A particle swarm optimisation-based grey prediction model for thermal error compensation on CNC machine tools

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Abstract

Thermal errors can have a significant effect on CNC machine tool accuracy. The thermal error compensation system has become a cost-effective method of improving machine tool accuracy in recent years. In the presented paper, the Grey relational analysis (GRA) was employed to obtain the similarity degrees between fixed temperature sensors and the thermal response of the CNC machine tool structure. Subsequently, a new Grey model with convolution integral GMC(1, N) is used to design a thermal prediction model. To improve the accuracy of the proposed model, the generation coefficients of GMC(1, N) are calibrated using an adaptive Particle Swarm Optimisation (PSO) algorithm. The results demonstrate good agreement between the experimental and predicted thermal error. Finally, the capabilities and the limitations of the model for thermal error compensation have been discussed.

1 Introduction

Serious attention has been paid to the influence of temperature changes on the accuracy of the CNC machine tools [1-3]. Temperature gradient can be caused by variation of ambient temperature, self-generated heat in machine ball-crews, spindle motors, etc. Thermal errors are yet more complex since they represent a response to the interaction between environmental changes and internally generated heat. There are three primary approaches to mitigate these thermal errors which can be categorised as [4]: elimination or avoidance, reduction of generated heat, and compensation strategies. Elimination or avoidance strategies try to eliminate any change in dimensions due to temperature changes. These strategies are best or, rather, can only be implemented during the design stage of the machine tool. Some examples of these strategies are use of symmetry in machine design, choice of materials and use of direct feedback [4]. Reduction of generating heat strategies tend to directly cool the heat sources, for instance, through on-machine cooling systems. However, the end-user must also be responsible for improved environmental temperature control [2], or good operating practices such as spindle warm up. Compensation approaches tend to compensate for any change in the size and the shape of the machine structure due to temperature gradients. They can be implemented during any stage of the machine tool design. Many compensation techniques have been explored to reduce thermal errors in a direct or indirect way [5].

Numerical techniques such as a finite-element method and finite-difference method [1] are powerful tools in modelling the thermal characteristics. However, building a numerical model can be a great challenge due to problems of establishing the boundary conditions and accurately obtaining the characteristic of heat transfer [6]. Therefore, testing of the machine tool is still required to calibrate the model for successful application of these techniques.

The data driven models are behavioural models that are based on historical data to predict the thermal error of machine tool. Contrary to the numerical models, they are not based on explicit physical equation definitions but on experimental database which is capable of reflecting the relationship between inputs and outputs. Data driven techniques for thermal error modelling can be divided into two categories: statistical techniques such as regression methods, linear polynomial models, etc., and Artificial Intelligence (AI) techniques such as artificial neural networks (ANNs), fuzzy systems, etc.

Abdulshahed *et al*. [7] employed an adaptive neuro fuzzy inference system (ANFIS) to forecast thermal error compensation on CNC machine tools. Two types of ANFIS model were built in this paper: using grid-partitioning and using fuzzy c-means clustering. According to the results, the ANFIS with fuzzy c-means clustering produced better results, achieving up to 94 % improvement in error with a maximum residual error of ± 4 μm. In another work [8] they built a thermal model by integrating ANN and GMC(1, N) models. The thermal model can predict the Environmental Temperature Variation Error (ETVE) of a machine tool with reduction in error from over 20 μm to better than ± 3 μm.

Nevertheless, robust solution for both principle-based and some of data driven models require the measurement of temperature and related thermal error components that have to be obtained by time-consuming experiments. This is difficult to achieve in a working machine shop, because of the prohibitively costly downtime required to conduct the experiments.

Appropriate selection of input variables is an important task in modelling. In fact, not all input variables are equally important; some may have no significant effect on the system being modelled. There are many approaches which have been proposed to reduce the number of sensors. Abdulshahed *et al*. [9] proposed a thermal model merging Grey model GM(0, N) and ANFIS model. A thermal imaging camera was also used to record temperature distributions a across the spindle-carrier structure of the machine tool. Each pixel can be considered as a possible temperature measurement point. The Grey model and fuzzy c-means clustering were applied to minimise the number of temperature points and select the most suitable sensor locations for modelling. Grey relational analysis (GRA) provides an alternative approach to identifying the similarity degree among factors, or to determining the optimal temperature sensors for modelling with less experimental data.

