Estimation of volumetric errors of a machining centre fabricated with conformance to geometric tolerances of ISO 10791 series

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Abstract

Machine-specific ISO standards are internationally well-accepted documents which are broadly used to clarify technical aspects during the trading of different types of machine tools and often are considered as the technical annex of a contract between two sides, manufacturer (seller) and end-user (purchaser). Therefore, these standards are mainly used for pre-acceptance and final acceptance tests to check the accuracy and performance of the machine tool and whether it complies with the agreed technical specifications. Among them, ISO 10791 series address different aspects of accuracy of machining centres as the most applied machine tools in industries. ISO 10791 series specify the tolerances for "general-purpose normal accuracy" machining centres. Regardless of this emphasis, it is seen that some contracts are signed without attention to the required machine tolerances which might fulfil dimensional and geometrical tolerances of the workpieces intended to be finished with that machining centre.
In this study, based on the positioning tolerances specified in ISO 10791-4:1998 and the geometric tolerances indicated in ISO/FDIS 10791-2:2022 for vertical machining centres (VMC), probable volumetric errors of a 3-axis C-frame VMC with cross table are investigated for all linear axes equal to 500 mm. In other words, the paper will show that if a VMC with that specified structural loop is fabricated with full conformance to ISO 10791 series, what volumetric errors and its tool path accuracy are achievable. This is a useful indicator for machine tool purchasers and users and the method is applicable to all machine sizes.

1 Introduction

Quantifying volumetric performance of a CNC machine tool in its working volume can provide useful information to its manufacturer for compensation purposes and optimization of the design and also to its end user to enable them to produce more accurate workpiece.
ISO 230-1:2012 [1] defines the "volumetric accuracy for three linear axes, V_{XYZ}" in its clause 3.8.12 by classifying it in three translational deviations range along each linear X, Y and Z-axes: $V_{XYZ,X}$, $V_{XYZ,Y}$, $V_{XYZ,Z}$. These ranges along three linear axes are studied in this work by applying the tolerance specified in ISO 10791-4:1998 [2] and ISO/FDIS 10791-2:2022 [3] assuming various combination of them by synthesising each set of error motions with Homogeneous Transformation Matrices (HTM) method. The rotational volumetric accuracies addressed in ISO 230-1:2012 are important aspects when we want to analyse the performance of the machine in using various tools with different tool lengths which influence the abbe-offset lengths. In this study, the point which is the intersection of the spindle gauge plane and the spindle average axis of rotation of a 3-axis machine is investigated on which the rotational volumetric accuracies including $V_{XYZ,A}$, $V_{XYZ,B}$ and $V_{XYZ,C}$ have no effect. Thus, any assembly errors of the spindle such as squareness of the spindle axis to X and Y-axes and also parallelism of the spindle axis to Z-axis, will have minimal effect on the analysis. Therefore, these deviations and specified tolerances in ISO/FDIS 10791-2 were not included in the analysis.
ISO 230-6:2002 [4] provides a procedure and defines positioning of body diagonals of the working volume of a machine tool as an index for directly evaluating volumetric performance of those production machinery. In machine-specific standards published by ISO, no tolerance has been specified for volumetric performance of machining centres while their linear axes are commanded to move along a linear trajectory e.g. body diagonals.
Fletcher et al. [5] compared direct methods for evaluation of the volumetric performance of machine tools with error synthesis methods. Since, by indirect approach such as HTM, the full assessment of the volumetric accuracy of the machine is obtainable from common measurements on a machine, in this work we apply this methodology.

2 HTM Method for error modelling and computing volumetric deviations and correspondig vectors

As Donmez [6] explained how to use HTM method on a 2-axis turning centre to extract its volumetric errors and Okafor and Ertekin [7] applied this procedure on a 3-axis VMC, the same methodology is used in this study to compute volumetirc deviation at any coordinate point and its correcponding deviation vector.
Equations in HTM method vary depending on the kinematic chain of the machine under test. ISO/FDIS 10791-2 addresses 12 different configurations of VMCs and their corresponding designations based on their structural loops and

resultant kinematic chains. In this study, a VMC with [w X’ Y’ b Z (C) t] kinematic chain is investigated which is depicted in Figure 1.

