

A methodological approach for implementing a digital metrological twin for multi-axis machine tools

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Abstract

This paper presents a methodological approach for developing a Digital-Metrological Twin (D-MT) tailored to a five-axis machining center (Mikron HSM 600U). The work addresses a major industrial need: ensuring metrological traceability during in-situ measurements on the Machine Tools. Current approaches are often limited, as traceability is only guaranteed through post-process inspection on coordinate measuring machines or by restricting control to simple prismatic geometries. To overcome these limitations, the proposed D-MT concept—introduced within the framework of the European 23IND12 [ADAM](#) project—is a traceable virtual replica of a measuring instrument or a freeform material standard combining robust models and parametrization methods to evaluate measurement uncertainty and to support quality control in advanced manufacturing. The D-MT integrates a thermo-elasto-geometric model that accounts for both geometric errors and thermally induced structural deformations. Geometric errors are modeled following ISO 230-1 conventions through analytical formulations, while thermal effects are represented through a semi-empirical approach coupling beam-theory estimations with Auto-Regressive with eXogenous input (ARX) models identified from sensor data. The D-MT will be deployed in a real-time hardware–software environment based on a Scalexio LabBox equipped with acquisition boards, where signals are acquired and processed within a MATLAB/Simulink application and subsequently compiled for real-time execution using dSPACE's ConfigurationDesk. A distinctive element of this methodology is the conception of a dedicated material standard, derived from sensitivity analyses of simulated geometric and thermal deviations within the machine's working volume. This artefact will materialise representative error-affected surfaces, enabling both the calibration of D-MT parameters and the evaluation of measurement uncertainty through reproducibility tests. These developments are expected to enhance the overall accuracy of the machine and contribute to the implementation of traceable, industry-ready measurement processes.

Key words: Digital-Metrological Twin, Thermal errors, Metrology, Geometric errors, Compensation strategy, Material Standard, Calibration

1. Industrial context

In today's manufacturing industry, there is an increasing demand for high-end components with tight dimensional tolerances, particularly in sectors such as biomedical, aerospace, automotive, etc. To reach these requirements, multi-axis CNC machine tools are employed. However, these machines are affected by different sources of errors. In this context, metrology plays a dual role: verifying part conformity and quantifying deviations in the machine's behavior. While several approaches have been developed to integrate metrological verification into the production process, full metrological traceability is currently only achievable through post-process inspection in controlled environments using traceable equipment. To reduce time and costs, modern industry increasingly favours in-situ and on-process measurement systems integrated directly into the machine tool, although these do not yet ensure traceability. Such systems are often associated with IoT sensors and real-time parameter adaptation, technologies that converge towards an innovative concept with the potential to transform manufacturing: the Digital Twin (DT) [1]. According to ISO 23247-1:2021 [2], a DT is "a virtual model of a physical entity in the manufacturing domain, enabling real-time synchronization and interaction with the real entity for decision-making purposes." This approach therefore relies on accurate modelling of the real system to ensure both fidelity and predictive capability.

In literature, numerous studies have characterised geometric errors of machine tools [3], often employing high-precision instruments such as laser interferometers with dedicated material standards for the full calibration of the kinematic chain. Similarly, a wide range of thermal error compensation models can be found, ranging from simplified analytical approaches—where the machine structure is discretised and thermal deformations are estimated using heat transfer laws [4]—to more complex data-driven methods, including neural networks trained on large empirical datasets [5]. Although these geometric and thermal compensation strategies are well established in both research and industry, they typically focus on the kinematic chain during machining operations, overlooking the fact that the same chain is also used for measurement purposes. As a result, existing approaches rarely address the evaluation of measurement uncertainty during in-situ inspection, which remains a key requirement for acceptance in industrial quality control. Recent studies on DTs for machine tools [6], [7] have shown promising results, but they remain focused on specific error sources and do not yet address traceability or uncertainty evaluation. To overcome these limitations, this work proposes an integrated D-MT framework that combines geometric, elastic and thermal models with a dedicated material standard serving as a physical reference for calibration and verification.