In this paper, the GRA model is used to determine the major sensors influencing thermal errors of a small vertical milling machine (VMC), which is capable of simplifying the system prediction model. A PSO-based Grey prediction model for thermal error compensation is developed by adopting PSO to calibrate GMC(1, N) model. It is then used to predict thermal error on a small VMC on the basis of the selected sensors.

2 Material and methods

2.1 Modelling the thermal error using a Grey model

The Grey systems theory, established by Deng in [10], is a methodology that focuses on solving problems involving incomplete information or small samples. The technique can be applied to uncertain systems with partially known information by generating, mining, and extracting useful information from available data so that system behaviours and their hidden laws of evolution can be accurately described. It uses a Black-Grey-White colour to describe complex systems [11]. GM(1, N) is the most widely used implementation in literature [11], which can establish a first-order differential equation featured by comprehensive and dynamic analysis of the relationship between system parameters. The accumulated generating operation (AGO) is the most important characteristic of the Grey system theory, and its benefit is to increase the linear characters and reduce the randomness of the samples. Based on the existing GM(1, N) model, Tien [11] proposed a GMC(1, N) model, which is an improved Grey prediction model. The modelling values by GM(1, N) are corrected by including a convolution integral. Traditionally, these models have been calibrated by the least square method. However, due to the nonlinearity of the problem, the least square solution may not provide a satisfactory solution.

2.2 The particle swarm optimization (PSO)

The particle swarm optimization (PSO) algorithm was introduced by Eberhart *et al*. [12] as an alternative to other evolutionary techniques. The PSO algorithm is inspired by the behaviours of natural swarms, such as the formation of flocks of birds and schools of fish. The advantages of the PSO algorithm is that it does not require the objective function to be differentiable as in the gradient decent method, which makes few assumptions about the problem to be solved. Furthermore, it has a simple structure and its optimisation method illustrates a clear physical meaning. PSO consists of a population formed by individuals called particles, where each one represents a possible solution of the problem. Each particle tries to search the best position with time in D-dimensional space (solution space). During flight or swim, each particle adjusts its ”flying” or “swimming” in light of its own experience and its companions’ experience, including the current position, velocity and the best previous position experienced by itself and its companions. Therefore, instead of using the standard algorithms, a PSO algorithm is employed to optimise the Grey model parameters.

2.3 GMC (1, N) and its learning algorithm

In this section, we illustrate the main steps of GMC(1, N) and discuss its learning algorithm using PSO. The model can reveal the long-term trend of data and, by driving the model by the AGO, rather than raw data, can minimise the effect of some of the random occurrences. Therefore, the first step for building GMC(1, N) is to carry out 1-AGO (first-order Accumulated Generating Operation) to the data, so as to increase the linear characteristics and reduce the randomness from the measuring samples. PSO algorithm, with capability to optimise complex numerical functions, is adopted to calibrate the GMC(1, N) model. Finally, an IAGO (inverse Accumulated Generating Operation) is performed to predict the thermal error and generate the final compensation values. The modelling detail is described as follows:

Step 1: Consider the original data series as:
$X_1^{(0)} = \left\{x_1^{(0)}(1+r),\ x_1^{(0)}(2+r), \dots, x_1^{(0)}(n+r)\right\}$, and
$X_i^{(0)} = \left\{x_i^{(0)}(1), x_i^{(0)}(2), \dots, x_i^{(0)}(n), \dots, x_i^{(0)}(n+m)\right\}$, where $i = 2,3,\dots,N$, *r* is the period of delay, *n* gives the length of original data series and *m* denotes the number of entries to be predicted.

