Analysis of the advantage of measuring straightness when characterising volumetric performance of machine tools by body diagonal kinematic tests

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Abstract

Testing the volumetric performance of machine tools is an increasingly demanding need in industry due to the growing complexity of workpiece geometries with tighter tolerances, and the development of more sophisticated machine tools with additional attachments and axes. Various techniques and methods have been devised to quantify the volumetric performance of machine tools when they are commanded to move with simultaneous motion of their axes. Some of these methods have been standardised and addressed in the ISO 230 series.

ISO 230-6:2002 standard addresses only positioning error of body and face diagonal trajectories. This paper explores whether it is valuable to measure the straightness errors of the diagonal trajectories from different perspectives including conceptual, numerical and practical aspects. It conceptually discusses why it is necessary to quantify diagonal straightness deviations alongside the diagonal positioning deviation for a nominal target position. Additionally, for the numerical analysis, this study investigates the sensitivity of these three diagonal errors through half a million simulations over more than 1.9 million possible combinations under various error profiles for each error motion of the linear axes of a 3-axis vertical machining centre (VMC). Moreover, nearly 600 practical body diagonal tests have been made on a 3-axis VMC with the same kinematic chain using a Renishaw XM-60 laser system to assess the feasibility and ease of setup for this kinematic test. The findings support measurement of diagonal straightness errors to supplement the tests specifies in ISO 230-6:2002.

1 Introduction

ISO 230-6:2002 [1] indicates that diagonal positioning tests can be used to evaluate the volumetric performance of machine tools. In addition to positioning errors, ISO 230-1:2012 [2] addresses diagonal straightness errors for diagonal trajectories resulting from the interpolation of the machine's linear axes. This standard categorises this test as a kinematic test employing the simultaneous motion of linear axes along a linear trajectory. However, only mentioning the term "face/body diagonal straightness" without fully defining the straightness components, including their directions and signs, may cause confusion for the users of the standard who aim to apply this concept to assess the volumetric performance of a machine along a linear diagonal tool path.

Kinematic test K3 in ISO 10791-6:2014 [3] specifies conditions and tolerance of a test to evaluate the straightness of linear motion over a length of 100 mm with interpolation of two linear axes of machining centres in their three main planes XY, YZ and ZX. This machine-specific standard defines three different angles for face diagonal trajectories, including angles 3, 45 and 87 degrees in the horizontal plane. However, it suggests testing the machine at two angles in their vertical planes, angles 45 and either 3 or 87 degrees without explaining the rationale for these angles which determine the face diagonal trajectory orientation. Furthermore, it only requires measuring the straightness of motion in the plane where the diagonal displacement and straightness under test lie. In other words, this test overlooks the second straightness of motion which is perpendicular to both positioning and the specified straightness in this test.

ISO 13041-5:2015 [4] addresses this kinematic test with the same concept for turning machines and turning centres with different configurations, including horizontal machines in AK3, vertical machines in BK3, and vertical inverted machines in CK3. Both of these machine-specific standards suggest using a linear displacement sensor such as a dial gauge along with a mechanical straight artefact such as straightedge to conduct this test. To orientate the straightedge in the desired direction, these standards advise utilising a sine bar or a swivelling vice.

Over the past years, the applicability of diagonal tests has been controversial. While Wang and Liotto [5] highlighted the complementary benefits of step diagonal tests with mathematical formulations, Chapman [6] theoretically explained the limitations of these tests. Svoboda [7] conducted some experiments demonstrating the Wang's method in not as effective as he claimed. Ibaraki et al. [8, 9] explained that under specific condition, Wang's method can give valid results, including a good alignment of the laser system. Even though all these researchers studied diagonal positioning tests from different perspectives, none of them addressed the straightness of diagonal trajectories.

Dashtizadeh et al. [10] previously defined the straightness components of face and body diagonal trajectories. Based on this, in the present research, we will conceptually explain why diagonal positioning error alone is insufficient to evaluate volumetric performance of a machine tool. Additionally, to assess the effect of any linear error motion of all machine's axes on the diagonal positioning and straightness errors, we will present the results of a statistical analysis based on the simulation of numerous machine tools with various error motions of their linear axis. This statistical analysis is based on half a million permutations.

2 Diagonal positioning and straightness errors

To specify the directions of the two components of diagonal straightness, a vertical plane parallel to Z-axis is crossed over the diagonal tool path in which the entire diagonal trajectory lies. Figure 1 depicts this vertical plane. As shown in this figure, the first diagonal straightness deviation, e_{S1D}, is defined in that plane and is perpendicular to the trajectory itself. Direction of the second straightness deviation, e_{S2D}, is determined based on the cross product of the unit vector of these two vectors. Dashtizadeh et al. [10] provided a detailed explanation of determining the direction and sign of both straightness components.

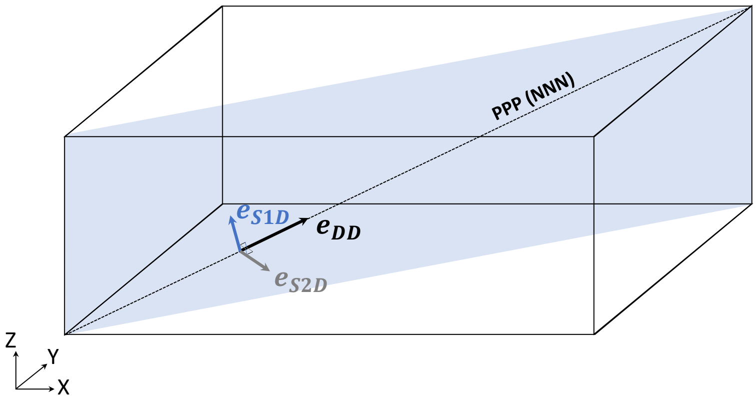

Figure 1: Direction of diagonal positioning and straightness deviations at a target position for a body diagonal trajectory [10]

ISO 230-6:2002 [1] outlines how target positions should be selected and how the data (positioning deviation at any target position) are processed to obtain the positioning error, E_{DD}. While ISO 230-2:2014 [11] deals with positioning tests for single linear axes, ISO 230-6 technically applies the same procedure but along diagonal trajectories. Consequently, the same procedure can be applied to the straightness deviations at any target position and over the diagonal trajectory to derive the straightness errors, E_{S1D} and E_{S2D}.

3 Conceptual analysis of diagonal deviations

The measured diagonal positioning deviation along the trajectory, as well as the two diagonal straightness deviations at a target position, are decomposed elements of a volumetric deviation vector. Figure 2 demonstrates this volumetric deviation vector, *V*, in a magnified sketch. In other words, by projecting volumetric deviation vector, *V*, onto the diagonal trajectory, the diagonal positioning deviation, e_{DD}, at that target position is obtained. As the directions of straightness components, S1D and S2D were defined previously, their deviation vectors can be determined by projecting *V* onto those two directions. Therefore, e_{S1D} and e_{S2D} are computed based on the magnitude and orientation of the volumetric deviation vector for any target position.