This paper is structured as follows: Section 2 presents the modelling of the machine tool, including geometric, elastic, and

thermal aspects. Section 3 describes the architecture of the digital twin. The conclusions is presented in Section 4.

2. Modeling of the machine tool

The first step toward developing a reliable and robust D-MT is the accurate modeling of the machine tool in its fundamental aspects: geometric, thermal, and elastic. The machine tool used in this study is a Mikron HSM600U five-axis high-speed machining center equipped with a Heidenhain iTNC530 numerical control. Its kinematic chain can be described as [w C' B' X' f Y Z (C1) t]. Machine tools are inherently subject to various sources of error that affect their positioning accuracy and machining performance.

2.1. Geometric model

Geometric errors—commonly referred to as kinematic errors in literature—arise from undesired deviations in the relative motion of machine tool components. These include backlashes, assembly inaccuracies, and other structural imperfections that affect the positioning of the tool. The standard reference for these error definitions is ISO 230-1:2012 [8].

A common approach for modeling geometric errors involves the use of homogeneous transformation matrices (HTM) to describe the machine's kinematic chain [9], [10]. These matrices define the relative position and orientation between successive reference frames, from the machine coordinate system (MCS) to the workpiece coordinate system (WCS) (and viceversa). The following equation represents the geometric model applied through a sequence of homogeneous transformation matrices that describe the machine tool's nominal kinematic chain:

$$P_{nom/MCS} = \begin{bmatrix} a & b & c & -p_x \\ d & e & f & -p_y \\ g & h & i & -p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c(C) & s(C) & 0 & 0 \\ -s(C) & c(C) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \dots \times \begin{bmatrix} c(C1) & s(C1) & 0 & 0 \\ -s(C1) & c(C1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & -J_x \\ 0 & 1 & 0 & -J_y \\ 0 & 0 & 1 & -J_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}_{WCS} \quad (1)$$

The presented homogeneous matrices correspond to the transformations illustrated in Figure 1, describing the coordinate transformation from the machine coordinate system (MCS) to the workpiece coordinate system (WCS).

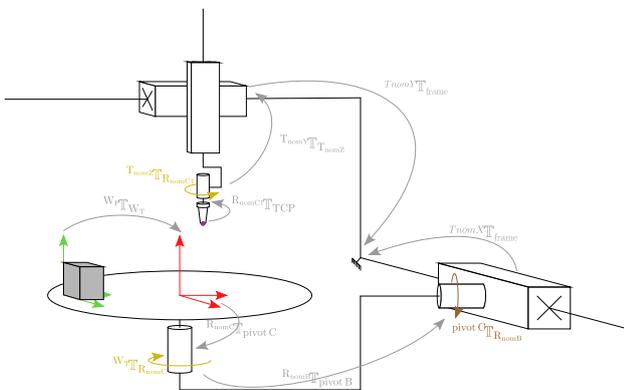


Figure 1. Graphical representation of rigid-body transformations from MCS to WCS within the kinematic chain of the considered machine tool.

However, the above model is nominal and does not take into account the geometric errors present in the kinematic loop. According to ISO 230-1:2012 [8], six independent error motion components can be associated with each machine axis: three

linear error motion and three angular error motions. When applied to a linear or rotary axis, these errors assume different geometric meanings. Their representation can be integrated within the HTM-based formulation, as they correspond to small rotations and translations that can be approximated to first order. Under the – previously and carefully verified – assumption of small displacements, the transformation matrices for motion errors or location (i.e. position) and orientation errors of average lines or reference straight line for rotary axis and linear axis respectively can be simplified on the rotation components. As example, for X-axis affected by error motion can be expressed as:

$$T_{TRX} = \begin{bmatrix} 1 & -E_{CX} & E_{BX} & E_{XX} \\ E_{CX} & 1 & -E_{AX} & E_{YX} \\ -E_{AX} & E_{BX} & 1 & E_{ZX} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

This model becomes extremely complex due to the large number of geometric errors that can be defined in a five-axis machine tool (71 in total for 5-axis machine-tools, including motion, positioning and assembly errors). Nevertheless, some of these components can be neglected according to the recommendations provided in ISO 230-1:2012.