Step 2: The above sequences of each variable are processed using 1-AGO to obtain the 1st-order AGO sequences as follows:
$X_1^{(1)} = \left\{x_1^{(1)}(1+r), x_1^{(1)}(2+r), \dots, x_1^{(1)}(n+r)\right\}$, and
$X_i^{(1)} = \left\{x_i^{(1)}(1), x_i^{(1)}(2), \dots, x_i^{(1)}(n), \dots, x_i^{(1)}(n+m)\right\}$,
where $X^{(1)} = \sum_{j=1}^{t} x^{(0)}(j),\ t = 1,2,\dots,n+m.$

Since the details of GMC(1, N) can be found in [11], this paper only briefly mentions the core equations of this method.

$$\frac{dX_1^{(1)}(t+r)}{dt} + b_1 X_1^{(1)}(t+r) = b_2 X_2^{(1)}(t) + b_3 X_3^{(1)}(t) + \dots + b_N X_N^{(1)}(t) + u, \quad (1)$$

where $t = 1,2,\dots,n+m$, b_1 is the development coefficient, $b_i, (i = 2,3,\dots,N)$ the driving coefficient, and u is the Grey control parameter. Therefore, time response sequences can be obtained.

$$\hat{X}_1^{(1)}(t+r) = x_1^{(0)}(1+r)e^{-b_1(t-1)} + \frac{1}{2} \times e^{-b_1(t-1)} \times f(1) + \sum_{\tau=2}^{t-1}\left[e^{-b_1(t-\tau)} \times f(\tau)\right] + \frac{1}{2} \times f(\tau), \quad \text{where } f(\tau) = \sum_{j=2}^{N} b_j X_j^{(1)}(\tau) + u. \quad (2)$$

To calculate the coefficients b_1, b_i and u, the PSO can be used to calibrate the equation (2). Then, the Grey model is optimised until the performance is satisfactory. Finally, the optimal corresponding coefficients are used as the Grey model coefficients to predict the thermal error. The calibrating process of GMC(1, N) can be summarised as follows:

In PSO algorithm, a particle refers to a coefficient in the model that changes its position from one move to another based on velocity updates. The mathematical description of the PSO algorithm is as follows: suppose that the search space is D-dimensional, and then the current position and velocity of the *i*th particle can be represented by $B_i = [b_{i1}, b_{i2}, \ldots, b_{iD}]^T$ and $V_i = [v_{i1}, v_{i2}, \ldots, v_{iD}]^T$ respectively, where $i = 1, 2, \ldots, M$ and M is the number of particles in the swarm.

Particle *i* can remember the best position so far, which is known as the local best position $Pbest_i = [pbest_1, pbest_2, \ldots, pbest_{iD}]^T$. It can also obtain the best position that the whole swarm establish, known as the global best position $\mathrm{G}best_i = [\mathrm{g}best_1, \mathrm{g}best_2, \ldots, \mathrm{g}best_{iD}]^T$. The first position and velocity of Particle *i* are randomly initialised by the uniformly distributed variables. Afterwards, particle *i* adjusts its velocity of iteration *k+1* according to the local and global best positions, as well as the velocity and position of iteration *k*, as follows:

$$V_i(k+1) = \omega V_i(k) + c_1 R\big(Pbest_i(k) - B_i(k)\big) + c_2 R\big(\mathrm{G}best_i(k) - B_i(k)\big) \quad (3)$$

where ω is the inertia factor which is used to manipulate the impact of the previous velocities on the current velocity, c_1 and c_2 are the self-confidence factor and the swarm-confidence factor, respectively. R is a uniformly distributed random real number that can take any values between 0 and 1. With the updated velocity, the position of particle *i* in the iteration *k+1* can be obtained as follows:

$$B_i(k+1) = B_i(k) + V_i(k+1) \quad (4)$$

The fitness of particle is measured using a fitness function that quantifies the distance between the particle and its optimal solution as follows:

$$f(B_i) = \sum_{k=1}^{N} \left[\hat{x}^{(0)}(k) - x^{(0)}(k)\right],^2$$

where f is the fitness value, $\hat{x}^{(0)}(k)$ is the target output; and, $x^{(0)}(k)$ is the predicted output based on model parameters (particles) updating.

