# A compliant continuously variable transmission mechanism

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## Abstract

Continuously Variable Transmission (CVT) is widely applicable in several robotic and precision systems. A compliant CVT mechanism is presented here which works based on the warping of twisting beams. It is demonstrated that the twist on one side of a beam can be transmitted by sectional warping and propagating across a rotational constraint in the middle of the beam to produce a reverse twist on the opposite side. As with a lever mechanism, where the ratio of input to output is proportional to the position of the middle hinge, in the proposed compliant CVT the output over input ratio is proportional to the location of the middle constraint. We have demonstrated the idea of this concept and its relation to the twisting beam's warping constant, as well as its functionality for transmission ranges of 1:4 to 4:1 with Finite element analysis (FEA). An average kinematic performance of 93% is revealed for output/input angles when compared to an ideal transmission situation for the whole range of ratios. The presented compliant CVT is monolithic and can be manufactured in tiny sizes, as well as the fact that it is free of backlash and other issues associated with multi-element conventional CVTs. Therefore, it is well suited for precision and small-scale applications.

## Introduction

The majority of known continuously variable transmissions (CVTs) utilize two pulleys with variable diameters coupled to one another as input and output. Srivastava reviewed these belt and chain-based variable transmissions [1].

However, for small-scale applications, such as sensors or micro-robots, these conventional systems are inadequate because of fabrication and assembly problems. Moreover, conventional CVTs operate on the basis of contacts between numerous connected elements, which introduces inherent problems such as backlash. The contact-related errors can reduce the applicability of these conventional variable transmission solutions for precision applications such as MEMS and micro-manufacturing [2].

A compliant transmission mechanism is already capable of resolving several of the concerns listed above. Due to the fact that these mechanisms operate via element deflection rather than contact, issues such as backlash are not present. Additionally, these systems are predominantly monolithic, light-weight and easily fabricatable on small scales. As a result, these mechanisms can be used to construct devices such as tiny aerial robots [3], watch oscillators [4], and precision systems.

To achieve rotation to rotation transmission, compliant elements such as twisting beams with relatively low torsional stiffness can be used. Earlier, these torsionally compliant twisting beams were employed to morph structure's shape [5], and also to design monolithic revolute joints [6]. However, in these concepts, mainly the bending to torsion stiffness ratio of the beam is of high importance.

To utilize these twisting beams for transmission mechanisms, warping as an intrinsic property of a beam with open thin-walled sections can be used. Warping is usually regarded as an undesirable behavior, and designers try to minimize or avoid it. There are only a few applications, such as adjusting the effective angles of airplane wings [7] that take advantage of warping behavior.

The purpose of this paper is to present a concept for a compliant continuously variable transmission mechanism that operates by twisting torsional beams and transmitting motion via warping. It is demonstrated that the rotation of one beam side can be transmitted by warping along the beam and propagating across a rotational constraint in the middle of the beam to cause the opposite side to rotate in reverse. The ratio of the rotation angles at the input and output sides is dependent on the location of the intermediate rotational constraint. As a result, it is demonstrated that by repositioning the middle constraint of a twisting beam with a high warping constant, a compliant mechanism with a continuously variable transmission ratio can be created. This concept has been demonstrated and validated using finite element analysis.

## Concept Overview

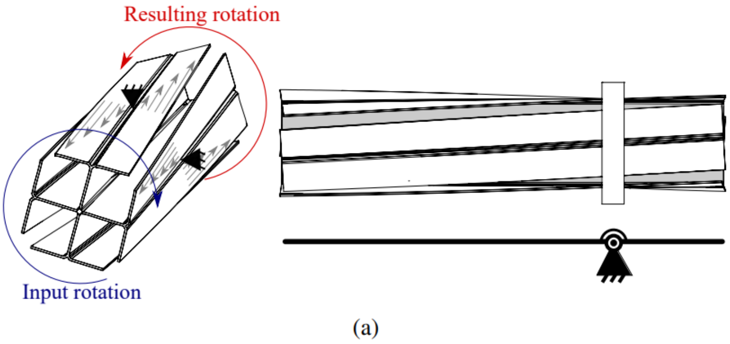
Warping is defined as cross-sections out-of-plane distortion along the direction of the beam's longitudinal axis. This happens when a beam with a high warping constant undergoes torsional deformations due to twisting between an input and a rotational constraint. These longitudinal displacements can propagate along the beam length. Therefore, the rotational constraint perpendicular to the beam's length can not block them. This means that the warping induced displacements can propagate the sectional twist across the rotational constraint. This behavior results in reverse twisting of the output of the beam compared to the input twist. Here, this principle is used as a transmission between input and output. The schematic of the mechanism and the warping-induced displacements are shown in Fig. 1(a).

