Mixed H_2/H_∞ Controller Optimization for a Graviational Wave Telescope

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Introduction

Gravitational waves (GW) are ripples in space-time, induced by interactions between large interstellar objects. The information that these signals carry can lead to new insights about the universe and our understanding of physics (Schutz, 1999). The detection of gravitational waves is not trivial and is done by telescopes such as LIGO (Aasi & et.al., 2015), VIRGO (Acernese & et.al., 2004) and in the future the Einstein Telescope. All these telescopes rely on a similar operating principle, namely that of a laser interferometer. Such an interferometer measures the presence of a GW by sending a laser through two long perpendicular arms that generally have a length in the order of a few kilometers. The lasers are reflected by mirrors at the end of the arms, and interfere at the base of the telescope. The measured interference pattern can potentially be used to observe a GW passing by through earth, since a GW disturbs the space-time and thus the lasers, which on its turn induces non-destructive interference between the reflected laser beams.

The disturbance induced by the GW is very small and lies below the noise floor caused by the seismic activity of the earths surface. The mirrors should hence be isolated from the earths surface by a stacked combination of active and passive vibration isolation systems. Typically, the controllers for these type of suspensions are designed via classical methods, such as loop-shaping, which can be time consuming and require manual tuning when sensors, actuators or the telescopes noise budget changes. In this paper, we combine optimal control methods and Dynamic Error Budgetting (Jabben, 2007) techniques to aid controller design for a payload suspension by utilizing a method that is more flexible to cope with possible changes in requirements or the system dynamics. This allows for quick evaluation of the potential performance of a specific telescope configuration, without relying on the experience of the designer.

Problem Formulation

Optical resonators, or Fabry-Perot cavities, that resonate the light are installed in the arms. The resonators increase the effective travelling distance of the laser, which increases the telescopes sensitivity. The mirrors that make up these resonators manipulate the direction of the laser beams and are isolated by multiple vibration isolation stages, to limit the influence of seismic noise up to the required sensitive range of the telescope. The vibration isolation system typically consists of an active isolation platform, which compensates for a part of the seismic disturbance. The isolation platform is followed by a set of consecutive pendulums, that are suspended from a large inverted pendulum that is connected to the platform (Bersanetti & et.al., 2021). The first set acts as a passive filter. The pendulums after the passive stage make up the payload suspension. The payload suspension is an actively controlled multi-pendulum system that compensates for any residual noise, based on the actual output that is measured by the photodiode at the base of the telescope. Figure 1 shows a simplified ideal physisical model of a 3DoF payload suspension that is actuated at the two upper stages, together with a schematic representation of the laser interferometer. The mirror stage is not actuated, since it is expected that the last stage does not provide sufficient roll-off, such that the DAC noise disturbs the telescope output beyond acceptable limits. The mass m_3 denotes the mirror stage of the pendulum. The interference of the laser is measured by a photodiode at the base of the arms.

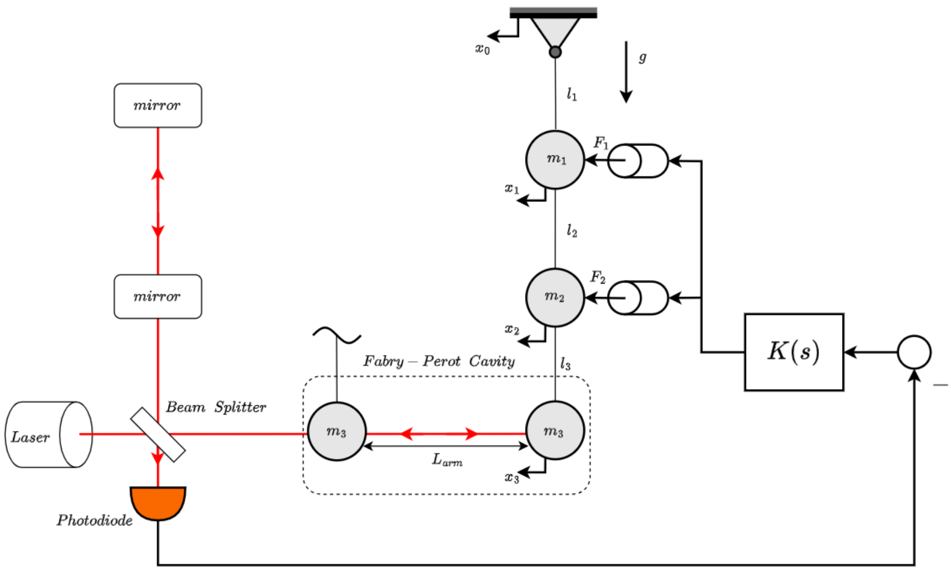

Figure 1: Schematic overview of the laser interferometer, including an IPM of the suspension.

