Application of GUM Supplement 2 to uncertainty estimation of five-axis CNC machines geometric error parameters identification using the SAMBA method

A. Los, J. R. R. Mayer
Department of Mechanical Engineering, Ecole Polytechnique de Montreal, Montreal, Canada

Abstract

This paper investigates the uncertainty estimation of the machine tool identified geometric errors parameters. The analysed quantities are obtained through the simulation of the scale enriched reconfigurable uncalibrated master balls artefact (SAMBA) calibration process. The uncertainties are estimated using two approaches described in the GUM Supplement 2: the generalized uncertainty framework (GUF) and adaptive Monte Carlo method (MCM). The associated coverage factors are calculated. The results obtained by both approaches are compared and examples of estimated probability density plots are presented. The results show that in case of investigated (iterative) measurement the GUF cannot be validated with MCM. However, the latter method gives an opportunity to calculate uncertainty of identified errors.

1 Introduction

Computerized numerical control (CNC) machines are widely used for machining parts, including those with high complexity and quality demands. In order to fulfil those requirements machines have to be checked and calibrated periodically. Various methods have been developed for indirectly measuring and estimating machine parametric errors, such as 2- and 3-D artifacts [1-3] or the ball-bar [4].

In this paper the uncertainty on identified CNC machine geometric errors is investigated through the simulation of SAMBA [5] measurement. Master balls and the scale bar are probed in different indexations of the machine rotary axis. The obtained sets of data are used to identify machine geometric errors parameters using a Jacobian matrix. All the errors are calculated simultaneously so the measurement model is a multi-output function.

Until December 2011 the Guide to the Expression of Uncertainty in Measurement (GUM) [6] was offering methods for calculation of the uncertainty on a single output measurement. Neither the general, nor the MCM, could be applied in accordance with GUM for a multi-output model was in subject. GUM Supplement 2 (GUM S2) [7] gives now the opportunity to calculate the uncertainty for multi-output measurement results with, both GUF and MCM. The latter has been applied by Eichstadt et al. [8] for efficient uncertainty estimation in a challenging case of dynamic measurement. The MCM uncertainty estimation results are compared with two other memory-efficient approaches developed by the authors. In 2011 Andolfatto et al. [9] proposed calculation of CNC machine tool link errors uncertainty through the MCM based on GUM Supplement 1 (GUM S1) [10] This approach did not include the covariance of the input and output data, unlike in GUM S2.

2 Machine geometric errors parameters

CNC machines geometric error parameters include two groups of errors: inter-axis link geometric error parameters (location errors) and inter-axis joint motion errors (component errors) [11]. Each machine axis is affected by six joint kinematic and up to three link errors, which are major causes for position and orientation inaccuracies of the tool and workpiece. Thus they should be identified and corrected or compensated.

2.1 Machine topology and SAMBA

In this paper a five-axis machine with WCBXFZYT topology (Figure 1) is investigated. The artefact (SAMBA) consists of four master balls of diameter 12.7 mm and a scale bar with a length 304.6686 mm.

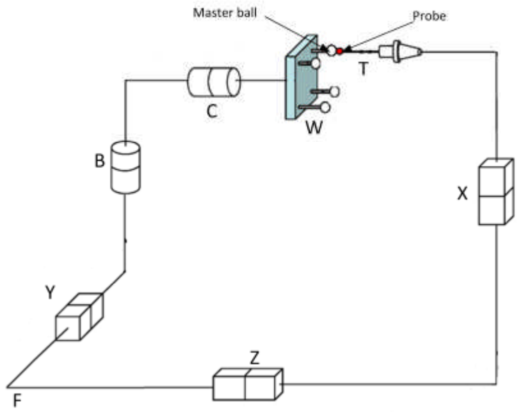

Figure 1: Five-axis CNC machine model with the topology WCBXFZYT;
W - workpiece, T - tool, F - machine foundation, B, C – rotary axes around the Y and Z axes respectively, X, Y, Z – machine linear axes [12]

2.2 Errors identification

Thirteen geometric error parameters (eight link errors, two spindle location errors and three positioning linear error terms) that are simulated in the machine model and later identified are listed in Table 1.