In a twisting beam, if there are no longitudinal constraints for the warped parts of the section, the beam only undergoes uniform torsion upon a twist at input [8], as shown in Fig. 1(b). This uniform torsion results in a constant twist angle along the whole beam. At the same time, the presence of the prismatic mid-beam rotational constraint (the gray ring) leads to a zero twist angle for this middle section. Therefore, considering the constant twist angle along the beam, the ratio between output twist angle $\theta_{Output}$ (red ring) to input twist angle $\theta_{Input}$ (blue ring) is theoretically equal to the ratio of the distance between mid constraint to output ring $OM$, and mid constraint to input ring $IM$ (see Fig. 1(b)).

$$\frac{\boldsymbol{\theta}_{\mathbf{Output}}}{\boldsymbol{\theta}_{\mathbf{Input}}} = \frac{\boldsymbol{OM}}{\boldsymbol{IM}} \qquad \text{Eq. 1}$$

In an ideal case, changing the mid constraint location directly affects the output to input ratio, exactly like what happens to mechanical advantage when the location of the middle joint of a lever changes. This analogy is illustrated schematically in Fig. 1(a).

In this mechanism, the warping-induced longitudinal displacement of the sections transfers the motion and plays the role of the rigid bar in the lever mechanism.



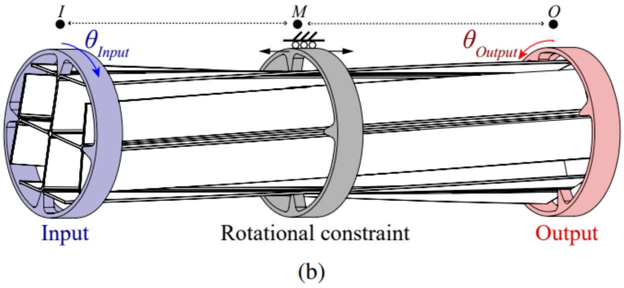


Figure 1: (a) The warping-induced displacements (vectors) play the role of the rigid bar in the lever mechanism and propagate the rotations. The mechanism works like a lever, where the ideal output/input ratio is dependent on the position of the middle pivoting point. (b) Uniform torsion causes a constant twist angle for the beam. The distances between the middle constraints and the beam's sides specify the transmission ratio as indicated in Eq. 1.

## Methods

To realize this concept and to understand the effect of warping on the transmission behavior of the proposed mechanism, three open thin-walled cruciform beams with equal lengths 200 mm and different sectional dimensions are selected. As illustrated in Fig.2, the web sizes of all beams are identical, and the flange widths are selected to have three sizes.

The warping constant of a cross-section is a measure of the effort needed to reduce warping [8]. Similarly, the torsional constant of a section is a measure of the effort needed to make an angle of twist in the beam.

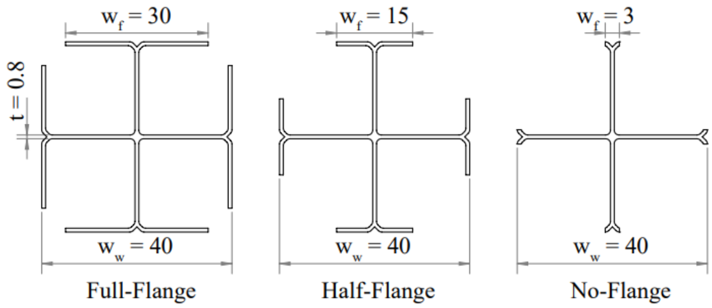


Figure 2: The cross-sectional dimensions of three beams. All dimensions are in (mm).

The warping and torsional constants of these sections as the most important parameters for the proposed concept are shown in Table 1.

Table 1: The St. Venant torsional constant $J$ and warping constant $C_w$ for three beams with full-flange, half-flange, and no-flange.

| | Full-flange | Half-flange | No-flange |
|---|---|---|---|
| $J$ [m$^4$] | 3.4e-11 | 2.4e-11 | 1.5e-11 |
| $C_w$ [m$^6$] | 2.8e-12 | 1.7e-13 | 1.6e-15 |

*Finite Element Modeling (FEM)*

To analyze the output to input ratio for the mentioned beams with the finite element method (FEM), the ANSYS Parametric Design Language (APDL) has been used. The eight-node shell element (shell 281) is selected as the base element and the material properties are set based on additive manufactured Polylactic acid (PLA) [9].

The beam is constrained at five points, one at the middle of the input section with three transnational constraints, and four at the tips of the webs at the position of prismatic middle rotational constraint, where points on horizontal webs are constrained in *Y* translation, and points on vertical webs are constrained in *X* translation.

To realize the input moment, four free forces are applied to the end points of webs at input section. The constraints and forces are shown in Fig.3.