Step 3: Update the velocity and position of each particle based on equations (3) and (4). Adjusting the model parameters in equation (2):

Step 4: If the value of the error meets the requirement of the model, or a pre-determined number of epochs are passed, then the model calibration will end if not, then return to Step 3.

Step 5: Export the optimal solution B_i.

Step 6: 1- IAGO can be applied to obtain the predicted values. The mathematical expression is as the following:

$$\hat{x}_1^{(0)}(t+r) = \hat{x}_1^{(1)}(t+r) - \hat{x}_1^{(1)}(t-1+r), \text{ and } \hat{x}_1^{(0)}(1+r) = \hat{x}_1^{(1)}(1+r).$$

2.4 Grey relational analysis method

Grey relational analysis (GRA) is a method to capture the correlations between the reference factor and other compared factors of a system with a relatively small amount of data [11]. On the basis of Deng's initial models of grey incidences, Liu *et al*. [13] proposed a new type of GRA model to investigate the closeness of connection between sequences using the geometric shapes of the sequences. The GRA model can be summarised as follows:

Step 1: Assume sequences:

$X_i = \left(x_i(1), x_i(2), x_i(3), \ldots x_i(n)\right)$, is a sequence of data representing a system's characteristics, and,

$X_j = \left(x_j(1), x_j(2), x_j(3), \ldots x_j(n)\right)$, is a sequence of relevant factor.

Step 2: The initial point zeroing images are:

$$X_i^0 = \left(x_i^0(1), x_i^0(2), x_i^0(3), \ldots x_i^0(n)\right),$$

$$X_j^0 = \left(x_j^0(1), x_j^0(2), x_j^0(3), \ldots x_j^0(n)\right),$$

where, $x_i^0(k) = x_i(k) - x_i(1),\; x_j^0(k) = x_j(k) - x_j(1),\; k = 1,2, \ldots, n.$

Step 3: The grey similitude degree is calculated as follows:

$$\varepsilon_{ij} = \frac{1}{1 + \left|s_i - s_j\right|}, \qquad \text{where } s_i - s_j = \int_1^n \left(X_i^0 - X_j^0\right) dt.$$

The similitude degree of the GRA model is used to measure the geometrical shape similarity between sequence X_i and X_j. The ε_{ij} is called the similitude degree of X_j with respect to X_i. According to the above equations, the similitude degree ε_{ij} between thermal error of CNC machine tool and the various temperature sensors can be calculated. The bigger ε_{ij} the greater impact on thermal error and on the contrary the smaller ε_{ij}.

3 Experimental work

To verify the applicability of the proposed model, an example simulating the machining of six parts is investigated. The experiments were performed on a small vertical milling centre (VMC) and utilised a Renishaw OMP40-2 spindle-mounted probe to monitor distortion. It has a stated unidirectional repeatability of 1.0 μm at 480 mm/min with a 50 mm stylus. The test consists of simulating the machining of six parts which are machined individually at a datum point on the table. When a part is finished the table moves to the next datum point to start machining the next part. Each part excites the X, Y and Z axes simulate milling operations. This allows heat to be generated from spindle, motors and axes movement. A probing routine is run before the first machining operation to create a datum baseline for the test on four corners of granite square (see Figure 1). Probing routines are run after the third part and sixth part to measure the drift of the tool in the X, Y and Z axes. The thermal data were measured using twenty eight temperature sensors placed in strips at the carrier, spindle boss, axes motors, axes ballscrews nut, and ambient temperature sensors were placed around the machine to pick up the ambient temperature. A general overview of the experimental setup is shown in Figure 1.

