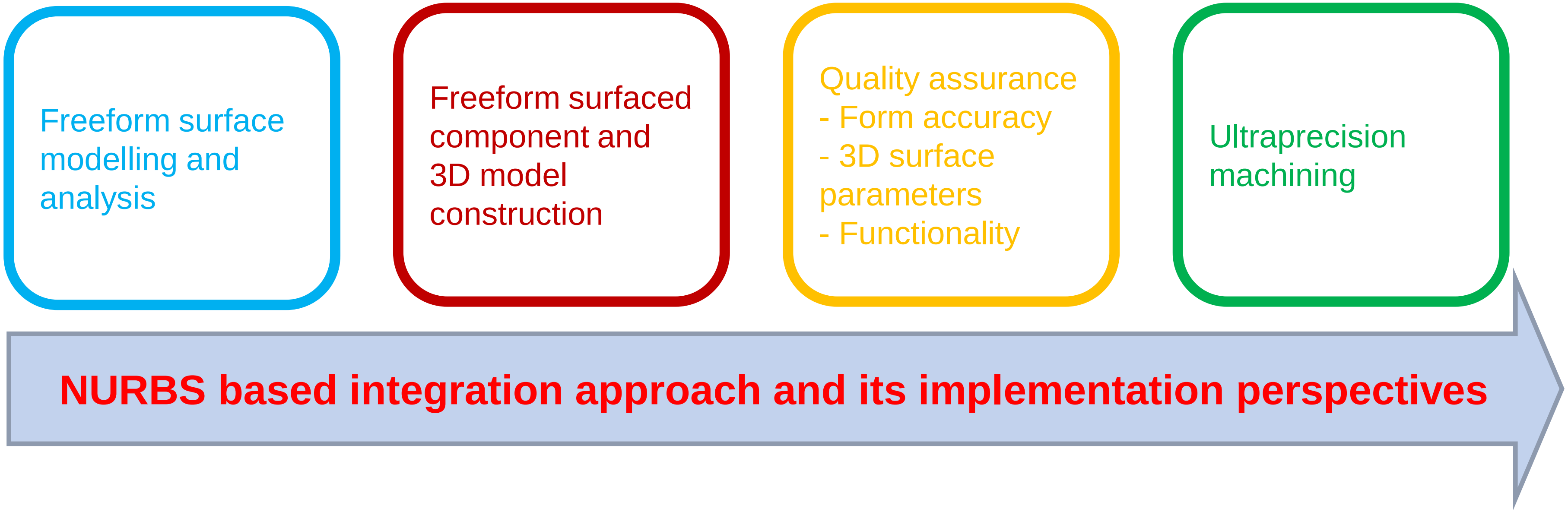
- Near-eye
- Head-worn
- Smart glass



Transportation

- Heads-up displays (HUDs)

NURBS based approach to freeform surface design and 3D model construction/manufacturing



- To develop the NURBS based integrated approach for design and modelling of freeform surfaced components and devices.
- To design the engineering-feasible quality assessment strategy for freeform surfaces in the aspects of both functional and surface topography.
- To investigate ultraprecision machining of freeform surface and the underly issues in processes and the process chain.

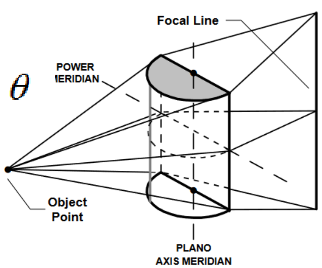
Vari-focal lens surface modelling/construction

Spherical Power Formula

$$F_S = \frac{n' - n}{r}$$

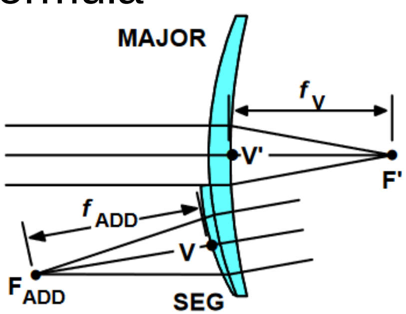
Cylinder Power Formula

$$F_\theta = F_{SPH} + C \cdot \sin^2 \theta$$



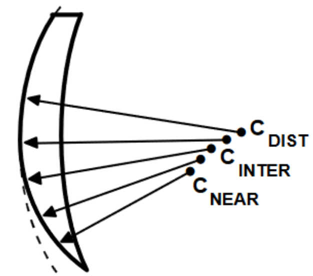
Additional Power Formula

$$Near - Power = \frac{(n-1) * 1000}{Sph + Add}$$



Lens Construction Formula

$$\frac{1}{r(u)} = \frac{1}{r_F} + \left(\frac{1}{r_N} - \frac{1}{r_F}\right) \sum_{i=m}^{m+l-1} c_i (u+L)^i$$