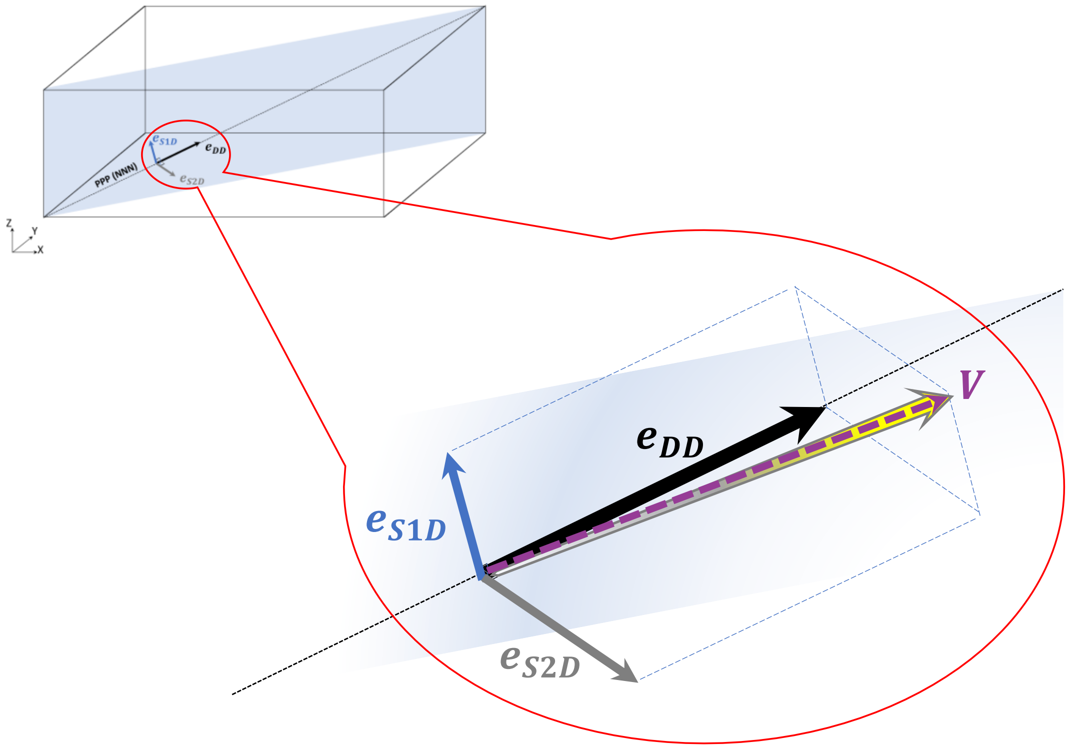

Figure 2: Volumetric deviation vector at a target position and its decomposed diagonal positioning and straightness vectors along the main body diagonal trajectory

An imaginary plane, P_{DD}, passing through the endpoints of the vectors V and e_{DD} is shown in Figure 3. This plane is parallel to the plane containing both straightness deviation vectors, e_{S1D} and e_{S2D}. As depicted in this figure, other volumetric deviation vectors whose endpoints are located in the plane P_{DD}, have identical positioning deviation vector, e_{DD}, even though their straightness deviation vectors, e_{S1D} and e_{S2D}, are each different. For example, the volumetric deviation vector V_4 has identical e_{DD} positioning deviation while its two straightness deviation vectors are much smaller than e_{S1D} and e_{S2D}. Similarly, V_3 has the identical positioning deviation vector to e_{DD} whereas its both straightness deviation vectors are larger than e_{S1D} and e_{S2D}. In other words, there can be an infinite number of volumetric deviation vectors at the same target position with positioning deviation vectors identical to e_{DD} but with entirely different straightness deviation vectors compared to the shown e_{S1D} and e_{S2D}. Therefore, to quantify the volumetric performance of a machine tool along a diagonal trajectory, the positioning deviation alone does not seem sufficient. It is evident that the positioning error is the resultant of these positioning deviations, as recommended by ISO 230-2: five target positions per meter for linear axes shorter than 2 meters.

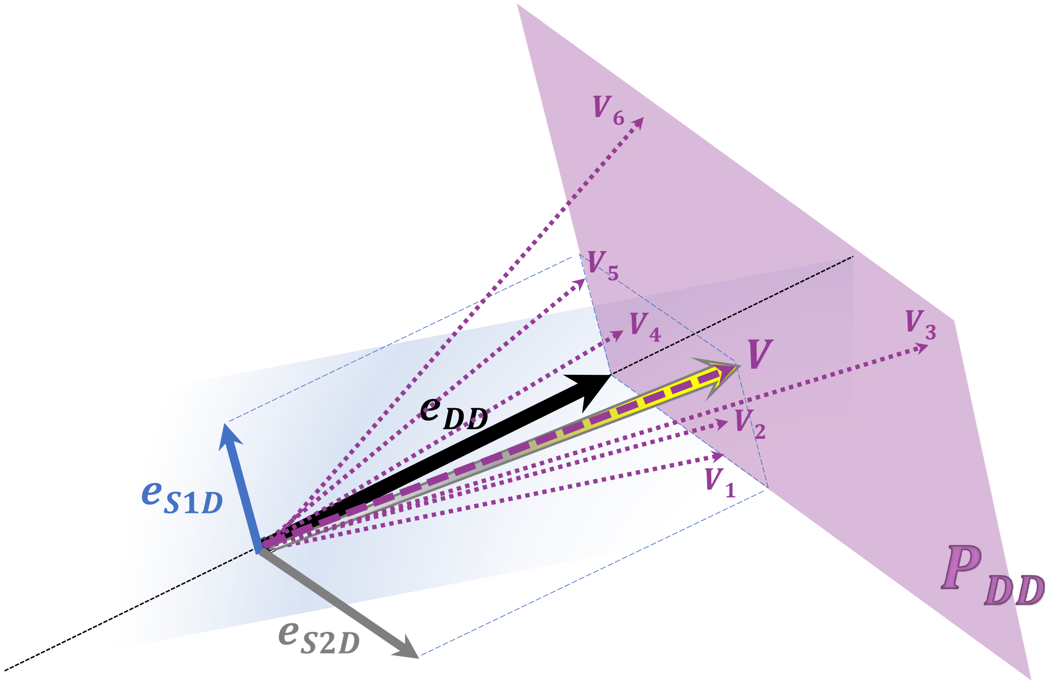

Figure 3: Different volumetric deviation vectors at a target position leading to the identical diagonal positioning deviation of e_{DD} with non-equal straightness deviations of e_{S1D} and e_{S2D}

4 Numerical analysis of diagonal errors

In the last section, we discussed the concept of volumetric deviation vectors and the necessity of quantifying diagonal straightness deviations, e_{S1D} and e_{S2D} along with the diagonal positioning deviation, e_{DD}. Here, we will numerically investigate how different error motions of machine tools can influence the diagonal errors, E_{DD}, E_{S1D}, and E_{S2D}. To assess the sensitivity of these diagonal errors to changes in different error motion, simulations were carried out and their results were statistically analysed. Dashtizadeh et al. [12] conducted simulations to investigate the volumetric errors of machining centres built in conformance with the geometric tolerances specified by ISO 10791 series. Those simulations were based on Homogeneous Transformation Matrices (HTM) to compute volumetric errors according to the pre-determined error profiles for all 21 geometric errors of a machining centre. To establish a basis for calculating the diagonal errors, we utilised the same concept from that research but employed different error profiles that do not comply with the ISO standard tolerances for machining centres. This approach, using exaggerated error values, allows for easier study of the changes and sensitivity of the responses, including diagonal positioning and diagonal straightness errors.

HTM methodology was previously developed by Donmez et al. [13] for 2-axis turning machines and by Okafor and Ertekin [14] for 3-axis machining centres.

4.1 Assumptions for the simulations

To simulate the body diagonal positioning and straightness errors, a 3-axis vertical machining centre with the kinematic chain of [w X' Y' b Z (C) t] was modelled using the HTM method. The configuration of this machine tool, along with the attached coordinate frames to its main components, is depicted in Figure 4. By applying HTM, deviation vectors at all target positions can be derived within the working volume of the machine tool under study. These computed deviation vectors are then used to extract any tool path, including the main body diagonal trajectory. Subsequently, the diagonal positioning error and two diagonal straightness errors are calculated for this trajectory.