Let $G(s)$ denote the transfer function of the suspension and $K(s)$ that of the controller. The actuators are modelled by gains K_a that represent the sizing of the actuators and are considered as a part of the optimization, such that the total open-loop plant is $P(s) = G(s)K_a$. Noise sources are often expressed as their open-loop equivalent, which is a common expression within gravitational wave disciplines. This open-loop equivalence concept is a result of how a GW is reconstructed from signals in the loop. To illustrate this, consider the feedback loop from the block-diagram that is shown in Figure 2. There are three inputs in the block diagram: the DAC noise u_d, the seismic disturbance x_d and a gravitational wave signal f_{GW} that enters the control loop similarly to the seismic noise, after the plant output[1]. The DAC noise u_d enters the feedback loop between the controller and the open-loop plant $P(s)$. Reconstructing a GW from signals in the loop conceptually yields the following:

$$\hat{x}(s) = \left(I + P(s)K(s)\right)^{-1} P(s)u_d(s) + (I + P(s)K(s))^{-1}(x_d(s) + f_{GW}(s)) \tag{1}$$

$$f_{GW}(s) = (I + P(s)K(s))\hat{x}(s) - P(s)u_d(s) - x_d(s) \tag{2}$$

From the last equation, we can see that the open-loop equivalent DAC noise is $P(s)u_d(s)$. The main aim of the control system is to attenuate the seismic disturbance, such that the Fabry-Perot cavity is stabilized. To achieve this, the RMS of the closed-loop seismic noise should be reduced to a value of less then $1 \cdot 10^{-13} m$. The actuators introduce DAC noise, which should not exceed the telescope's noise budget design. The sensitive range of the telescope is expressed as a noise budget, which is an open-loop equivalent spectrum that indicates the sensitive operating range of the telescope and may not be exceeded by any noise source (Moore, Cole, & Berry, 2014). Therefore, a limit must also be imposed on the open-loop equivalent DAC noise.

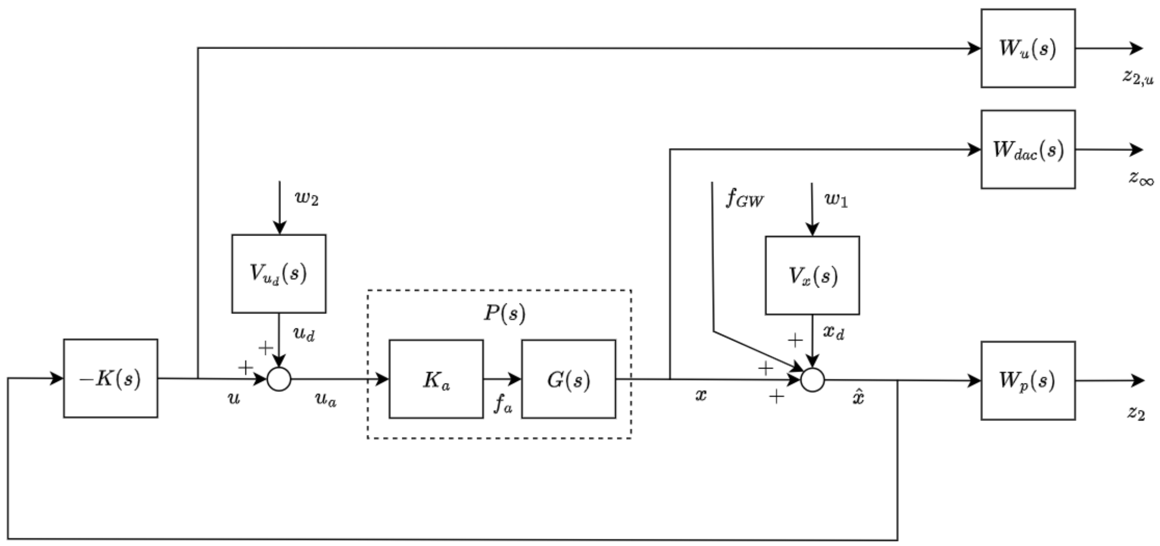

Figure 2: block diagram of a standard feedback loop including the suspension and controller.