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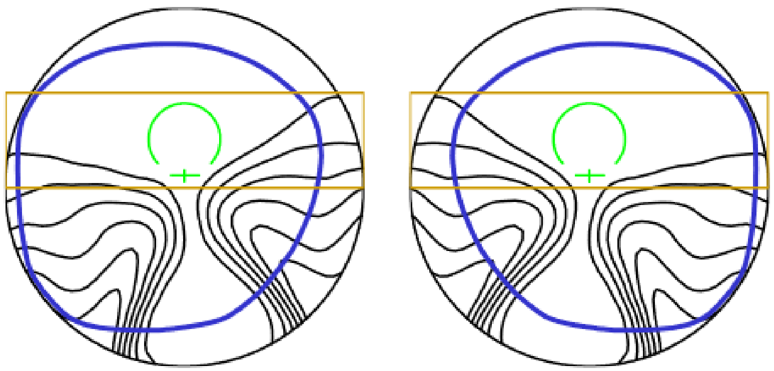
Lens Surface Design Plot



Advance algorithm for lens surface design

Prescription

Prescription
SPH
CYL
ADD

Algorithm for surface design

Algorithm 1: Radius of curvature of an arbitrary point of lens surface

Function *u Calculation*:

  Input: h, L

  $p = x - \frac{h}{2} + L$

  $g = \left(\frac{1}{2}\right)\left(p + \frac{y^2+h^2/4}{p}\right)$

  $u = \frac{h}{2} - L + g - (\operatorname{sgn} p)\left(g^2 - \frac{h^2}{4}\right)^{\frac{1}{2}}$

  return u

Def *Cn*:

  $c_1 = c_2 = c_3 = c_4 = 0$

  $c_5 = 56/h^5$

  $c_6 = -140/h^6$

  $c_7 = 120/h^7$

  $c_8 = -35/h^8$

Function Main:

  Input: Rd, Rr, Cn

  for *i in 1:8* **do**

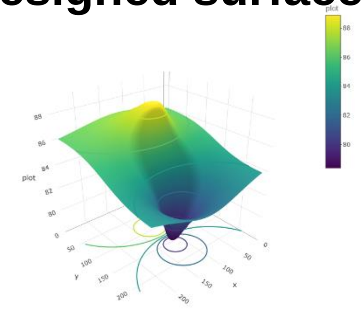
    $\frac{1}{r(u)} = \frac{1}{r_D} + \left(\frac{1}{r_R} - \frac{1}{r_D}\right)\sum_1^i c_i(u+L)^i$

  return $\frac{1}{r(u)}$

Data point

-40.000 -40.000 18.703617223142
-40.000 -39.750 18.557064263439
-40.000 -39.500 18.412118478382
-40.000 -39.250 18.268965825744
-40.000 -39.000 18.127790027962
-40.000 -38.750 17.988773092621
-40.000 -38.500 17.852093057227
-40.000 -38.250 17.717916161267
-40.000 -38.000 17.586189147990
-40.000 -37.750 17.456610238039
-40.000 -37.500 17.328870145372
-40.000 -37.250 17.202666579247
-40.000 -37.000 17.077738427256
-40.000 -36.750 16.954044096976

