

## Modal Identification for Multivariable Motion Systems\*

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### Abstract

Accurate parametric identification of multi-input multi-output (MIMO) models is vital for driving the future development of advanced mechatronic motion systems. It is essential that these models accurately reflect the physical properties of the system to generate interpretable, minimal-order models that enable effective control, offer crucial insights for the mechanical design and facilitate monitoring and diagnostics of machine dynamics.

The dynamical behaviour of mechanical motion systems is typically accurately described in a modal modelling framework [1]. However, most state-of-the-art parametric system identification methods do not impose this inherent modal structure. Integrating modal model structures offers substantial benefits, including enhanced computational efficiency, improved statistical optimality, and interpretability [2]. Recent advancements have enabled the estimation of modal structures in single-input single-output (SISO) systems [3], where the identified parameters have a direct interpretation of physical quantities, including resonance frequencies  $\omega_i$  and damping ratios  $\zeta_i$ . State-of-the-art advanced precision motion systems are generally MIMO, hence also require a parameterization of the mode shapes  $\phi_{l,i}$ ,  $\phi_{r,i}$ . This research extends these methodologies to the MIMO setting, where the model is parameterized as either a generally damped modal system or a proportionally damped modal system, i.e.,

$$G(s) = \sum_{i=1}^{n_{\text{rbm}}} \frac{\phi_{l,i} \phi_{r,i}^T}{s^2} + \sum_{i=1}^{n_{\text{flex}}} \frac{\phi_{l,i} \phi_{r,i}^T}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}.$$

To estimate these parameters, we present a non-linear estimation problem that is solved iteratively. The key aspect of this algorithm is the use of Refined Instrumental Variables (RIV), which ensures optimal statistical properties and robust performance. Additionally, the algorithm can process both time-domain and frequency-domain datasets, which extends its applicability across a broad range of identification scenarios.

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The developed methodology is experimentally validated on a next-generation wafer stage setup (Figure 1), which features four sensors and thirteen actuators operating in the out-of-plane direction. A 40<sup>th</sup> order model is estimated consisting of three rigid-body modes and seventeen proportionally damped flexible modes. The identified modal model together with the frequency-response measurement are presented in Figure 2.

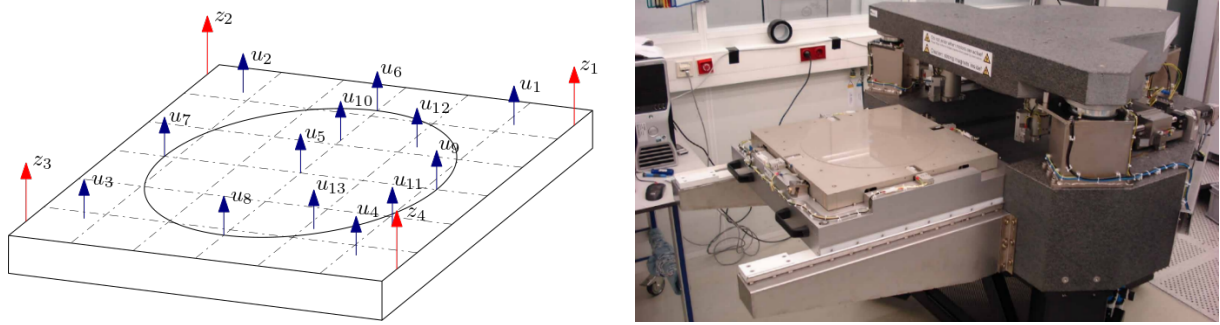


Figure 1: Schematic overview of the featured actuators  $u_i$  and sensors  $z_i$  (left) and over-actuated test rig (right).

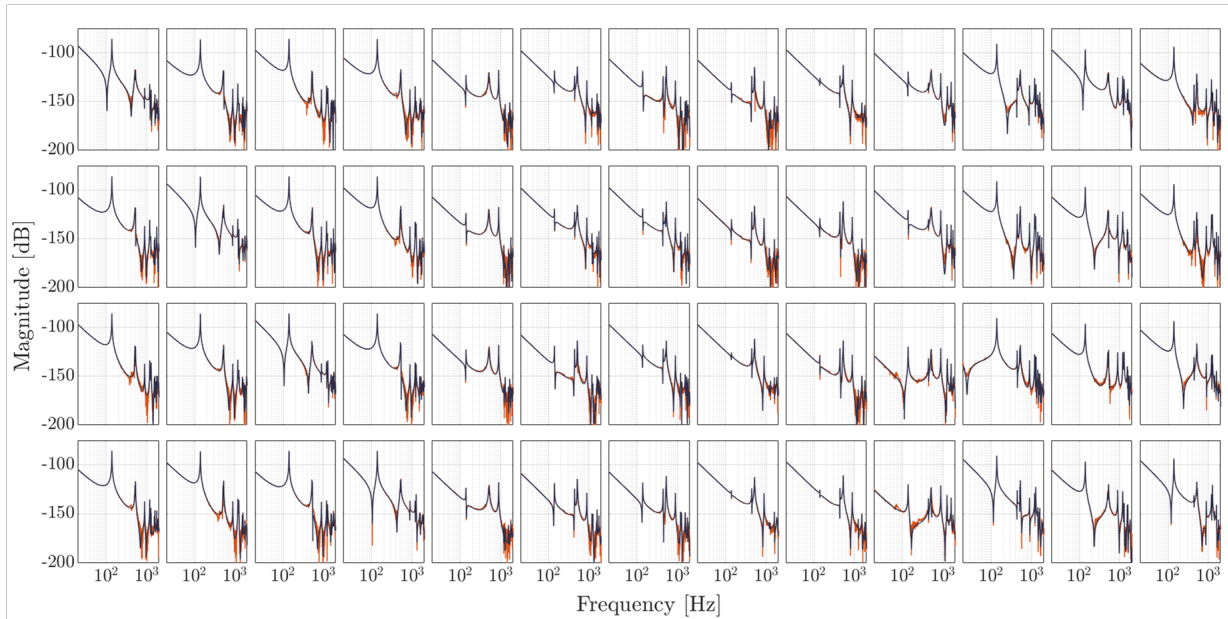


Figure 2: Identified parametric modal model (—) and the frequency-response measurement (—) from the 13 actuators  $u_i$  to the four sensors  $z_i$ . A 40<sup>th</sup> order parametric model is estimated in a modal structure, explicitly described by the eigenfrequencies, damping-ratios, and mode shapes. The close agreement with the frequency-response up till 2 [kHz] showcases the accuracy and effectiveness of the introduced method.

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- [3] R. A. González, K. Classens, C. R. Rojas, J. S. Welsh and T. Oomen, "Identification of additive continuous-time systems in open and closed-loop," arXiv: 2401.01263, 2024.