Optimization

The attenuation of the seismic disturbances is an energy based minimization. This can be interpreted as an H_2-optimal control problem, if the input signals are unit intensity white noise signals. Coloured noise signals that represent real disturbances can be included by adding input weighting filters $V(s)$ that contains models for the root of the actual noise spectra (Spanjer, Köroğlu, & Hakvoort, 2023).

[1] Note that this modelling choice implies that the suspension dynamics are part of the filter $V_x(s)$ that models the effect of the seismic noise at the mirror output, hence there exists a difference between x_0 at the base of the suspension and the disturbance x_d at the mirror.

Since the open-loop equivalence of the DAC noise should not exceed the noise budget design of the telescope, this requirement is captured best by an H_∞-constraint on the open-loop equivalent spectrum. Since an H_2-optimal controller generally results in a system that is marginally stable, an additional H_∞-constraint is included to guarantee sufficient margin against process variations. An H_2-constraint on the controller command $u(s)$ regulates this signal to a reasonable range for the DAC. Let $F_l(s)$ denote the linear fractional transformation of $G(s)$,$K(s)$ and K_a which is the closed loop that maps $\tilde{\boldsymbol{w}} \rightarrow \tilde{\boldsymbol{z}}$, with $\tilde{\boldsymbol{w}} = \left[u_{d,}\ x_d\right]^T$. $\tilde{\boldsymbol{z}} = [u, x, \hat{x}]^T$. The optimization is then mathematically stated as follows:

$$\Theta = arg\ min \left|W_p(s)F_l(s)V(s)\right|_2 \qquad s.t. \tag{3}$$

$$|W_{dac}(s)P(s)V(s)|_\infty < \gamma_d \qquad |W_{rob}(s)S(s)V(s)|_\infty < \gamma_r \qquad |W_u(s)F_l(s)V(s)|_2 < \gamma_u$$

$$K_{a,l} \leq K_a \leq K_{a,u}$$

Where $W_{dac}(s)$ and $W_{rob}(s)$ are H_∞ related weigting filters regarding the open-loop equivalent DAC noise and robustness constraint respectively. Since the DAC noise should stay below the noise budget design of the telescope, a logical choice for the filter $W_{dac}(s)$ would be the exact inverse of the sensitivity curve, possibly scaled to incorporate some safety margin. The robustness filter constrains the peaks of the sensitivity function $S(s) = (I + PK)^{-1}$, whose inverse is directly related to the modulus margin of the system. $W_{rob}(s)$ should thus be taken as the desired modulus margin. The filters $W_u(s)$ and $W_p(s)$ are filters related to the control signal and seismic disturbance performance outputs respectively. Since the optimizer will minimize the energy of the associated signals, the weighting filters are chosen as gains. The filter $V(s)$ is a diagonal transfer function matrix with models for the DAC noise and seismic noise. Besides the controller, the actuators are also considered as part of the optimization, such that they can be scaled to limit the DAC noise that is amplified by the actuators. These are bounded to a reasonable range, set by the lower limit $K_{a,l}$ and upper limit $K_{a,u}$. The associated cost function is inherently non-convex due to the mixed H_2/H_∞-optimization, hence a non-smooth optimization method (Apkarian & Noll, 2006) is utilized to solve for a suitable controller and actuator distribution.

Results

The results from the optimization are summarized in Figure 3 and Figure 4. From the left figure, it can be seen that the controller is able to supress the seismic noise close to specifications, with a closed-loop RMS of about $5.89 \cdot 10^{-13} m$. Actuation at only the two top stages is therefore not able to satisfy the required attentuation of the seismic noise. The figure on the right shows the open-loop equivalent DAC noise. Around 2Hz, the open-loop DAC noise touches the inverse of the weighting filter, indicating that the optimizer is able to size the actuators to meet the DAC noise requirement well, with $\gamma_d \approx 1$.