Table 1: Identified geometric errors parameters

Symbol	Description
AOB	out-of-squareness of the B-axis relative to the Z-axis
COB	out-of-squareness of the B-axis relative to the X-axis
XOC	distance between the B and C axes
AOC	out-of-squareness of the C-axis relative to the B-axis
BOC	out-of-squareness of the C-axis relative to the X-axis
BOZ	out-of-squareness of the Z-axis relative to the X-axis
AOY	out-of-squareness of the Y-axis relative to the Z-axis
COY	out-of-squareness of the Y-axis relative to the X-axis
EXX	positioning linear error term of the X-axis
EYY	positioning linear error term of the Y-axis
EZZ	positioning linear error term of the Z-axis
XOS	X offset of the spindle relative to the B-axis
YOS	Y offset to the spindle relative to the C-axis

Identifying those errors requires building kinematic models of the machine and calculating volumetric errors (difference between the tool position relative to the workpiece) [13]. The next step is defining from the homogenous transformation matrices the sensitivity Jacobian matrix *J*, which represents changes of the volumetric error $\delta\tau$ caused by the small changes in machine parameters δp described by the equation:

(1)

which can be solved for δp using the J^+ (pseudoinverse matrix of J):

$$\delta p = J^+ \cdot \delta\tau \tag{2}$$

After the first estimation parameters set δp_1 is used to calculate predictions of volumetric errors $\delta\tau_{pred,1}$. Those values are compared with until δp is smaller than a predefined threshold δp permissible.

3 Uncertainty estimation

In this paper the uncertainty of the output quantity Y (machine errors parameters) which estimate $y=(y_1,...,y_N)$ resulting from the uncertainty of the input quantity X with the estimate $x=(x_1,...,x_m)$ (volumetric errors) is estimated using the GUF and Adaptive MCM for multi-output measurements.

Both uncertainty estimation methods require calculating the coverage factors k_p (for a hyper-ellipsoid coverage region) and k_q (for a hyper-rectangle coverage region) for the coverage probability p. Another significant parameter is the output quantity correlation matrix R_y and its maximum eigenvalue λ_{max}.

3.1 Generalized Uncertainty Framework

GUF defines the uncertainty on the multi-output measurement result by the following equation:

$$U_y = C_x U_x C_x^T \tag{3}$$

where:

C_x is the sensitivity measurement matrix, for the analysed case $C_x = J^+$

U_x is the input data covariance (uncertainty) matrix,

$$U_x = \begin{bmatrix} u(x_1, x_1) & \dots & u(x_1, y_N) \\ \vdots & \ddots & \vdots \\ u(x_N, x_1) & \dots & u(x_N, x_N) \end{bmatrix}. \tag{4}$$

Each element (i,j) in U_x equals:

$$u(x_i, x_j) = r(x_i, x_j) u(x_i) u(x_j) \tag{5}$$

where $r(x_i, x_j)$ is the correlation coefficient associated with x_i and x_j.

3.2 Adaptive Monte Carlo Method

The application of MCM requires creating M arrays of the N input data by sampling their PDFs (and/or joint PDF). For each from the M arrays the m output quantities are obtained. The estimate of the output quantity is the average calculated using all the obtained values:

$$\tilde{y} = \frac{1}{M}(y_1 + \cdots + y_M) \qquad (6)$$

The associated covariance matrix is estimated by:

$$U_{\tilde{y}} = \frac{1}{M-1}\left((y_1 - \tilde{y})(y_1 - \tilde{y})^T + \cdots + (y_{\mathrm{M}} - \tilde{y})(y_{\mathrm{M}} - \tilde{y})^T\right) \qquad (7)$$

The mean and covariance matrix are used to calculate the coverage factors kq and kp. The detailed procedure is described in [7].

The number of MCM trails can be set before conducting calculations. In this paper the adapative MCM is performed, thus the number of MC trials depends on the results stability.

The first step of the Adaptive MCM is performing $h=10$ times $M=10^4$ MC trials. For each $r=1,...,h$ subsets the output quantity estimate y(r) and its standard deviation $s_y^{(r)}$, maximum eigenvalue $\lambda_{max}^{(r)}$ of the correlation matrix $R_y^{(r)}$, coverage factors $k_p^{(r)}$ and $k_q^{(r)}$ are calculated.

The set of $h=10$ estimates of the output quantities is used to calculate their standard deviations (for $j=1,\ldots,m$) as one of the stability parameters:

$$s_{y_j}^2 = \frac{1}{h(h-1)}\sum_{r=1}^{h}\left(y_j^{(r)} - y_j\right)^2 \qquad (8)$$

Similarly, the other stability parameters standard deviations (of results variances s_{uyj}, maximum eigenvalue $s_{\lambda max}$ and coverage factors s_{kp} and s_{kq}) are calculated. The values are multiplied by 2 and compared with their corresponding required numerical tolerances. If at least one of the doubled standard deviations is greater than its numerical tolerance, the value of h is increased by 1 and a new set of M output data is calculated. The procedure is repeated until the results reach demanded stability.