Designed surface



u Calculation

$$p = x - \frac{h}{2} + L$$

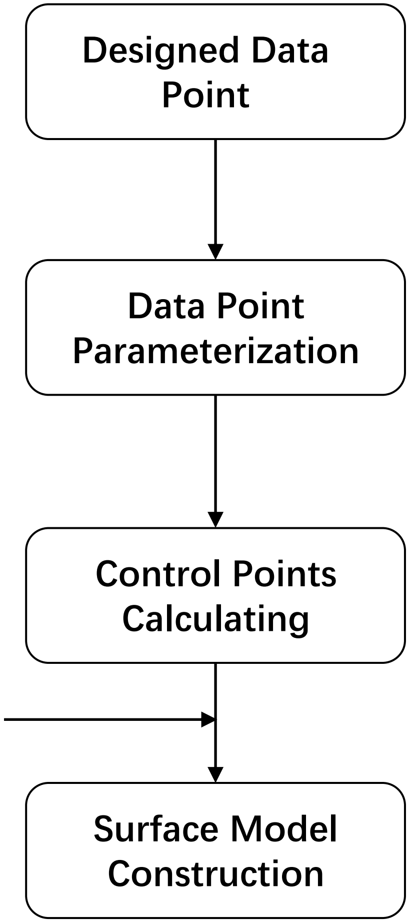
$$g = \left(\frac{1}{2}\right)\left(p + \frac{y^2 + h^2/4}{p}\right)$$

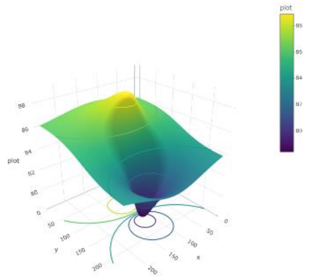
$$u = \frac{h}{2} - L + g - (sgn\ p)\left(g^2 - \frac{h^2}{4}\right)^{\frac{1}{2}}$$

$h = Vertical\ distance\ between\ distant\ point\ to\ near\ point$

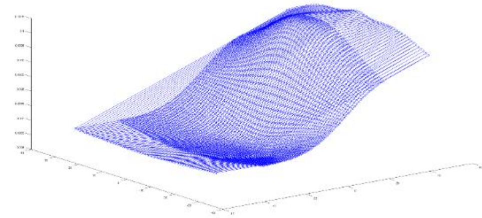
$L = Vertical\ distance\ between\ distant\ point\ to\ prism\ point$

Digital model construction and digital/virtual lens



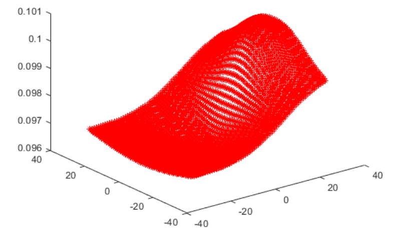


$$k_i = 1 + \frac{3}{2}\left(\frac{|\Delta_{p_{i-2}}|\theta_{i-1}}{|\Delta_{p_{i-2}}| + |\Delta_{p_{i-1}}|} + \frac{|\Delta_{p_i}|\theta_i}{|\Delta_{p_{i-1}}| + |\Delta_{p_i}|}\right)$$



NURBS Equation

$$p(u,v) = H\{p(u,v)\} = H\left\{\sum_{i=0}^{m}\sum_{i=0}^{n} d_{i,j}N_{i,k}(u)N_{j,l}(v)\right\}$$