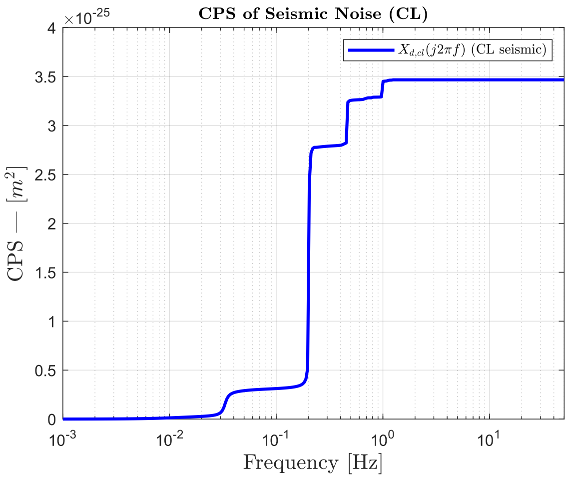

Figure 3: CPS of the closed-loop seismic noise.

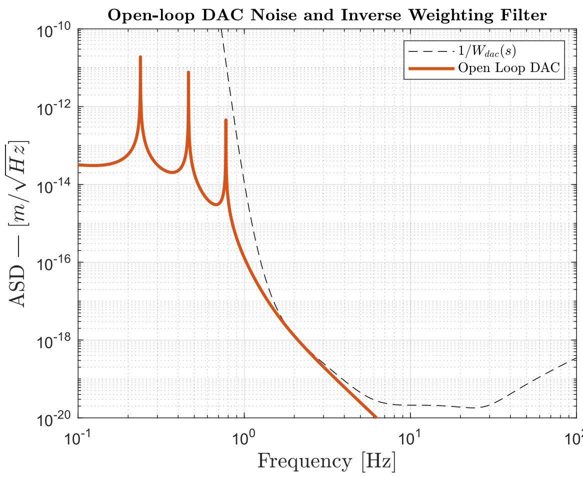

Figure 4: Open-loop equivalence of the DAC noise.

Finally, from the Nyquist plot that is shown in Figure 5, we can see from the green circle around the critical point that the robustness constraint allows to enforce the desired modulus margin of 0.1, as dictated by the robustness weighting filter. The robustness margin is allowed to be small, since the dynamics of the suspension can be identified with very high accuracy. The operating conditions are generally also very constant for this type of application.

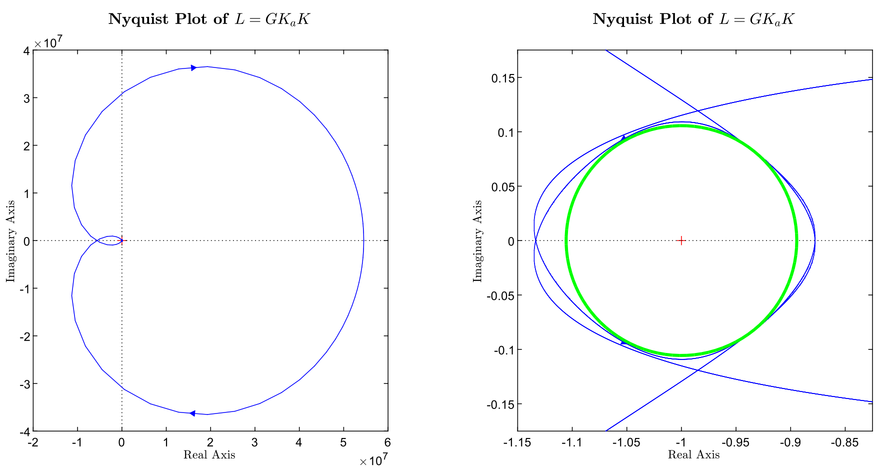

Figure 5: Nyquist plot of the loop gain and zoomed view around the critical point.

Conclusion

Summarizing, the problem formulation for the last active vibration isolation stage of the mirror suspension of a gravitational wave telescope can be casted in to a mixed H_2/H_∞-control problem. A non-smooth optimization method was utilized to solve the inherently non-convex problem. The seismic noise is not attentuated according to specifications, hence system configuration should be modified to design a control system that meets the requirements. Since the optimizer allows to tune parametrized dynamic systems, as demonstrated with the actuator distribution, this work can be extended with a simultaneous controller and suspension dynamics design for a more integrated design strategy that possibly improves the overall performance of the control system.

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