3.3 GUF validation by adaptive MCM

Comparison of the results obtained by GUF and MCM allows validating the former uncertainty method, so that in further work there is no need for performing time-consuming MCM. In order to do so, d – the absolute differences between the output quantity estimates (d_{y_j}) and parameters ($d_{u(y_j)}$, $d_{\lambda_{max}}$, d_{k_p}, d_{k_q}) have to be calculated for both methods using the eq. 9-13:

$$d_{y_j} = \left|y_j^{GUF} - y_j^{MCM}\right|, \qquad j = 1, \ldots, m \qquad (9)$$

$$d_{u(y_j)} = \left|u(y_j^{GUF}) - u(y_j^{MCM})\right|, \qquad j = 1, \ldots, m \qquad (10)$$

$$d_{\lambda_{max}} = \left|\lambda_{max}^{GUF} - \lambda_{max}^{MCM}\right| \qquad (11)$$

$$d_{k_p} = \left|k_p^{GUF} - k_p^{MCM}\right| \qquad (12)$$

$$d_{k_q} = \left| k_q^{GUF} - k_q^{MCM} \right| \tag{13}$$

Those values must be no larger than their corresponding numerical tolerances. The validation numerical tolerances should be at least five times greater than adaptive MCM numerical tolerances.

4 Simulation

In order to calculate uncertainty using the described methods various simulations were conducted in MATLAB. The master balls measurement data was generated for the following B- and C-axis positions for the master balls: *B=90* and *C=270*, *B=60* and *C=180*, *B=30* and *C=90*, *B=0* and *C=0*, *B=-90* and *C=-270*, *B=-60* and *C=-180*, *B=-30* and *C=-90 deg*. Scale bar measurement was generated for *B=C=0 deg*.

For both GUF and MCM the coverage probability was set to *p=0.95*. The input data covariance matrix U_x was estimated from repeated experimental probing measurement. The scale bar length was measured on a CMM machine, providing a length error variance $u_{scale\ bar}=0.0012\ mm$. MCM input data draws were generated from multivariate Gaussian distribution. The numerical tolerances set for Adaptive MCM and GUF validation are listed in Table 2.

Table 2: Demanded numerical tolerances

Adaptive MCM stability parameter	Numerical tolerance	GUF validation parameter	Numerical tolerance
y_1=AOB, $u(y_1)$, s_{y1},$s_{u(y1)}$	$0.5 * 10^{-6}$	d_{y1}, d_{u1}	$0.5 * 10^{-5}$
y_2=COB, $u(y_2)$, s_{y2},$s_{u(y2)}$	$0.5 * 10^{-6}$	d_{y2}, d_{u2}	$0.5 * 10^{-5}$
y_3=XOC, $u(y_3)$, s_{y3},$s_{u(y3)}$	$0.5 * 10^{-6}$	d_{y3}, d_{u3}	$0.5 * 10^{-5}$
y_4=AOC, $u(y_4)$, s_{y4},$s_{u(y4)}$	$0.5 * 10^{-5}$	d_{y4}, d_{u4}	$0.5 * 10^{-4}$
y_5=BOC, $u(y_5)$, s_{y5},$s_{u(y5)}$	$0.5 * 10^{-6}$	d_{y5}, d_{u5}	$0.5 * 10^{-5}$
y_6=BOZ, $u(y_6)$, s_{y6},$s_{u(y6)}$	$0.5 * 10^{-7}$	d_{y6}, d_{u6}	$0.5 * 10^{-6}$
y_7=AOY, $u(y_7)$, s_{y7},$s_{u(y7)}$	$0.5 * 10^{-6}$	d_{y7}, d_{u7}	$0.5 * 10^{-5}$
y_8=COY, $u(y_8)$, s_{y8},$s_{u(y8)}$	$0.5 * 10^{-6}$	d_{y8}, d_{u8}	$0.5 * 10^{-5}$
y_9=EXX, $u(y_9)$, s_{y9},$s_{u(y9)}$	$0.5 * 10^{-6}$	d_{y9}, d_{u9}	$0.5 * 10^{-5}$
y_{10}=EYY, $u(y_{10})$, s_{y10},$s_{u(y10)}$	$0.5 * 10^{-6}$	d_{y10}, d_{u10}	$0.5 * 10^{-5}$
y_{11}=EZZ, $u(y_{11})$, s_{y11},$s_{u(y11)}$	$0.5 * 10^{-4}$	d_{y11}, d_{u11}	$0.5 * 10^{-3}$
y_{12}=XOS, $u(y_{12})$, s_{y12},$s_{u(y12)}$	$0.5 * 10^{-4}$	d_{y12}, d_{u12}	$0.5 * 10^{-3}$
y_{13}=YOS, $u(y_{13})$, s_{y13},$s_{u(y13)}$	$0.5 * 10^{-6}$	d_{y13}, d_{u13}	$0.5 * 10^{-5}$
k_p, k_q, s_{kp}, s_{kq}	$0.5 * 10^{-1}$	d_{kp}, d_{kq}	0.5
λ_{max}	$0.5 * 10^{-1}$	$d_{\lambda max}$	0.5