Integrated digital assessment and implementation

Surface Design

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Lens Modelling

↓

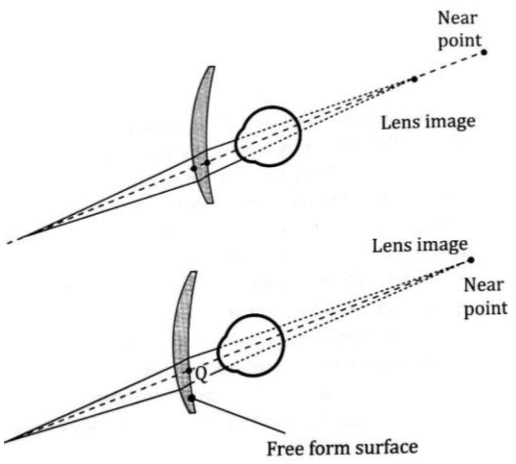
Digital Functional Assessment

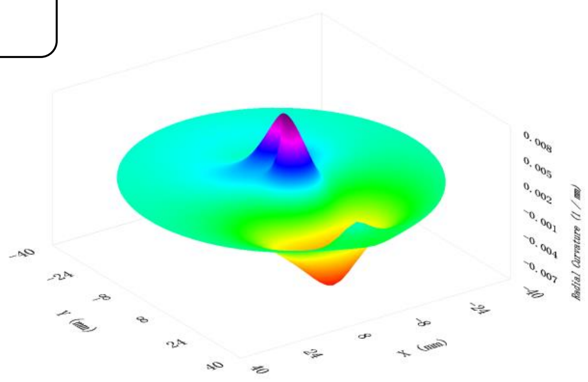
→ Ray Tracing

→ 3D Surface Parameters

↓

Machining Trials





Functional assessment

- ISO Standard combining with RMS value

ISO Standard

Table 1 — Tolerances on the focal power of lenses

Values in dioptres[1)]

Power of principal meridian with higher absolute focal power	Tolerance on the focal power of each principal meridian, A	Tolerance on the absolute cylindrical power B			
		⩾ 0,00 and ⩽ 0,75	> 0,75 and ⩽ 4,00	> 4,00 and ⩽ 6,00	> 6,00
⩾ 0,00 and ⩽ 3,00	± 0,12	± 0,09	± 0,12	± 0,18	—
> 3,00 and ⩽ 6,00	± 0,12	± 0,12	± 0,12	± 0,18	± 0,25
> 6,00 and ⩽ 9,00	± 0,12	± 0,12	± 0,18	± 0,18	± 0,25
> 9,00 and ⩽ 12,00	± 0,18	± 0,12	± 0,18	± 0,25	± 0,25
> 12,00 and ⩽ 20,00	± 0,25	± 0,18	± 0,25	± 0,25	± 0,25
> 20,00	± 0,37	± 0,25	± 0,25	± 0,37	± 0,37

Table 2 — Tolerances on the direction of cylinder axis

Absolute cylindrical power D	⩽ 0,50	> 0,50 and ⩽ 0,75	> 0,75 and ⩽ 1,50	> 1,50
Tolerance on the axis °	± 7	± 5	± 3	± 2

Table 3 — Tolerances on the addition power for multifocal lenses

Values in dioptres

Value of the addition power	⩽ 4,00	> 4,00
Tolerance	± 0,12	± 0,18

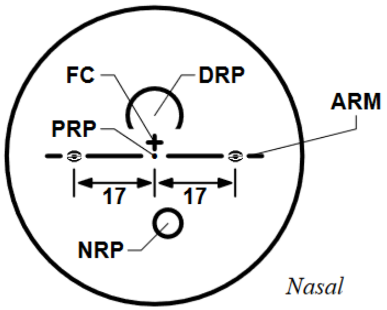
Table 4 — Prismatic tolerance

Values in prism dioptres

Prismatic power	Lenses		
	Single vision	Multifocal	
		Horizontal	Vertical
⩾ 0,00 and ⩽ 2,00	$\pm(0{,}25+0{,}1\times S_{max})$	$\pm(0{,}25+0{,}1\times S_{max})$	$\pm(0{,}25+0{,}05\times S_{max})$
> 2,00 and ⩽ 10,00	$\pm(0{,}37+0{,}1\times S_{max})$	$\pm(0{,}37+0{,}1\times S_{max})$	$\pm(0{,}37+0{,}05\times S_{max})$
> 10,00	$\pm(0{,}50+0{,}1\times S_{max})$	$\pm(0{,}50+0{,}1\times S_{max})$	$\pm(0{,}50+0{,}05\times S_{max})$

NOTE S_{max} is the focal power, in dioptres, in the meridian of higher absolute power.