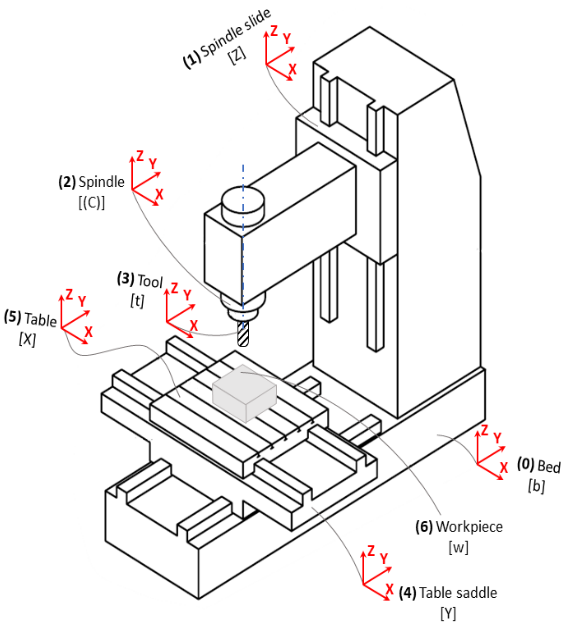

Figure 4: Vertical machining centre with kinematic chain of w X' Y' b Z (C) t (modified from ISO 10791-2 [15])

To conduct the simulations, certain error profiles were assumed to be associated with the linear axes of the above-mentioned machining centre, based on engineering knowledge from testing machine tools. X-axis, Y-axis and Z-axis travels were considered equal, each ranging from 0 to 500 mm. Furthermore, 11 equally-spaced target positions were considered on each axis. For the simulation of body diagonal trajectories, five error profiles for positioning and five error profiles for straightness errors of X-axis, Y-axis and Z-axis were generated and named as listed in Table 1:

Table 1: factors and levels for design of experiments and statistical analysis

Error motion (factors) DOE levels	Positioning	Straightness
Level 1	Zero, all deviations are zero (ideal axis), named "Zero"	Zero, all deviations are zero (ideal axis), named "Zero"
Level 2	Positive Progressive with total error of 0.050 mm, named "0.05PPrg"	Positive Hump with total error of 0.050 mm, named "0.05PHmp"
Level 3	Positive Cyclic with total error of 0.050 mm, named "0.05PCyc"	Positive Cyclic with total error of 0.050 mm, named "0.05PCyc"

Level 4	Negative Progressive with total error of 0.050 mm, named “0.05NPrg”	Negative Hump with total error of 0.050 mm, named “0.05NHmp”
Level 5	Negative Cyclic with total error of 0.050 mm, named “0.05NCyc”	Negative Cyclic with total error of 0.050 mm, named “0.05NCyc”

Graphical representation of all above-mentioned error profiles for positioning and straightness for one linear axis is depicted in Figure 5 and Figure 6, respectively. For simplicity, all angular error motions were assumed to be zero, although it is very unlikely to happen. Furthermore, all three squareness errors of linear axes of the machine were considered ideal (zero squareness). Additionally, the assembly errors of the tool spindle on the machine structure (Z-axis in this case), including its squareness to X-axis and Y-axis were assumed to be zero (ideal situation). Its parallelism to Z-axis in both YZ and ZX planes was also considered zero (ideal situation).

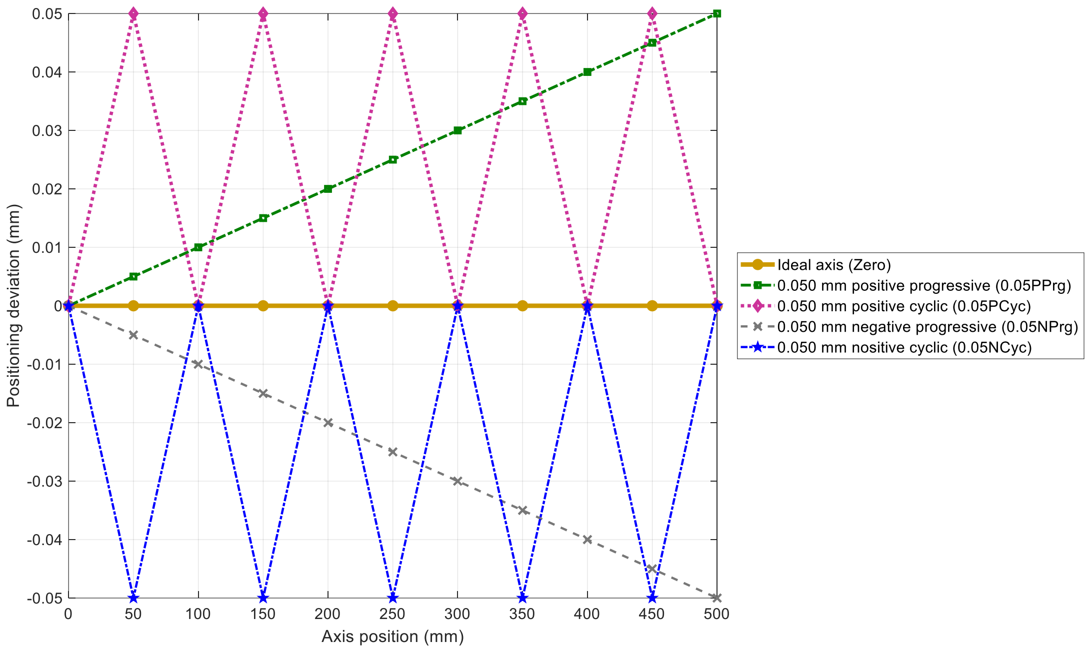

Figure 5: Positioning error profiles of all three axes used for simulations and statistical analysis

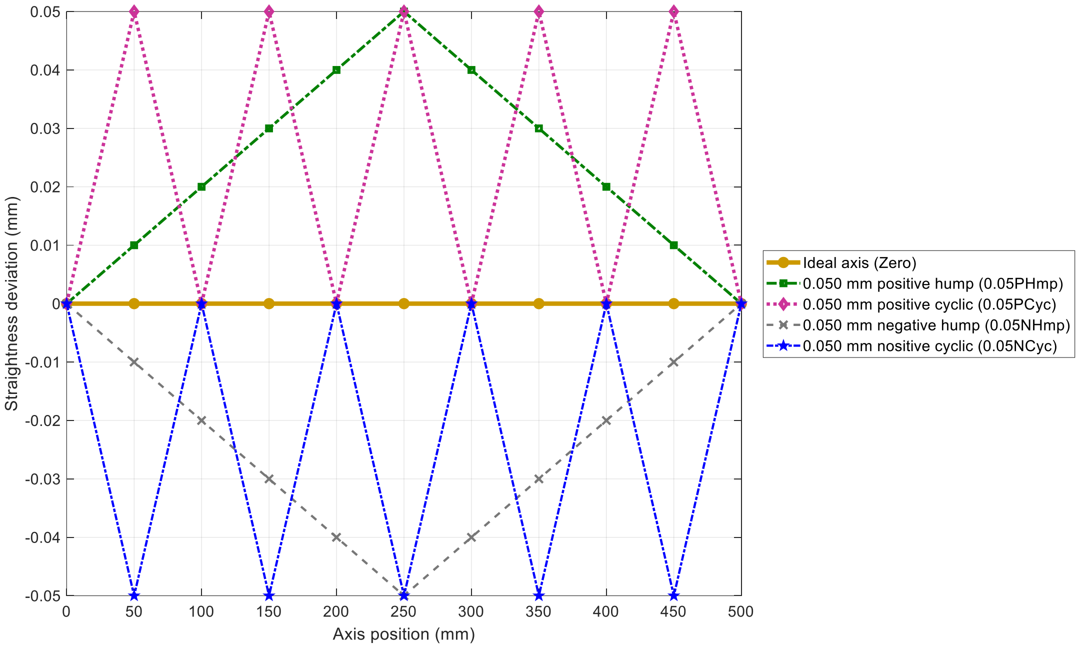

Figure 6: Straightness error profiles of all three axes used for simulations and statistical analysis

To simulate all possible combinations of the error profiles, we would have needed to conduct 5^3 × 5^3 × 5^3 = 1,953,125 permutations. However, due to the limitations of IT resources, we carried out 500,000 permutations to obtain a good sample of the entire population.