5 Results

The results of the uncertainty calculation by GUF and Adaptive MCM are presented in Tables 3-5. Although the identified parameters vector has *m=34* output quantities, just thirteen identified error parameters are presented as the subject of the analysis. However, the remaining output values, which do not contain the values of error parameters, as being part of the identification, are estimated for each MCM trial and verified for their stability.

Table 3 contains the identified parameters values estimated through GUF and MCM for four different numbers of trials. The mean values of the parameters obtained by MCM give very good estimation comparing to those obtained by GUF. The absolute difference between them is smaller than their numerical tolerances (Table 2).

Table 3: Estimated output quantities using GUF and MCM

Method	GUF	MCM	MCM	MCM	Adaptive MCM
M		10^4	2×10^5	3×10^5	10^5
y_1=AOB	1,410E-04	1,40991E-04	1,41000E-04	1,40999E-04	1,40998E-04
y_2=COB	1,610E-04	1,60991E-04	1,60997E-04	1,61000E-04	1,60997E-04
y_3=XOC	1,120E-04	1,11957E-04	1,11957E-04	1,11992E-04	1,11971E-04
y_4=AOC	5,110E-03	5,11152E-03	5,11061E-03	5,11006E-03	5,11048E-03
y_5=BOC	5,410E-05	5,40681E-05	5,40888E-05	5,40967E-05	5,40923E-05
y_6=BOZ	5,510E-04	5,50994E-04	5,50997E-04	5,50999E-04	5,50996E-04
y_7=AOY	3,120E-04	3,11994E-04	3,11996E-04	3,11999E-04	3,11996E-04
y_8=COY	3,410E-04	3,40985E-04	3,41002E-04	3,40999E-04	3,40999E-04
y_9=EXX	3,610E-04	3,60987E-04	3,60993E-04	3,60999E-04	3,60994E-04
y_{10}=EYY	3,320E-05	3,31311E-05	3,31513E-05	3,31947E-05	3,31679E-05
y_{11}=EZZ	2,110E-03	2,12578E-03	2,11742E-03	2,11027E-03	2,11452E-03
y_{12}=XOS	2,210E-03	2,20763E-03	2,20829E-03	2,20919E-03	2,20833E-03
y_{13}=YOS	2,220E-04	2,21954E-04	2,21963E-04	2,21996E-04	2,21974E-04

The uncertainty values are listed in Table 4 and depicted in

Figure 2. Given the different orders of magnitude of the results the y-axis is set to logarithmic. For most of the output quantities the uncertainty values have the same order of magnitude and the difference between GUF and MCM does not exceed their numerical tolerance. The exception is y_{11}, where the difference between GUF and MCM uncertainty is greater than the numerical tolerance. Figure 3 shows the probability density functions obtained by GUF and MCM (based on histogram) for the most (y_6) and the least (y_{11}) matching case. For both of the parameters the mean values are basically the same for GUF and MCM. The uncertainty $u(y_{11})$ estimated by GUM is more than 2.5 times greater than estimated by MCM, while the uncertainty $u(y_6)$ has very close values for, both GUM and MCM.