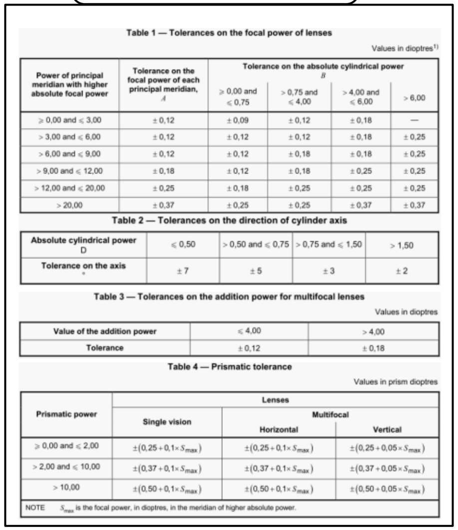
Three Reference Points



Functional assessment

- ISO Standard combining with RMS value

ISO Standard



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> 3,00 and ⩽ 6,00	± 0,12	± 0,12	± 0,12	± 0,18	± 0,25
> 6,00 and ⩽ 9,00	± 0,12	± 0,12	± 0,18	± 0,18	± 0,25
> 9,00 and ⩽ 12,00	± 0,18	± 0,12	± 0,18	± 0,25	± 0,25
> 12,00 and ⩽ 20,00	± 0,25	± 0,18	± 0,25	± 0,25	± 0,25
> 20,00	± 0,37	± 0,25	± 0,25	± 0,37	± 0,37

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Tolerance on the axis °	± 7	± 5	± 3	± 2

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Values in dioptres

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> 2,00 and ⩽ 10,00	$\pm(0{,}37 + 0{,}1 \times S_{max})$	$\pm(0{,}37 + 0{,}1 \times S_{max})$	$\pm(0{,}37 + 0{,}05 \times S_{max})$
> 10,00	$\pm(0{,}50 + 0{,}1 \times S_{max})$	$\pm(0{,}50 + 0{,}1 \times S_{max})$	$\pm(0{,}50 + 0{,}05 \times S_{max})$

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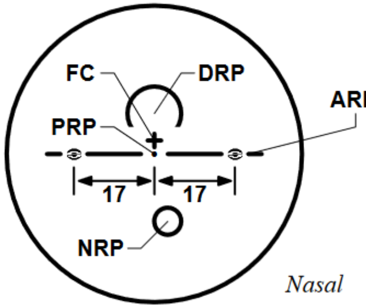
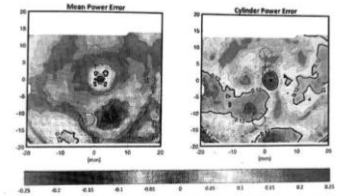
Ray Tracing

Three Reference Points



Lens assessment result



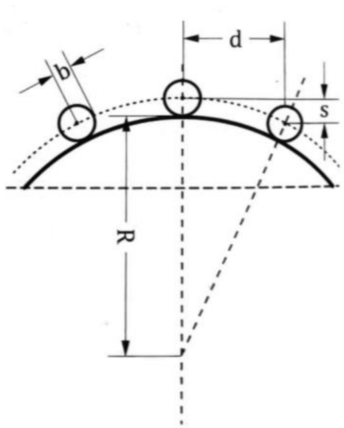
DRP				NRP				Prism			
Theoretical		Measured		Theoretical		Measured		Theoretical		Measured	
H	C	H	C	S_H	C	H	C	Prism	Axis	Prism	Axis
0.04	−0.01	0.00	−0.04	2.17	−0.35	2.12	−0.37	0.48	270	0.46	277

Root-Mean-Square Deviation

$$S_q = \sqrt{\frac{1}{MN}\sum_{j=1}^{N}\sum_{i=1}^{M}\eta^2(x_i, y_j)}$$

Functional assessment

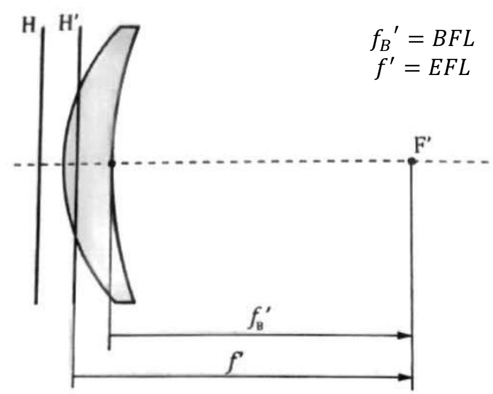
- Ray tracing algorithms



$$R = \frac{d^2 + s^2}{2d} \mp b$$

Material refractive index

Radius of curvature → Local refractive power → Back focal length (BFL)