The nominal and biased direct geometric models are jointly used to evaluate the volumetric error, defined as the deviation between the nominal and actual relative positions of the tool with respect to the workpiece. This quantity is typically employed in geometric error compensation models, but it can also be exploited to analyse the influence of geometric errors on the machine's working volume. A simulation study is currently being carried out to evaluate the volumetric error along representative joint trajectories, assuming different plausible distributions of geometric errors—such as linear, parabolic, and sinusoidal distributions—along each machine axis (as illustrated in Figure 2 and Figure 3). The objective is to generate a dataset of volumetric errors that highlights the machine regions most susceptible to geometric deviations under various realistic scenarii. These results will then be used to guide the conception of the material standard, which will materialise the critical surfaces and points that are recurrently affected across the simulated conditions.

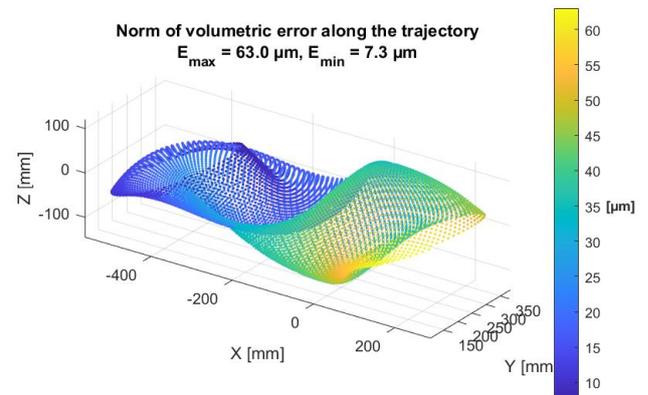


Figure 2. Effect of X-axis error motions on a representative X–C (linear) and B (sinusoidal) multi-axis trajectory. Linear shape functions are assigned to form errors with amplitudes between 50 µm and 100 µm.

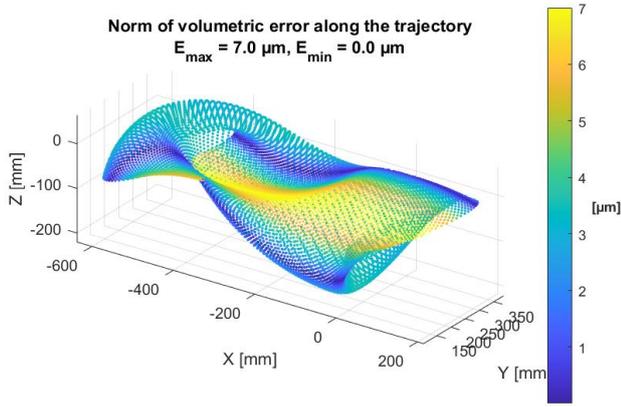


Figure 3. Effect of C-axis error motion on a representative X–C (linear) and B (sinusoidal) multi-axis trajectory. The dominant contribution to the volumetric error arises from the radial error motions E_{XC} and E_{YC} , both modeled with a sinusoidal shape of $10\ \mu\text{m}$

Building upon these results, a sensitivity analysis of the simulated volumetric errors will be performed to quantify the influence of each geometric error parameter E_{ij} or E_{i0j} on the Tool Centre Point (TCP) deviation along the trajectories. The geometric Jacobian, also referred to as the Influence Matrix by Bringmann & Knapp [11], allows defining a local sensitivity coefficient as:

$$S_{E_{ij}}(\mathbf{q}) = \left\| \frac{\partial \Delta \mathbf{p}_{TCP}(\mathbf{q})}{\partial E_{ij}} \right\| = \left\| \mathbf{J}_{E_{ij}}(\mathbf{q}) \right\| \quad (3)$$

Which quantifies the influence of E_{ij} or E_{i0j} on the TCP deviation at the configuration \mathbf{q} of the machine tool. These sensitivity indicators will be employed to identify error-dominant regions within the workspace, providing the basis for the design of the reference material standard.