4.2 Results of a sample simulation

This section presents the details and outputs for a single simulation to demonstrate the concepts and how the values of the diagonal positioning and two diagonal straightness errors are computed. This simulation is based on the following error profiles named scenario X(0.05NPrg/0.05NHmp/0.05PHmp) Y(0.05PCyc/0.05NCyc/0.05NHmp) Z(0.05PPrg/Zero/0.05PCyc):

X-axis positioning (e_{XX}): 0.05NPrg, X-axis straightness along Y (e_{YX}): 0.05NHmp, X-axis straightness along Z (e_{ZX}): 0.05PHmp;

Y-axis positioning (e_{YY}): 0.05PCyc, Y-axis straightness along X (e_{XY}): 0.05NCyc, Y-axis straightness along Z (e_{ZY}): 0.05NHmp;

Z-axis positioning (e_{ZZ}): 0.05PPrg, Z-axis straightness along X (e_{XZ}): Zero, Z-axis straightness along Y (e_{YZ}): 0.05PCyc.

Figure 7 demonstrates separate deviation vectors along X, Y, and Z-axes (in different colours) at different target positions on the diagonal trajectory. The golden straight line represents the nominal trajectory.

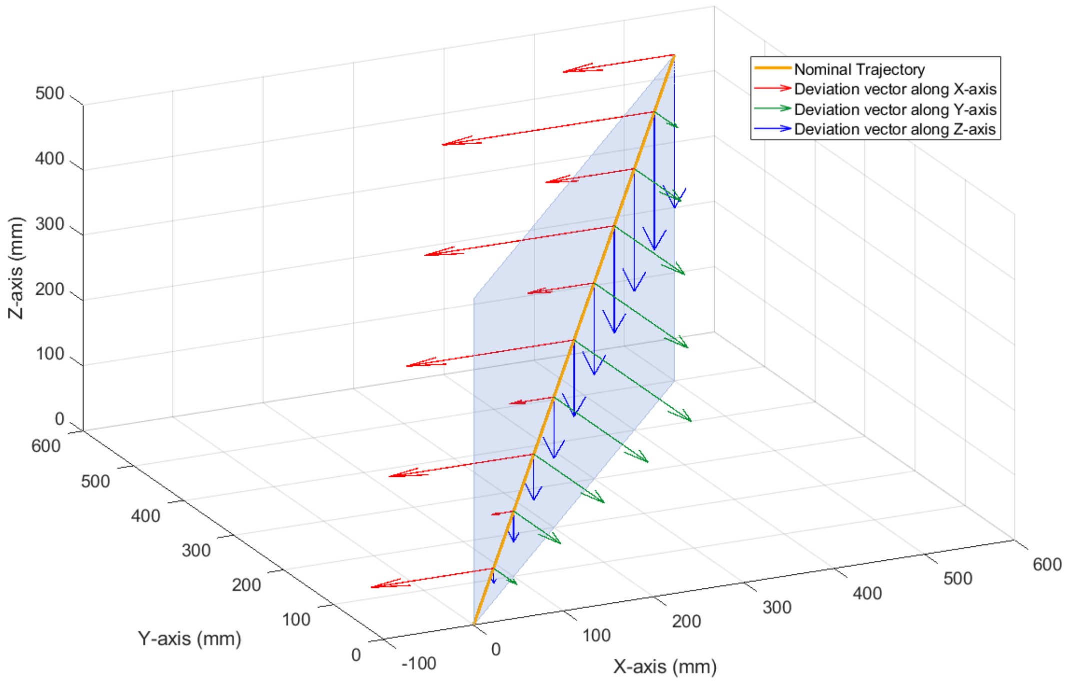

Figure 7: 3D deviation vectors along X, Y, and Z axes for the diagonal trajectory utilising scenario X(0.05NPrg/0.05NHmp/0.05PHmp) Y(0.05PCyc/0.05NCyc/0.05NHmp) Z(0.05PPrg/Zero/0.05PCyc)

Combining deviation vectors along X, Y, and Z-axes at any target position enables the computation of the volumetric deviation vector at that specific target position. Figure 8 shows the volumetric deviation vectors at different target positions on the diagonal trajectory for the scenario X(0.05NPrg/0.05NHmp/0.05PHmp) Y(0.05PCyc/0.05NCyc/0.05NHmp) Z(0.05PPrg/Zero/0.05PCyc).

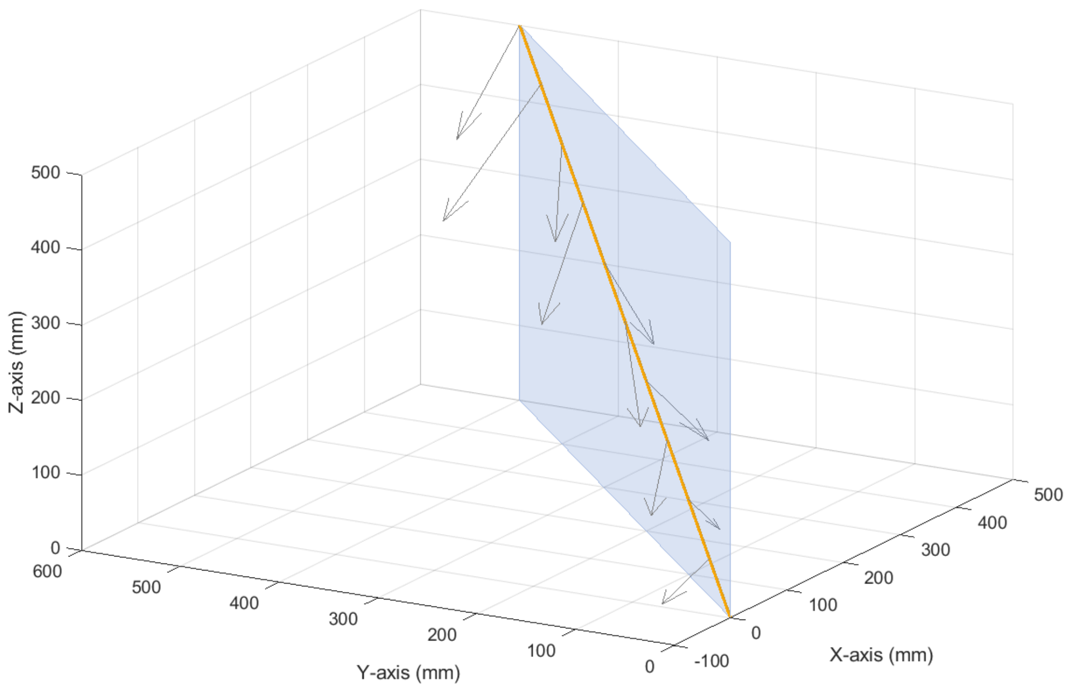

Figure 8: Volumetric deviation vectors for target positions along the diagonal trajectory utilising scenario X(0.05NPrg/0.05NHmp/0.05PHmp) Y(0.05PCyc/0.05NCyc/0.05NHmp) Z(0.05PPrg/Zero/0.05PCyc)

By projecting deviation vectors along the diagonal trajectory, the positioning deviation at any target position along the diagonal trajectory is derived. Figure 9 shows the derived positioning deviations versus diagonal trajectory. Regardless of the definitions given in ISO 230-2:2014 for different parameters of positioning, this article considers peak-to-peak value (range) of the deviations as an indication of diagonal positioning error.