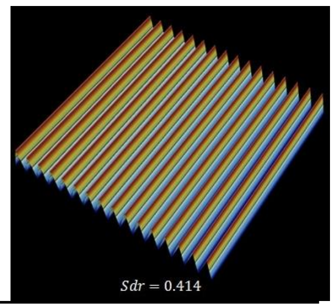
Functional assessment

- 3D surface parameters

Hybrid Parameters

S_{dr}

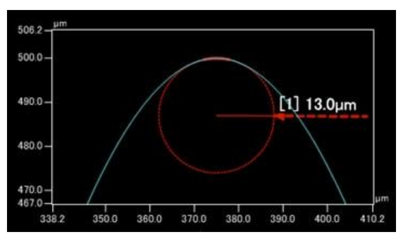
Developed interfacial area ratio



$$\frac{\sum_{j=1}^{N-1}\sum_{i=1}^{M-1}A_{ij}-(M-1)(N-1)\Delta x\cdot\Delta y}{(M-1)(N-1)\Delta x\cdot\Delta y}$$

S_{sc}

Arithmetic mean summit curvature of a surface



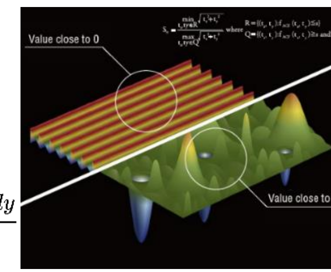
$$S_{sc}=-\frac{1}{2}\frac{1}{n}\sum_{k=1}^{n}\left(\frac{\partial^2 z(x,y)}{\partial x^2}+\frac{\partial^2 z(x,y)}{\partial y^2}\right)$$

Spatial Parameters

S_{tr}

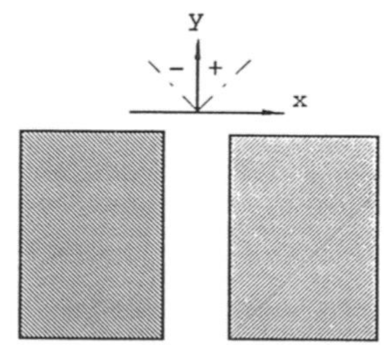
Texture aspect ratio

$$f_{ACF}(t_x,t_x)=\frac{\iint_A z(x,y)z(x-t_x,y-t_y)dxdy}{\iint_A z(x,y)z(x,y)dxdy}$$



S_{td}

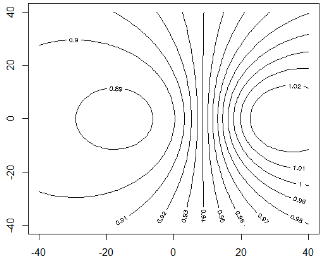
Texture direction



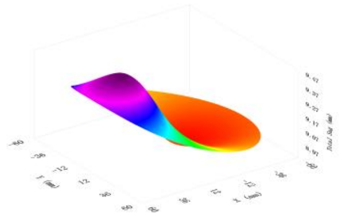
$$S_{td}\begin{cases}-\beta, & \beta\le\frac{\pi}{2}\\ \pi-\beta, & \frac{\pi}{2}<\beta\le\pi\end{cases}$$