Table 4: Estimated output quantities uncertainties using GUF and MCM

Method	GUF	MCM	MCM	MCM	Adaptive MCM
M		10^4	2×10^5	3×10^5	10^5
$u(y_1)$	9,47E-07	8,36E-07	8,34E-07	8,33E-07	8,34E-07
$u(y_2)$	9,58E-07	8,62E-07	8,56E-07	8,57E-07	8,55E-07
$u(y_3)$	4,00E-06	1,00E-05	9,89E-06	9,89E-06	9,90E-06
$u(y_4)$	1,66E-04	1,78E-04	1,75E-04	1,75E-04	1,74E-04
$u(y_5)$	1,05E-06	2,54E-06	2,51E-06	2,51E-06	2,51E-06
$u(y_6)$	9,60E-07	9,97E-07	9,92E-07	9,92E-07	9,90E-07
$u(y_7)$	1,54E-06	1,42E-06	1,41E-06	1,41E-06	1,40E-06
$u(y_8)$	1,13E-06	1,27E-06	1,26E-06	1,26E-06	1,27E-06
$u(y_9)$	1,55E-06	1,38E-06	1,37E-06	1,37E-06	1,37E-06
$u(y_{10})$	4,32E-06	1,14E-05	1,12E-05	1,12E-05	1,12E-05
$u(y_{11})$	6,97E-04	1,99E-03	1,96E-03	1,96E-03	1,96E-03
$u(y_{12})$	5,06E-04	9,62E-04	9,58E-04	9,59E-04	9,56E-04
$u(y_{13})$	4,91E-06	9,58E-06	9,42E-06	9,42E-06	9,43E-06

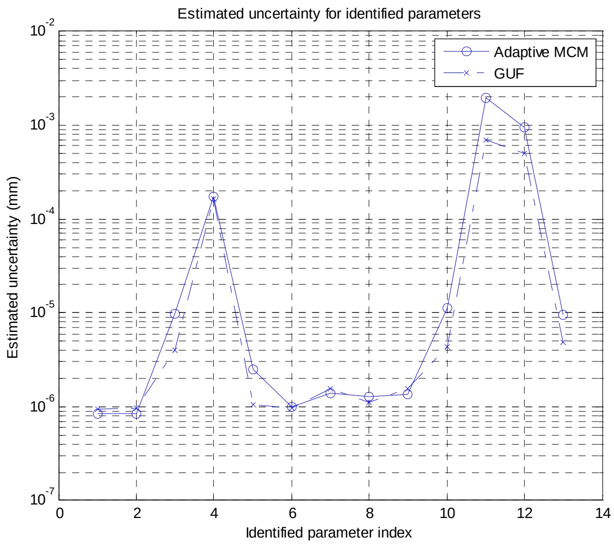

Figure 2: Estimated uncertainty for identified machine geometric errors parameters

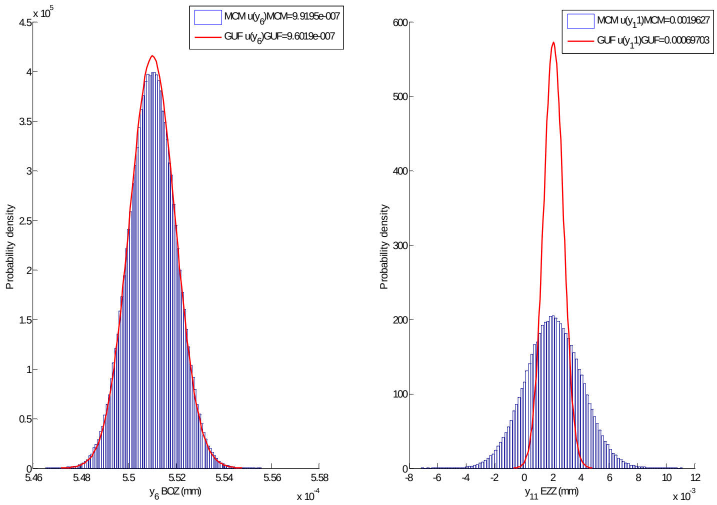

Figure 3: Probability density of y_6 and y_{11} estimated by GUF and MCM

The coverage factors and maximum eigenvalues of the correlation matrix are listed in Table 5. The coverage factors have very close values for GUF and for all four MCM cases. The $d_{k_p} = 0.0015$ and $d_{k_q} = 0.0836$ are smaller than required numerical tolerance (Table 2). The maximum eigenvalues of the correlation matrix for GUF and MCM differ significantly $d_{\lambda_{max}} = 8.48$. Thus the GUF method cannot be validated with MCM.

Table 5: Coverage factors and maximum eigenvalue calculated for GUF and MCM.