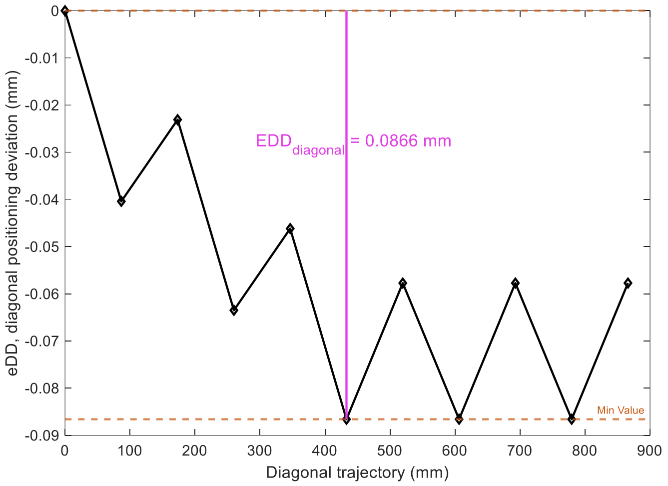

Figure 9: Diagonal positioning error (E_{DD}) plot utilising scenario X(0.05NPrg/0.05NHmp/0.05PHmp) Y(0.05PCyc/0.05NCyc/0.05NHmp) Z(0.05PPrg/Zero/0.05PCyc)

By crossing a vertical plane from the diagonal trajectory (perpendicular to XY plane in this case) shown in transparent blue colour, the deviation vectors are projected into that plane and perpendicular to the diagonal trajectory. Figure 10 illustrates the projected deviation vectors in the vertical plane and perpendicular to the diagonal trajectory, defining straightness 1 deviations (e_{S1D}) for the same scenario of error profiles.

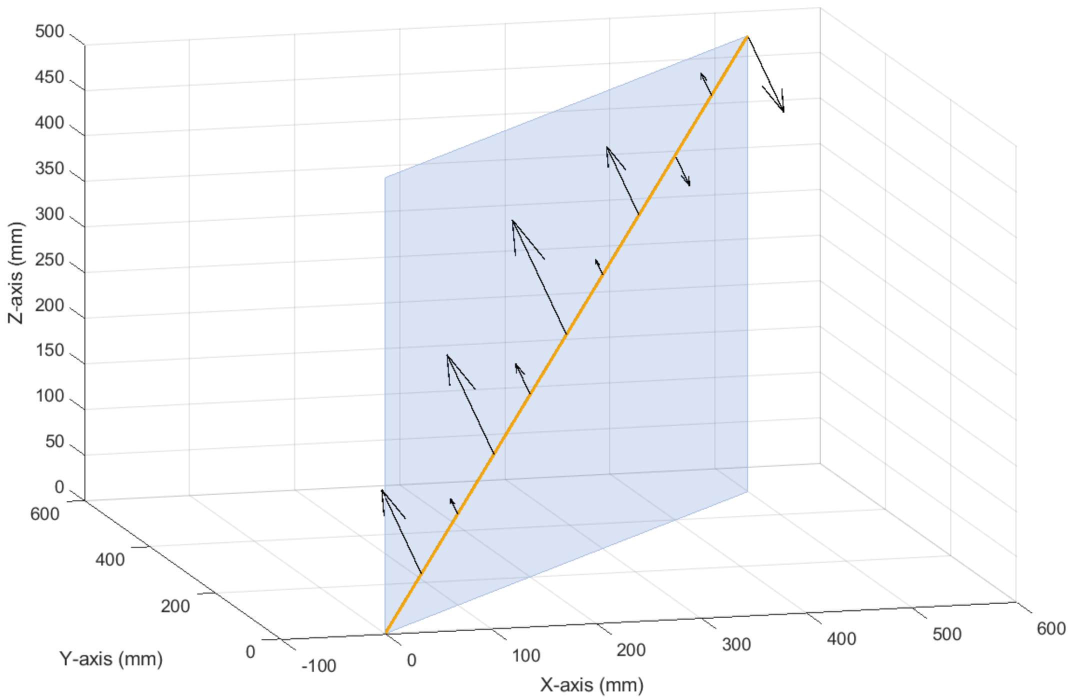

Figure 10: Straightness 1 deviation vectors (e_{S1D}) along the diagonal trajectory before slope removal utilising scenario X(0.05NPrg/0.05NHmp/0.05PHmp) Y(0.05PCyc/0.05NCyc/0.05NHmp) Z(0.05PPrg/Zero/0.05PCyc)

Applying the end-points method according to ISO 230-1:2012 to remove the slope, Figure 11 is derived, showing the straightness 1 deviations (e_{S1D}) in vertical plane for the body diagonal for the same scenario of error profiles.

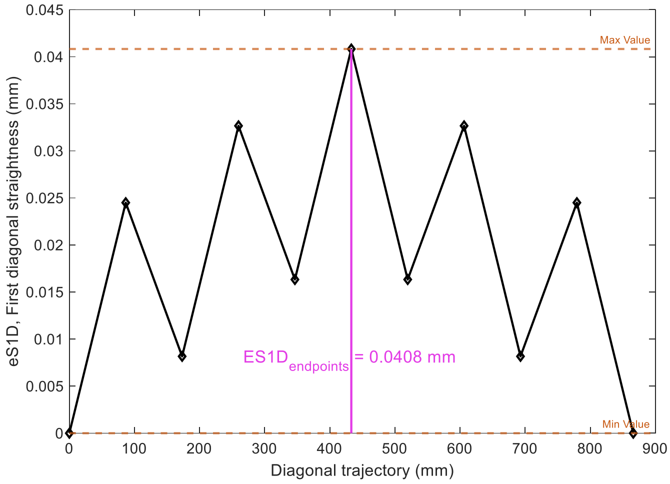

Figure 11: Diagonal straightness 1 error (E_{S1D}) plot utilising scenario X(0.05NPrg/0.05NHmp/0.05PHmp) Y(0.05PCyc/0.05NCyc/0.05NHmp) Z(0.05PPrg/Zero/0.05PCyc)

The direction of the straightness 2 deviations (e_{S2D}) is computed using cross product of directions of body diagonal positioning (e_{DD}) and diagonal straightness 1 (e_{S1D}), which is mutually perpendicular to both. Figure 12 shows straightness 2 deviations (e_{S2D}), perpendicular to the vertical plane, and consequently perpendicular to the body diagonal trajectory.