2.2. Elastic model

In most studies on machine tool errors, the structural elements and kinematic chain are idealised as perfectly rigid bodies. However, this assumption neglects static load elastic errors, caused by structural deformations under static loads such as the weight of the machine components, clamping forces, or cutting forces. To take into account these effects, an elastic model is introduced to represent the machine's mechanical behaviour. Figure 4 shows the CAD model of the five-axis machine-tool, highlighting the main mechanical components of each linear axis. These elements consist of carriages sliding on guideways, which can be approximated by beam-like structures using the beam theory. Due to the structural constraints and the kinematic architecture of the machine, each axis can therefore be modelled as a beam subjected to bending and torsional loads and with the appropriate section, material, and boundary conditions. Nevertheless, this approximation must comply with the assumptions of beam theory—particularly the slender-beam hypothesis—which may not be valid for all structural components of the machine. To perform this modeling, a Finite Element Method (FEM) approach will be adopted using a suitable software platform (e.g. 3DEXperience).

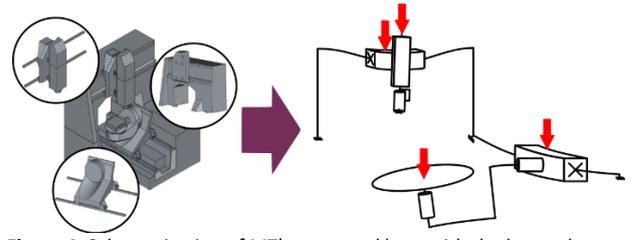


Figure 4. Schematization of MT's structural loop with the beam theory.

2.2. Thermal model

Thermal errors in machine tools have been investigated for decades due to their significant impact on machining accuracy. They are defined as the result of thermo-elastic deformations of the structural loop caused by internal and external heat sources acting on the machine tool (see Figure 5). Heat exchange between the machine, the surrounding environment, internal and external heat sources induces structural deformations, which account for approximately 70–75% of the total machine tool errors [12] and as indirect effects on geometric errors.

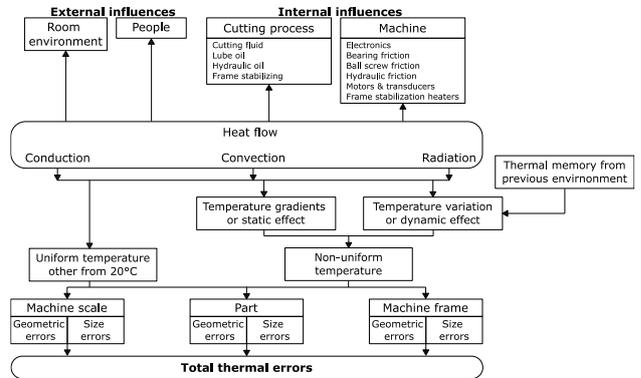


Figure 5. Schematic representation of the different heat sources and their effects on the machine tool structure and temperature field [Bryan, 1990].

Numerous approaches have been proposed in literature to model the thermal behaviour of machine tools, ranging from simplified analytical formulations to empirical and data-driven methods. Among the latter, techniques based on machine learning or deep learning are particularly popular. While these models have shown high prediction accuracy—up to 95% [13]—they often require a large number of thermal sensors (e.g. up to 50), which makes them impractical in industrial settings. In production environments, it is crucial to minimize sensor usage and avoid modifying the machine structure.