Ultraprecision machining

Lens Surface Design



Lens Model Construction



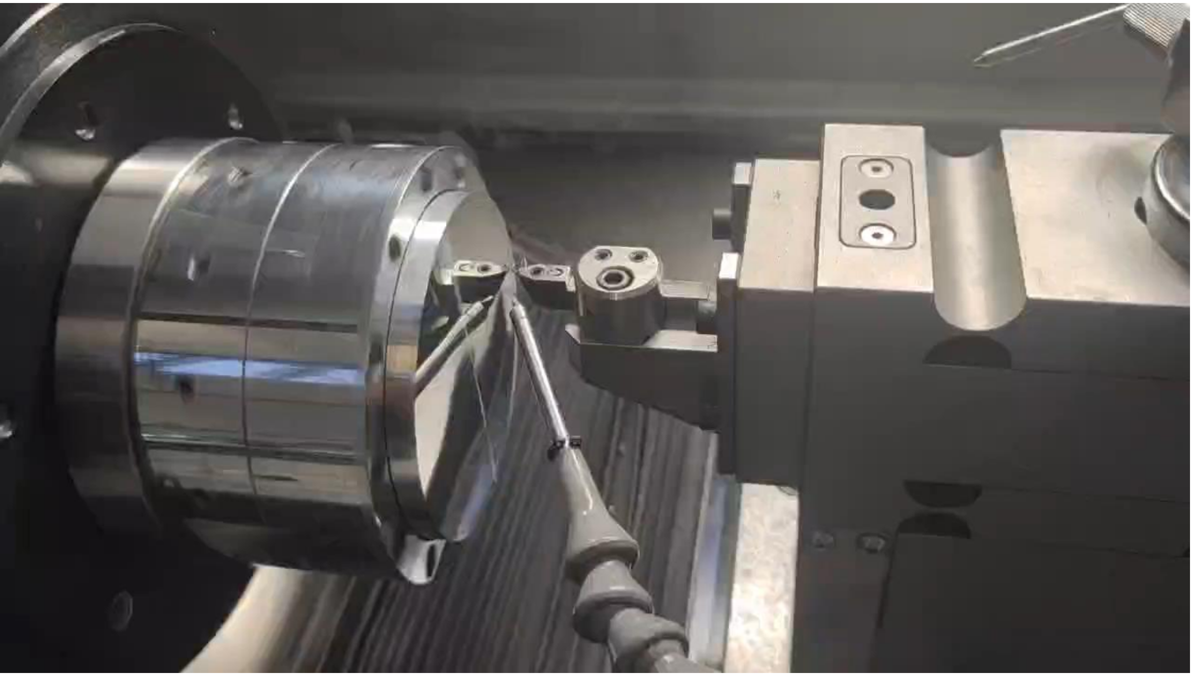
Functional Assessment

- ISO Standard
- 3D surface parameters
- Ray tracing

Integrated 'digital' process

Surface Model → CNC Software → Toolpath → Ultraprecision Machining

Ultraprecision machining trials



- Diamond turning machine (UPL250)

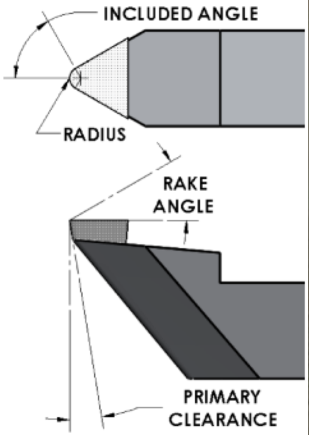
- Diamond cutting tools:
  - Tool radius (mm): 0.335
  - Included angle (deg): 60.0°
  - Rake angle (deg): 0°
  - Primary Clearance: 15.0°

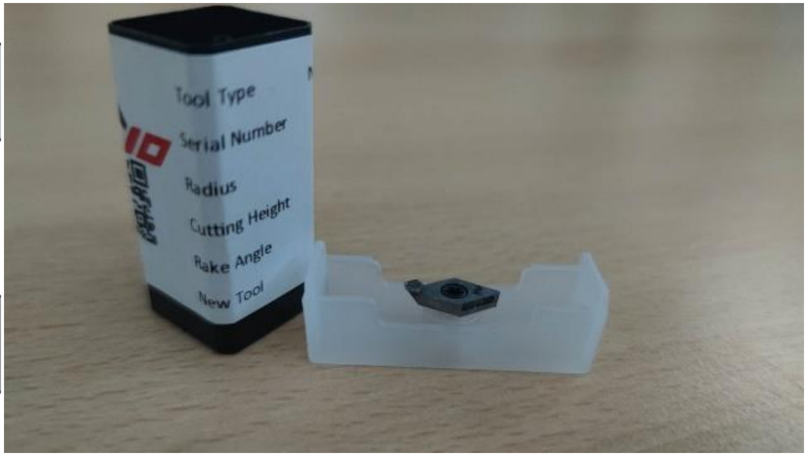
- Materials: Al and PMMA

- Process: slow tool servo (STS) mode

- Radial feed(mm/rev): 0.1

- Total cycle time: 9.1 mins





Concluding remarks

- High precision and personalized design and manufacturing
- Integrated digital lens assessment and quality assurance
- Ultraprecision machining trials - evaluation and validation of the NURBS-based integration approach

Thank you!