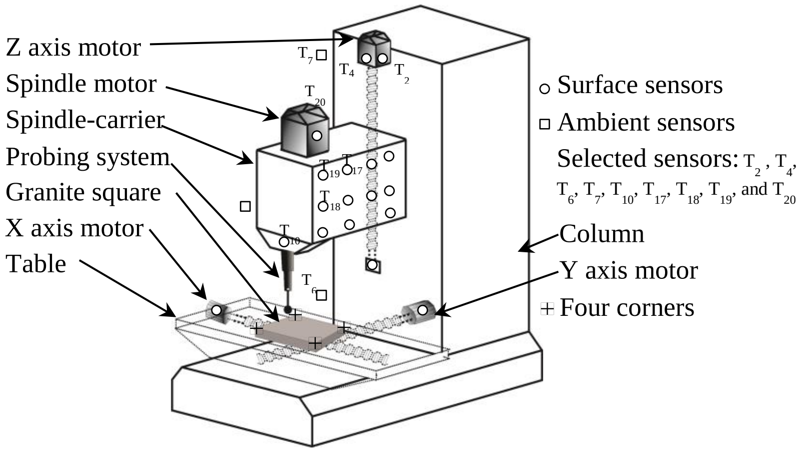

Figure 1. A general overview of the experimental setup.

The machine was examined by running the spindle at a speed of 9 000 rpm (except for the periods of probing), and a feedrate of 5 000 mm/min for 200 minutes to excite the thermal behaviour. The high rotational speed brings a larger thermal displacement for the spindle carrier. Moreover, the higher feedrate generates larger frictional heat at the interface points, and the motor temperature also increases with the higher feedrate. Temperature of measured points grows gradually until the equilibrium state is reached. The temperature sensors were measured simultaneously every 10 seconds. The maximum drift of the X-axis is 20 μm, the Y-axis is 18 μm, and the Z-axis is 58 μm. In this paper, the thermal drift of the Z-axis was investigated as an example for the modelling, and potential error compensation.

The representative temperature sensors for modelling were selected from each group (Surface sensors and ambient sensors) according to their influence coefficient value using GRA model, more details about similar Grey model is given in our work [7, 9]. The representative thermal sensors T10, T20, T2, T19, T4, T18, T17, T7, and T6, which are located on the spindle boss, spindle motor axes motors, carrier, and ambient, are selected as the thermal key sensors for modelling. The similitude degree of these temperature sensors are: $\varepsilon_{1T10} = 0.98$, $\varepsilon_{1T20} = 0.93$, $\varepsilon_{1T2} = 0.85$, $\varepsilon_{1T19} = 0.84$, $\varepsilon_{1T4} = 0.82$, $\varepsilon_{1T18} = 0.82$, $\varepsilon_{1T17} = 0.82$, $\varepsilon_{1T7} = 0.81$, $\varepsilon_{1T7} = 0.55$, and $\varepsilon_{1T6} = 0.5$, respectively.

4 Results and discussion

In order to optimise the GMC(1, N) parameters, the experimental data set was divided into two sets, one is being used for calibrating the model (approximately 10 %), and the rest for testing performance (approximately

90 %). Nine temperature sensors are used as inputs, and Z-axis displacement as output. In the PSO algorithm, the number of the particles is set to be 90 whilst the self-confidence factor and the swarm-confidence factor are $c_1=2$ and $c_2=2$, respectively. The inertia weight ω was taken as an adaptive decreasing function in iteration index k from 0.9 to 0.4. After 100 epochs, the total error was at an acceptable level (3 μm for testing dataset). The Grey model obtained using PSO algorithm is:

$$\frac{dX_1^{(1)}(t)}{dt}+5.31X_1^{(1)}(t)=72.12X_2^{(1)}(t)+61.04X_3^{(1)}(t)-26.07X_4^{(1)}(t)+66.34X_5^{(1)}(t)$$
$$-23.33X_6^{(1)}(t)-31.99X_7^{(1)}(t)+25.73X_8^{(1)}(t)-23.50X_9^{(1)}(t)+7.597X_{10}^{(1)}(t)-54.74.$$

The final GMC(1, 10) model being optimised and validated in this work has been tested next by a new testing dataset, not used during training stage. The individual variables are shown in Figure 2. Simulation results show that the thermal error in the Z direction can be significantly reduced from 58 μm to less than 4 μm using testing dataset (see Figure 3). Furthermore, this result shows that the PSO algorithm can act as an alternative training algorithm for Grey model that can be used for thermal error compensation.