Therefore, this work proposes a semi-empirical thermal error model based on a limited number of strategically placed temperature sensors. The approach relies on an Auto-Regressive model with exogenous inputs (ARX), which combines an auto-regressive component (AR) with an exogenous input model (X). In general, an ARX model represents a time series as a linear combination of its past values and a stochastic error term, together with a linear combination of a set of external input variables. In the context of thermal errors, this formulation is particularly suitable because the thermal drift depends primarily on its own past values—reflecting the thermal memory of the system—and, at the same time, on the temperature of specific machine components, which act as the exogenous inputs. The following equation defines the ARX model used for estimating the positioning error E_{XX} , where a_i , $b_{j,k}$, p , and q are the model parameters identified experimentally to ensure accurate prediction, where T_j denotes the temperature measured by the j -th sensor and $e(t)$ is the residual error.

$$E_{XX}(t) = E_{X0X} + \sum_{i=1}^p a_i E_{XX}(t - i) + \sum_{j=1}^N \sum_{k=1}^q b_{j,k} T_j(t - k + 1) + e(t) \quad (4)$$

Models of this kind have already been reported in literature [13] and have become increasingly popular in recent years. The key distinction of the present work lies in its metrological focus: the model is developed for measurement and verification processes carried out directly on machine tools, under operating conditions that differ significantly from those encountered during conventional machining.

3. Digital-Metrological Twin (DT)

The Digital Twin (DT) integrates all the models of the machine tool presented in the previous sections. Usually, both the thermal and elastic models are coupled into a single thermo-elastic model [14], [15], [16], since the parameters of the elastic model—such as stiffness—are material-dependent and vary with temperature. These coupled models are then integrated into the geometric model to describe the deformation of the machine's working volume as a function of the system state parameters.

The D-MT architecture is based on Guevel's developments [10], who used the dSPACE platform for real-time monitoring and compensation. In our case, signals from the machine's encoders are acquired through a Scalexio LabBox equipped with analog and digital I/O boards. These signals are processed to reconstruct the axis positions and apply error-compensated feedback to the numerical control (NC). The code of the application is written in MATLAB/Simulink and compiled for real-time execution using dSPACE's ConfigurationDesk. The application can be adjusted at runtime via parameter tuning in ConfigurationDesk, allowing for closed-loop compensation based on measured or estimated errors. This architecture has already been deployed on the five-axis machining center under investigation (see Figure 6 **Error! Reference source not found.**).

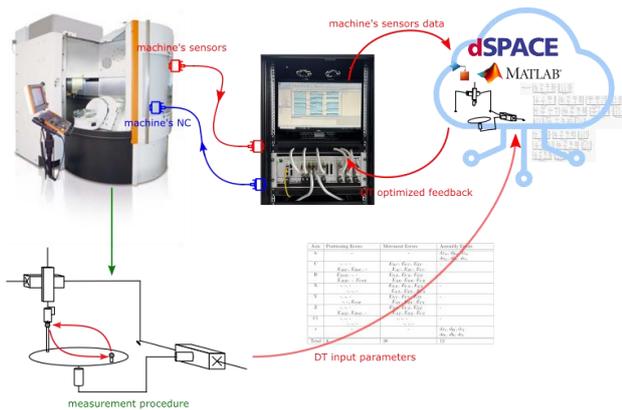


Figure 6. Digital twin architecture using dSPACE and Simulink.

4. Conclusions and perspectives

This work presents a methodological approach for the development of a digital-metrological twin (D-MT) for multi-axis machining. The proposed framework integrates a thermo-elasto-geometric model into a real-time architecture based on dSPACE hardware and software. Future work will focus on the design of a custom complex-shaped material standard, aimed at providing metrological traceability for measurements performed on the machine tool. This standard will support the

identification of geometric, elastic, and thermal errors and enable a parameterization strategy for the digital twin. The overall approach will be experimentally validated through the machining and analysis of dedicated test pieces. These developments will enhance the overall accuracy of the machine and contribute to the implementation of traceable, industry-ready measurement processes.

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