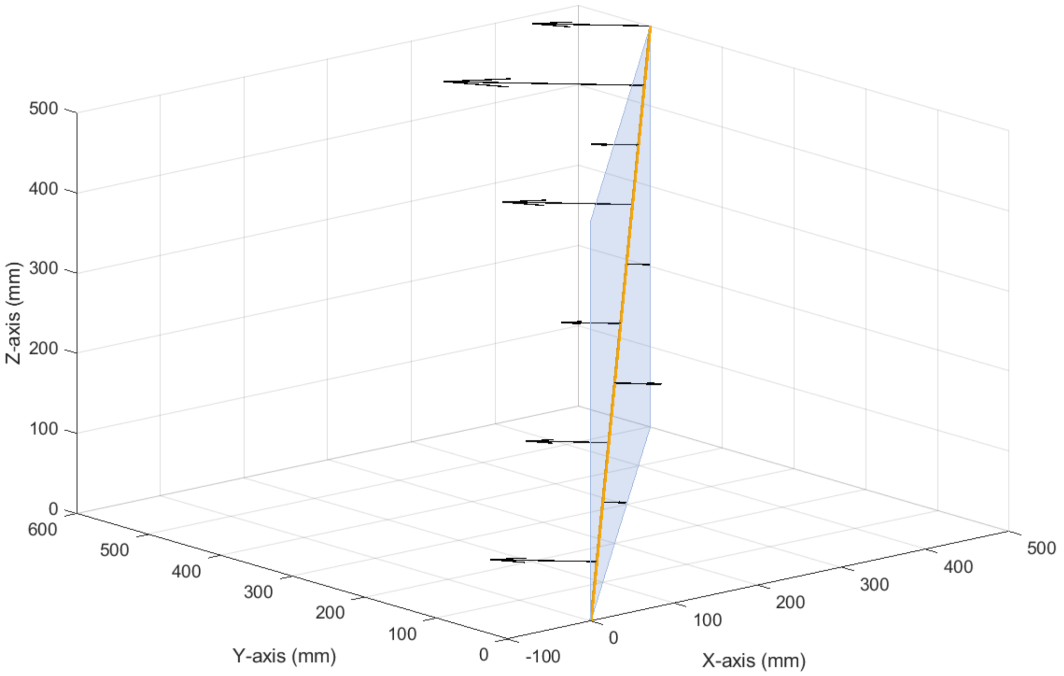

Figure 12: Straightness 2 deviation vectors (e_{S2D}) along the diagonal trajectory before slope removal utilising scenario X(0.05NPrg/0.05NHmp/0.05PHmp) Y(0.05PCyc/0.05NCyc/0.05NHmp) Z(0.05PPrg/Zero/0.05PCyc)

Again, by applying end-points method to remove the slope from straightness 2 deviations (e_{S2D}), Figure 13 is derived for the body diagonal with the same scenario of error profiles.

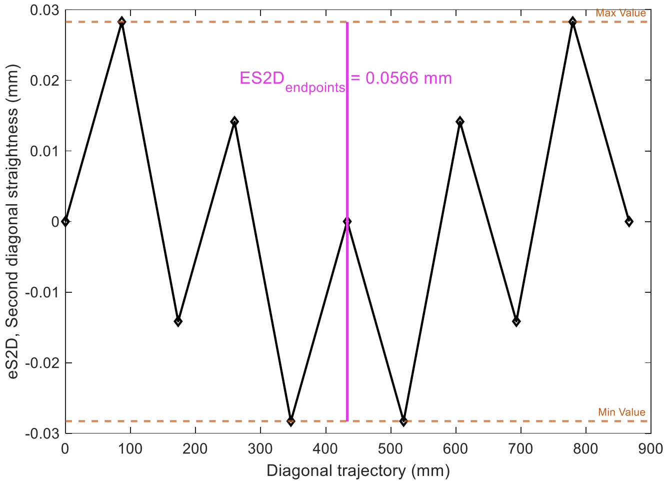

Figure 13: Diagonal straightness 2 error plot utilising scenario X(0.05NPrg/0.05NHmp/0.05PHmp) Y(0.05PCyc/0.05NCyc/0.05NHmp) Z(0.05PPrg/Zero/0.05PCyc)

Similar to the diagonal positioning error (E_{DD}), both straightness 1 (E_{S1D}) and straightness 2 (E_{S2D}) errors are computed as peak-to-peak values (range).

4.3 Results and analysis of statistical study

By executing half a million randomly selected permutations out of more than 1.9 million possible permutations, it is possible to visualise the responses E_{DD}, E_{S1D}, and E_{S2D} for those assumed error profiles (levels) for all nine error motions (factors) under study. Figure 14 demonstrates the histogram of these three diagonal errors along with some statistical measures of central tendency. As shown, all three diagonal errors are skewed to the left side of the diagram, indicating that occurrence of zero is probable according to the error profiles assumed for this study. It is also evident that in some combination of error profiles, the value of these diagonal errors could exceed 0.2 mm. As the levels designed for this study are significantly discrete from each other, the shape of the histograms does not fully follow a normal distribution.

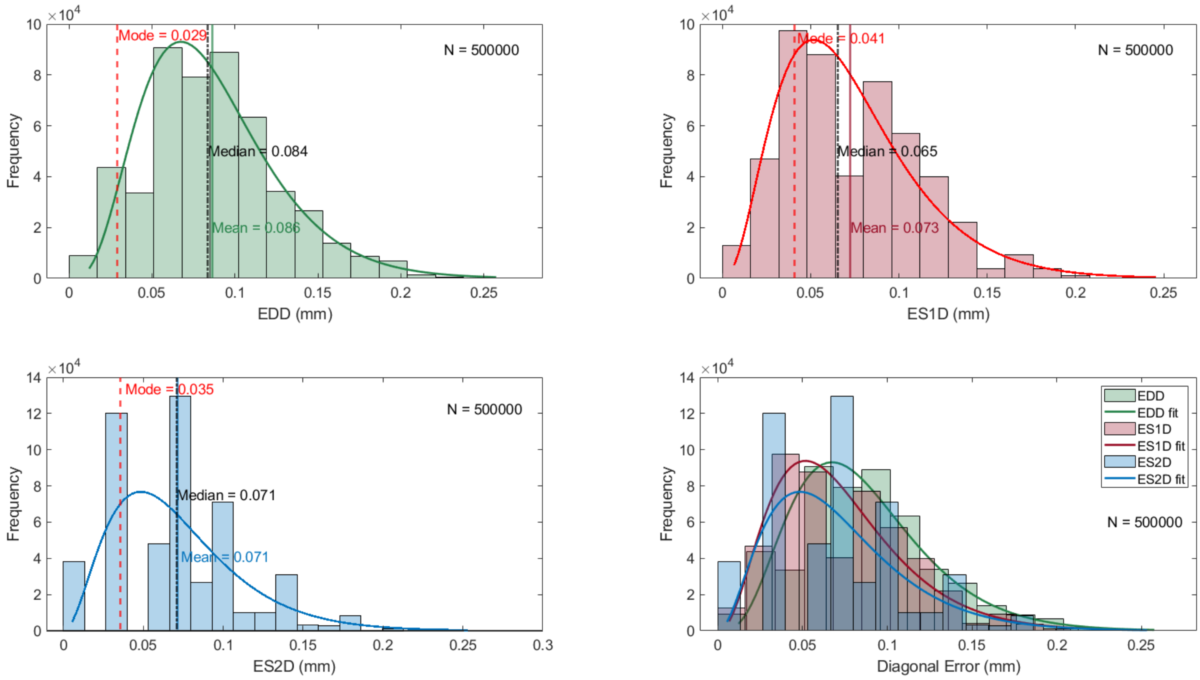

Figure 14: Histogram of E_{DD}, E_{S1D}, and E_{S2D} for 500k permutations with different scenarios

Figure 15 illustrates boxplot of the diagonal errors for all 500k permutations. This plot suggests that the middle 50% of the data for E_{DD}, E_{S1D}, and E_{S2D} lie within the range of “0.113-0.58 = 0.055mm”, “0.102-0.041 = 0.061 mm”, and “0.106-0.035 = 0.071 mm”, respectively. The red crosses in this plot represent outliers where both E_{DD} and E_{S1D} have some values significantly larger than the upper quartile.

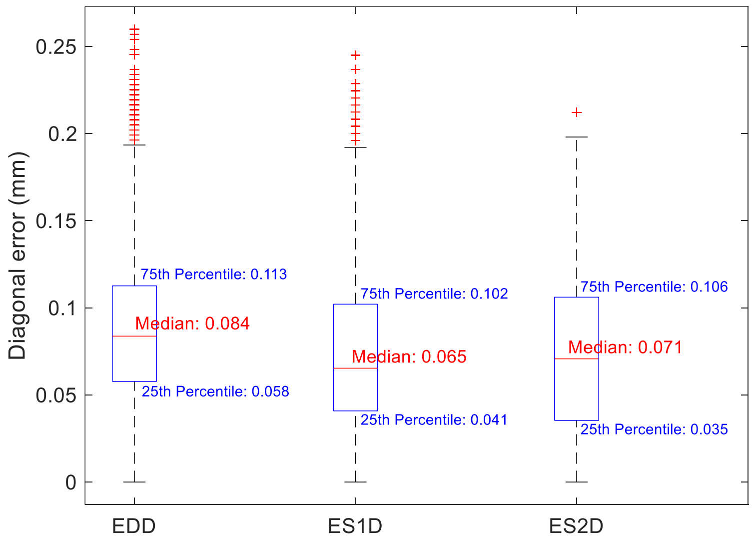

Figure 15: Boxplot of E_{DD}, E_{S1D}, and E_{S2D} for 500k permutations with different scenarios

Our observations on the results of every set of combination of error profiles show that in some cases, one or more than one of the diagonal errors could be approximately zero. In other words, although none of the axes error profiles are Zero, these error motions could cancel out the effect of each other.