Consequently, this paper develops a simple, less computationally intensive and low-cost approach based on Grey model and PSO algorithm to predict the thermal error compensation on CNC machine tools. In this work, this model has been used for prediction of the thermal error of a relatively simple structure with only a few calibrating samples. However, further work is required to validate the proposed model using disparate cycles on multiple machines.

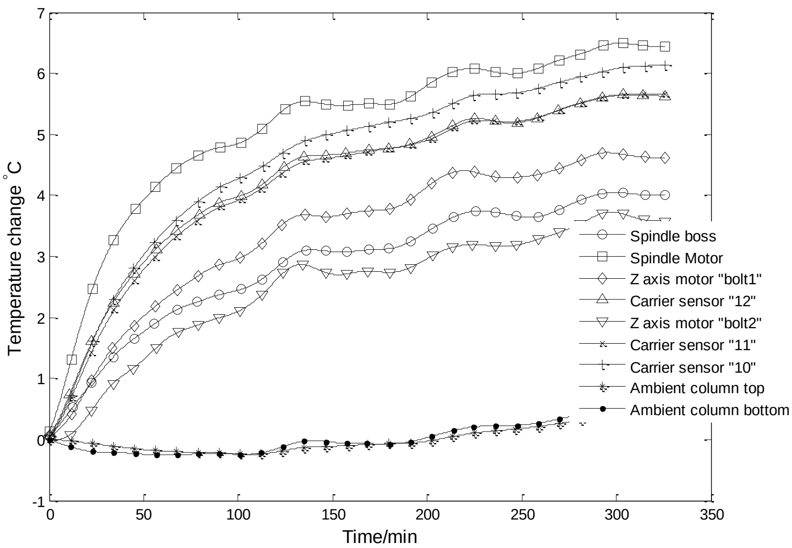

Figure 2. Measured temperature variation (model inputs).

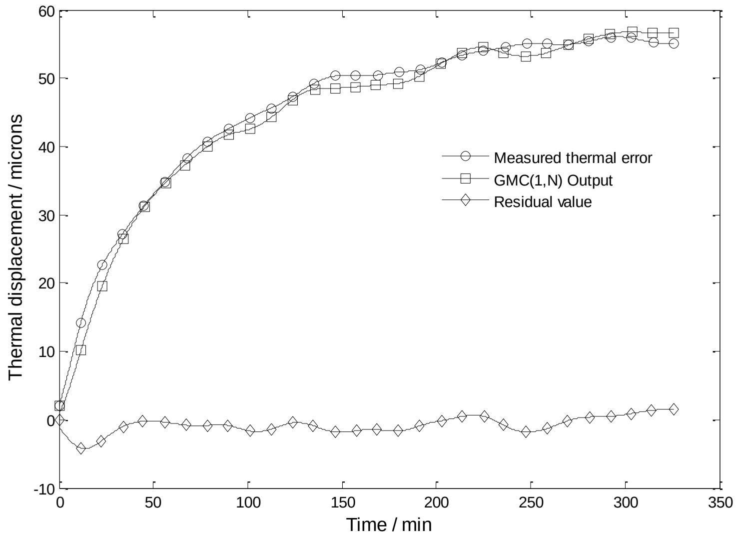

Figure 3. GMC(1, N) model output vs the actual thermal drift.

5 Conclusions

In this paper, a PSO-based Grey prediction model for thermal error compensation of a small vertical milling centre (VMC) is presented. Two main findings have been addressed in this paper. First of all, the optimal temperature sensors were determined through the GRA model. After calculating the similarity degrees between the thermal error and the temperature sensors, one sensor from each group is selected according to its similarity degree with the thermal distortion. The number of required temperature sensors was thus reduced from twenty eight to nine, which significantly minimised the computational time, cost and effect of sensor uncertainty. Secondly, the comparison between experimental results and predicted values of the Grey model show that there is an excellent agreement between the predicted thermal error and the experimental results with residual error of 4 μm. The results of this paper show that the PSO technique can act as an alternative calibration algorithm for Grey models that can be used for thermal error compensation. Further work is required to validate the model using disparate cycles on multiple machines.

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