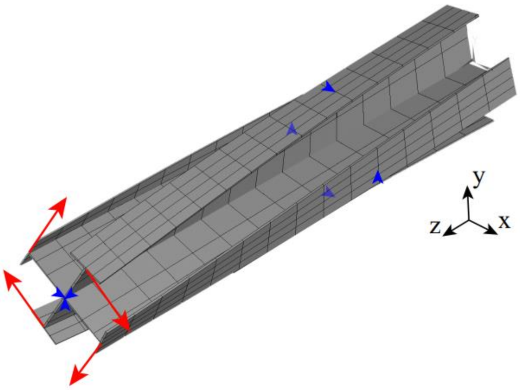
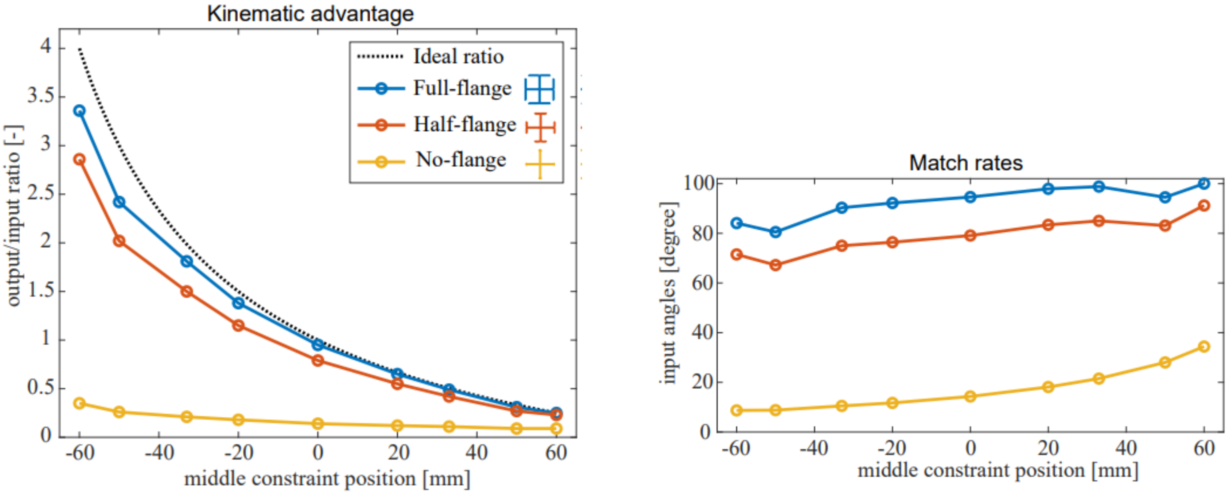


Figure 3: The model used for FEA and the rotational constraints imposed on it. Blue arrows show translation constraints, and red forces are making input moment.

## Results

The output/input rotations are measured in nine different locations of the middle constraint. The resulting ratios are shown for all three beams and with an input moment of 0.2 N.m in Fig. 4(a). The data points are shown with circles and connected to each other with straight lines.

The ideal ratio between input and output from Eq. 1 is also shown with a dotted line to give an impression of the maximum theoretically achievable ratio at each location of the prismatic middle constraint. The match rate between these theoretical ratios and actual ratios from FEM for each location is shown in Fig. 4(b) to indicate the kinematic performance of each beam on transferring rotation.



(a)

(b)

Figure 4: The figures depict data for all three beams. (a) Resulting in output/input ratios for different locations of the rotational constraint from the middle of the beam. The dotted line shows the ideal ratio based on Eq. 1. (b) The match rate with the ideal ratio for different locations of the prismatic rotational constraint.

## Discussion and Conclusion

It is shown that the proposed compliant mechanism can transfer the rotation with continuously variable transmission ratios by the relocation of the prismatic rotational constraint. This design can replace conventional CVTs in different applications, as the beam is lightweight and monolithic and can be manufactured in small scales.

It is shown that warping is a key feature in this transmission mechanism, and warping-induced displacement can transfer the rotation. Thus, only highly warpable beams can have high kinematic performance for transmission. Moreover, it is also shown that the transmission ratio doesn't have a major effect on this behavior and, therefore, on the mechanism's kinematic performance.

An average of 93% of kinematic performance is observed for a non-optimized high warping constant beam (full-flange) for ratios between 1:4 and 4:1.

Both highly warpable beams well followed the ideal line in the whole range of ratios.

This mechanism is presented mainly for applications where kinematics is the main focus. Transferring high moments with this mechanism can change the warping behavior to semi-restrained and lead to a non-uniform twist along the beam. However, it is still possible to analyze this and characterize the transmission behavior even for beams with high output resistance.

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