In terms of measurement, this means that if we measure these diagonal errors along the body diagonal of the machine, the measuring system would show a perfect zero line with no deviation or error.

Table 2 summarises all the zero occurrences for the 500k simulations for different conditions. Additionally, Figure 16 graphically shows the probability of these occurrences in percentage. As illustrated in row 5, the probability of E_{DD} being zero is approximately 1.81%. Since a maximum of one permutation in these simulations is for a machine with no geometric errors, this implies a 1.81% probability that a machine with $E_{DD} = 0$, indicating an ideal good machine, actually has large geometric errors.

Row 1 indicates that measuring all three diagonal errors would result in an improvement to 0.11% probability that a machine with errors was falsely reported as being ideal.

Table 2: Number of occurrences of zero for body diagonal errors for different conditions computed from 500k simulations

	Condition	Count	Probability (percentage) = 100×Count/N (N = 500,000)
1	E_{DD} & E_{S1D} & $E_{S2D} \approx 0$	540	0.11 %
2	E_{DD} & $E_{S1D} \approx 0$	1908	0.38 %
3	E_{DD} & $E_{S2D} \approx 0$	719	0.14 %
4	E_{S1D} & $E_{S2D} \approx 0$	3748	0.75 %
5	$E_{DD} \approx 0$	9044	1.81 %
6	$E_{S1D} \approx 0$	12797	2.56 %
7	$E_{S2D} \approx 0$	38156	7.63 %
8	E_{DD} or E_{S1D} or $E_{S2D} \approx 0$	54162	10.83 %
9	E_{DD} or $E_{S1D} \approx 0$	19933	3.99 %
10	E_{DD} or $E_{S2D} \approx 0$	46481	9.30 %
11	E_{S1D} or $E_{S2D} = 0$	47205	9.44 %

From Figure 16, it is observed that under certain conditions, either one of the three parameters, E_{DD}, E_{S1D} and E_{S2D} may show a zero line, or more than one of them may display zero changes. From a probabilistic point of view, it is more likely for E_{DD} to exhibit zero line rather than for all three parameters to simultaneously result in zero line. Therefore, it is recommended to measure these two diagonal straightness errors in addition to the diagonal positioning error. From the other side, even if all the diagonal errors for the main body diagonal exhibit zero, if the measurement is carried out along another diagonal trajectory rather than the main body diagonal, at least one of the diagonal error profiles will reveal some errors. The is because the error motions cancel out the effect of each other if each of them move over a certain length of their stroke, whereas if one of them moves a shorter or longer distance, they cannot cancel out the error motion of the other axes. Therefore, it is very unlikely to obtain zero outputs or small deviations for all three diagonal errors when measurements are carried out along two different trajectories that are not parallel with each other.

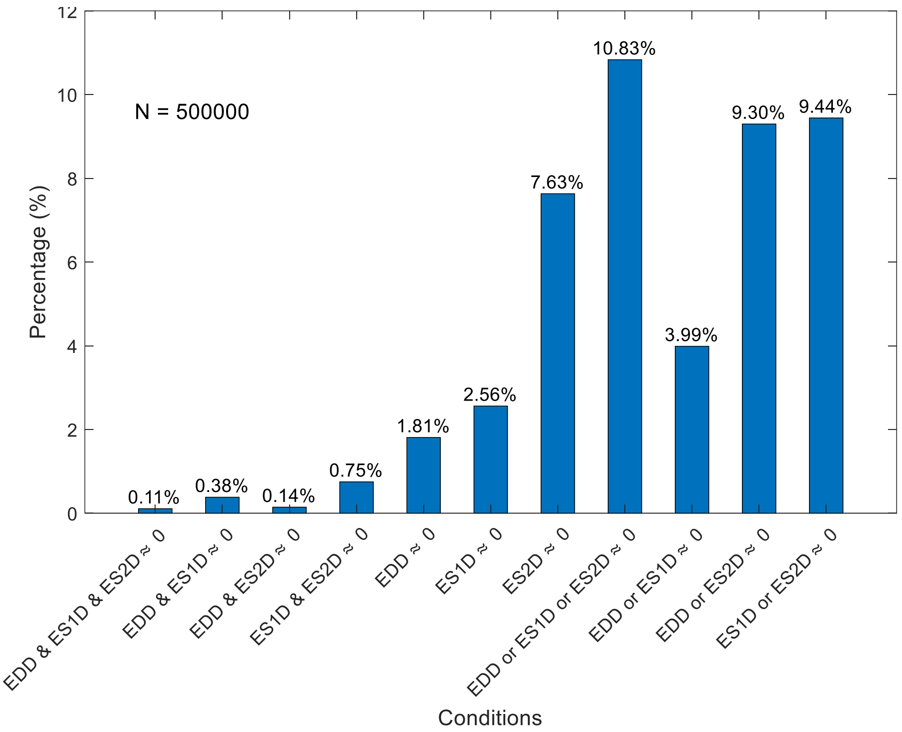

Figure 16: Bar chart of the probability of occurrence of zero for body diagonal errors under different conditions computed from 500k simulations

Comparative plots of the main effect of E_{DD}, E_{S1D} and E_{S2D} with respect to each individual error motion of the three linear axes, X, Y, and Z are shown in Figure 17. This figure shows that in all situations, the mean of E_{DD} is higher compared to the mean of E_{S1D} and E_{S2D}. However, its sensitivity to the changes of the value/profile of each error motion is always less than that of one of the diagonal straightness errors, E_{S1D} or E_{S2D}. This means that the sensitivity of response E_{DD} is consistently of second order for all factors (error motions). In other words, the sensitivity of either E_{S1D} or E_{S2D} is always higher than that of E_{DD}. The symmetry observed between the effects of 0.05PCyc and 0.05NCyc across all error motions suggests that the direction of cyclic errors (positive or negative) has a uniform impact on E_{DD}, E_{S1D}, and E_{S2D}. The same symmetry is also observed between 0.05PPrg and 0.05NPrg. Furthermore, 0.05PHmp and 0.05NHmp show a symmetrically uniform effect on all three responses, E_{DD}, E_{S1D}, and E_{S2D}.

Except all three error motions along Z-axis, EZX, EZY, and EZZ, which have no significant effect on E_{S2D}, all error motions have a significant effect on all three responses, E_{DD}, E_{S1D}, and E_{S2D}.