Method	GUF	MCM	MCM	MCM	Adaptive MCM
M		10^4	2×10^5	3×10^5	10^5
k_p	6.97	6.94	6.97	6.97	6.97
k_q	2.97	2.90	2.89	2.89	2.89
λ_{max}	13.86	22.34	22.23	22.25	22.235

From the Tables 3-5 it can be noticed that setting the number of MC trials greater than in adaptive MCM does not change the stability of the results.

6 Conclusions

The GUM S2 multi-output uncertainty measurement was applied for uncertainty calculation of the iterative machine geometric errors identification process. The results were obtained for GUF, MCM and adaptive MCM approaches. The adaptive MCM gave satisfying results and, although time-consuming, it can be used to calculate the uncertainty on the CNC machine errors identified parameters. Testing the procedure for number of trials smaller and larger than in adaptive MCM proved that its algorithm allows obtaining stabilized results without conducting too many simulations.

The GUF could not be validated with the MCM although the comparison of identified results, most of the uncertainties and coverage factors was acceptable. However, the difference between the correlation matrices maximum eigenvalues resulting from differences in correlation coefficients obtained for the identification results for each method do not allow validating the GUF method. The reason for this might be the iterative character of identification, which may need a more accurate representation of the measurement model than proposed in this paper.

7 Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada CANRIMT Strategic Network.

References

[1] B. Bringmann, A. Küng, and W. Knapp, "A Measuring Artefact for true 3D Machine Testing and Calibration," *CIRP Annals - Manufacturing Technology*, vol. 54, pp. 471-474, 2005.

[2] W. T. Lei and Y. Y. Hsu, "Error measurement of five-axis CNC machines with 3D probe ball," *Journal of Materials Processing Technology*, vol. 139, pp. 127-133, 2003.

[3] H. Schwenke, W. Knapp, H. Haitjema, A. Weckenmann, R. Schmitt, and F. Delbressine, "Geometric error measurement and compensation of machines-An update," *CIRP Annals - Manufacturing Technology*, vol. 57, pp. 660-675, 2008.

[4] H. Jae Pahk, Y. Sam Kim, and J. Hee Moon, "A new technique for volumetric error assessment of CNC machine tools incorporating ball bar measurement and 3D volumetric error model," *International Journal of Machine Tools and Manufacture,* vol. 37, pp. 1583-1596, 1997.

[5] J. R. R. Mayer, "Five-axis machine tool calibration by probing a scale enriched reconfigurable uncalibrated master balls artefact," *CIRP Annals - Manufacturing Technology*, vol. 61, pp. 515-518, 2012.

[6] GUM, "Evaluation of measurement data - Guide to the expression of uncertainty in measurement," ed, Joint Committee for Guides in Metrology, JCGM 100:2008.

[7] GUM, "Evaluation of measurement data – Supplement 2 to the "Guide to the expression of uncertainty in measurement" – Extension to any number of input quantities," ed, Joint Committee for Guides in Metrology, JCGM 102:2011.

[8] S. Eichstadt, A. Link, P. Harris, and C. Elster, "Efficient implementation of a Monte Carlo method for uncertainty evaluation in dynamic measurements," *Metrologia,* vol. 49, pp. 401-410, 2012.

[9] L. Andolfatto, J. R. R. Mayer, and S. Lavernhe, "Adaptive Monte Carlo applied to uncertainty estimation in five axis machine tool link errors identification with thermal disturbance," *International Journal of Machine Tools and Manufacture,* vol. 51, pp. 618-627, 2011.

[10] GUM, "Evaluation of measurement data – Supplement 1 to the "Guide to the expression of uncertainty in measurement" – Propagation of distributions using a Monte Carlo method," ed, Joint Committee for Guides in Metrology, JCGM 101:2008.

[11] T. O. Ekinci, J. R. R. Mayer, and G. M. Cloutier, "Investigation of accuracy of aerostatic guideway," *Int. J. of Machine Tools and Manufacture*, pp. 478-487, 2009.

[12] T. Erkan, J. R. R. Mayer, and Y. Dupont, "Volumetric distortion assessment of a five-axis machine by probing a 3D reconfigurable uncalibrated master ball artefact," *Precision Engineering,* vol. 35, pp. 116-125, 2011.

[13] Y. A. Mir, J. R. R. Mayer, and C. Fortin, "Tool path error prediction of a five-axis machine tool with geometric errors," *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture,* vol. 216, pp. 697-712, 2002.