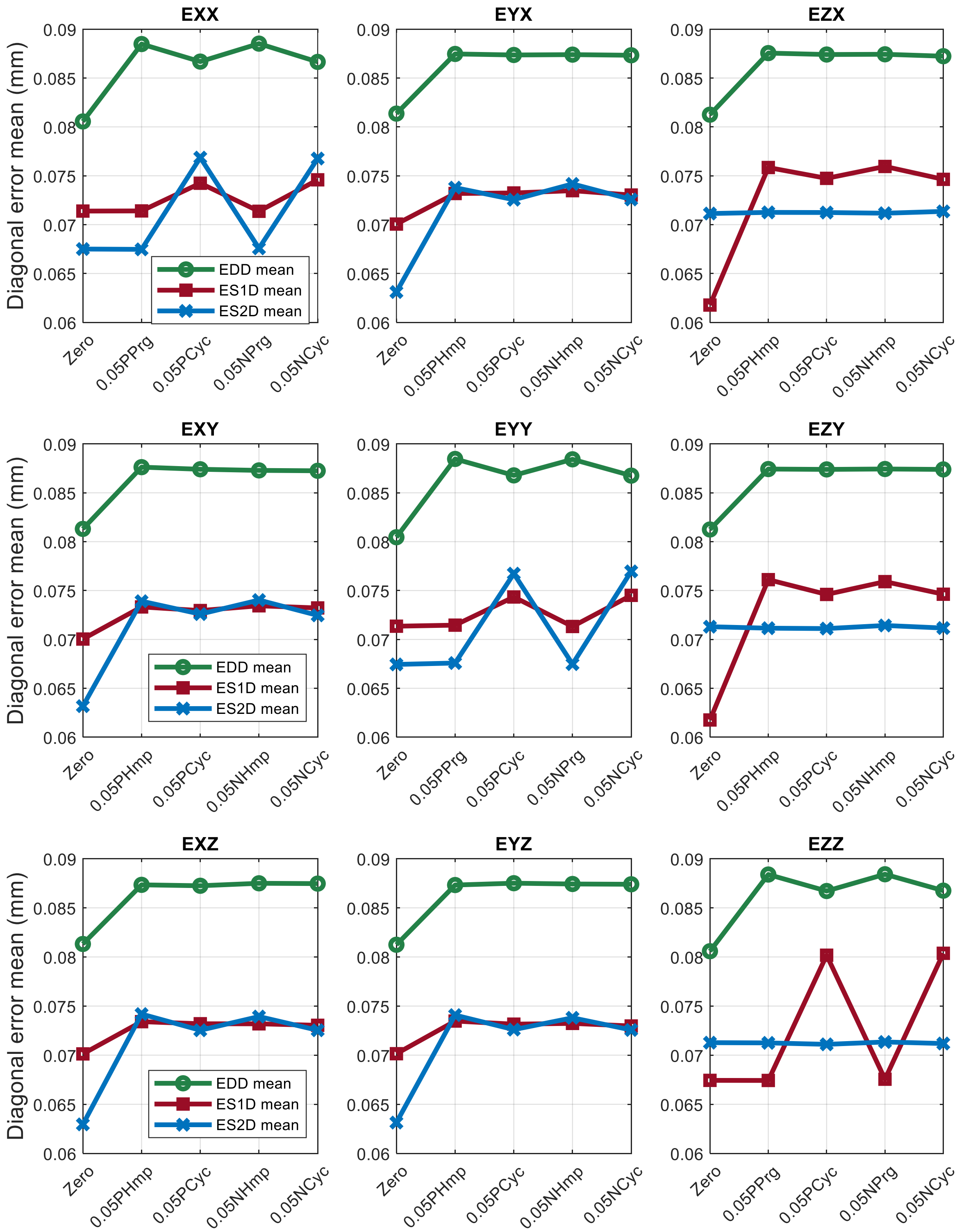

Figure 17: Comparative main effect plot of E_{DD}, E_{S1D}, and E_{S2D} computed from 500k simulations for all studied nine error motions

5 Practical aspects of diagonal tests

To investigate the practicality of diagonal tests and to check the ease of setup along body diagonal trajectories, approximately 600 tests were carried out at various spatial angles to generate different diagonal trajectories on a 3-axis vertical machining centre. The tests were conducted with the Renishaw XM-60 laser system, which enables direct measurement of diagonal positioning and both diagonal straightness errors. Figure 18 shows the body diagonal test setup using a simple angle plate to orient the laser's launch unit along the desired angle. Furthermore, two dial gauge arms were employed to align the receiver unit to the same angle along the pre-programmed diagonal trajectory on the machine's controller to make the setup very efficient.

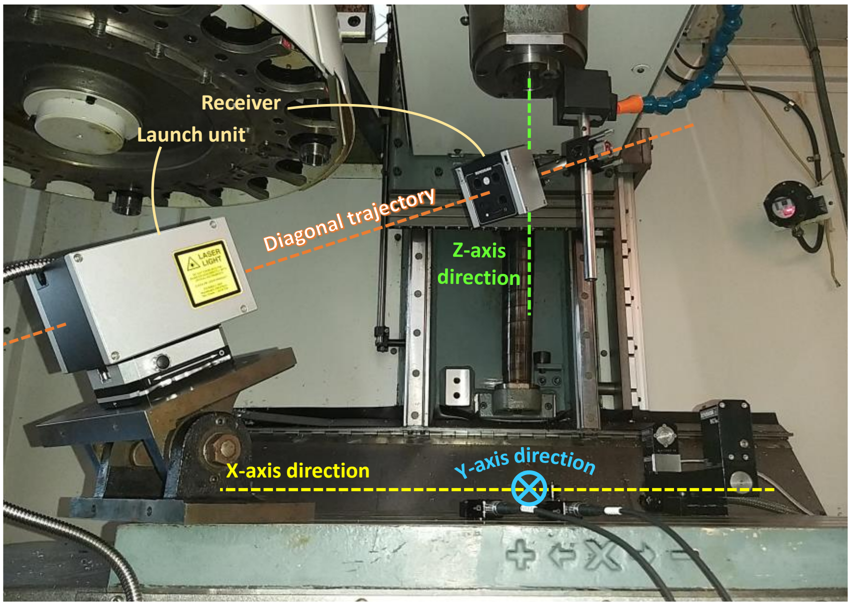

Figure 18: Practical test setup on Cincinnati 3-axis VMC

As previously explained [10], the current CARTO software developed by Renishaw does not fully support the notations according to ISO 230-6. However, by properly defining the direction of diagonal displacement (D) and the other two straightness directions (S1 and S2) as X, Y, and Z respectively, it is possible to directly obtain the measurement results for diagonal tests for both body or face diagonal trajectories.

These tests were conducted using the Renishaw laser system due to its availability. However, they could equally be carried out using other commercially available measuring instruments capable of simultaneously measuring diagonal positioning and both diagonal straightness errors. For measuring diagonal straightness specifically, other measuring instruments based on interferometry or Position Sensitive Detection (PSD) can also be used. The Instrumental uncertainty of some of these devices indicates that they are suitable for measuring diagonal straightness in the machine tools industry. Generally, the face and body diagonal straightness error of a machine tool should be larger than the straightness of a single axis of a machine tool; therefore, the same optical measuring system can be used for diagonal measurements as well.

6 Summary and conclusion

This paper conceptually explained why it is necessary to measure diagonal straightness errors along with the diagonal positioning error to achieve a comprehensive picture of the volumetric performance of a machine tool along its body diagonal trajectory. It also numerically presented how the terms related to diagonal straightness are derived. Furthermore, it statistically investigated the sensitivity of diagonal errors to each linear error motions. The findings revealed that either E_{S1D} or E_{S2D} is the most sensitive diagonal error with respect to each error motion. From practical point of view, it was demonstrated through near 600 tests over 7 working days that aligning laser system for body diagonal straightness measurements is fairly straightforward and can be achieved utilising a classic mechanical angle plate.

In near future, a comprehensive analysis of the practical body diagonal tests will be presented along with guidance on how to select the suitable direction of testing based on Pythagorean quadruples to obtain more detailed information about the volumetric performance of a machine tool.

Acknowledgements

The authors gratefully acknowledge the UK's Engineering and Physical Sciences Research Council (EPSRC) funding of the Future Metrology Hub (Grant Ref: EP/P006930/1) and UKRI-funded Advanced Machinery and Productivity Initiative (84646).

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