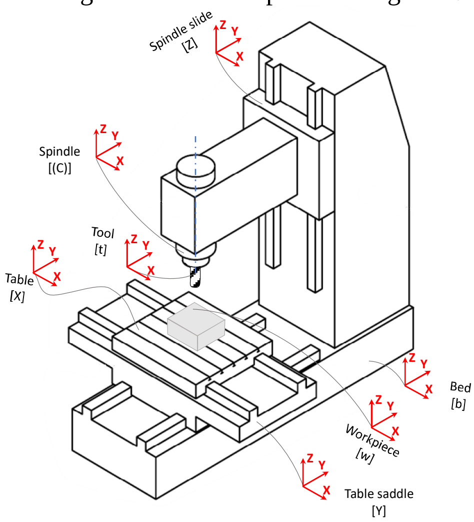

Figure 1: VMC with [w X’ Y’ b Z (C) t] structural loop and its coordinate frames (modified from ISO/FDIS 10791-2 [3])

To construct HTM equation (1) showing relative displacement between tool and workpiece, motions on the tool-side and motions on the workpiece-side are modelled using 4 by 4 matrices each of which specifies relative position and orientation of the moving component of the machine to its parent component.

$$ {}^{w}_{t}T = \left({}^{b}_{w}T^{-1}\right)\left({}^{b}_{t}T\right) \qquad (1)$$

A separate coordinate frame is attached to each component, and finally their relative position and orientation are calculated. This procedure is performed for all taget positions at which we have actual data. In this study, instead of actual measured data, simulated data generated by MATLAB in line with specified tolerances in ISO 10791 series.

3 Specified tolerances, categories and configurations for machining centres in ISO 10791 series

ISO 10791 series have 10 parts, each of which explains some aspects of performance of machinig centres and specifies tolerances for different types of these widely-used machine tools. In these standards, machining centres are classified based on the size of their linear axes into five categories including : less than 500 mm; between 500 and 800 mm; between 800 and 1250; between 1250 and 2000 mm; and larger than 2000 mm. Accordingly, these standards specify tolerances for general-purpose normal accuracy machining centres. ISO 10791-4:1998 [2] and ISO/FDIS 10791-2:2022 [3] (new revision of ISO 10791-2 will be published shortly) specify positioning and geometric tolerances for VMCs respectively. Table 1 shows the extracted tolerances from these two documents for different strokes of linear axes.

Table 1: Specified geometric tolerances for vertical machining centres according to ISO 10791 series for different length of linear axes up to 2000 mm (all linear tolerances are in mm, all angular tolerances are in mm/mm)

Geometric test items \ Tolerance values	Total tolerance				Local tolerance
	L ≤ 500	500 < L ≤ 800	800 < L ≤ 1250	1250 < L ≤ 2000	L=300
Positioning of all linear axes (E_{XX}, E_{YY}, E_{ZZ}): Range of the mean bidirectional positional deviation (M)	0.010	0.012	0.015	0.020	Not applicable
Straightness of all linear axes: X-axis: (E_{ZX} & E_{YX}) Y-axis: (E_{ZY} & E_{XY}) Z-axis: (E_{YZ} & E_{XZ})	0.010	0.015	0.020	0.025	0.007
Angularity of all linear axes: X-axis: (E_{BX}, E_{CX} & E_{AX}) Y-axis: (E_{AY}, E_{CY} & E_{BY}) Z-axis: (E_{AY}, E_{CY} & E_{BY})	0.060/ 1000	0.060/ 1000	0.060/ 1000	0.060/ 1000	0.016/ 1000 (E_{CZ}: 0.024/1000)
Squareness of linear axes: (E_{A0Y}, E_{A0Z}, E_{B0X}, E_{B0Z}, E_{C0X}, E_{C0Y})	0.040/ 1000	0.040/ 1000	0.040/ 1000	0.040/ 1000	Not applicable

Since the configuration shown in Figure 1 is mostly used for fabricating small-size VMCs, the length of 500 mm for all axes has been assumed for this study.

4 Assumptions, considerations and method for generating simulated values for error motions

In this study, we assumed that the VMCs with specified kinematic chain comply with rigid body behaviour. Therefore, changing the abbe-offset length of any axis does not result in non-rigid performance. In other words, geometric test results on a particular axis under test remain unchanging regardless of the position of the other axes which are not under test.

In order to implement HTM methodology for calculating volumetric deviations and vectors, we need to generate simulated error values for each error motion of the three axes of the machine under investigation. For this purpose, on a VMC with the same stroke of 500 mm for all axes, we assume 11 target positions equally-space distributed along any linear axes which means we have 11 target positions along individual X, Y and Z-axes. It should be noted that for linear axes equal or smaller than 2000 mm, ISO 230-2:2014 [8] specifies a minimum of 5 target positions per meter for performing positioning tests. Therefore, assuming 11 target positions over 500 mm is completely in line with this document and industrial practice. This assumption for the number of target positions is expanded to the other error motions including straightness and angular errors.

We also assumed that numerical compensations are active on the VMC. So, the output of simulated measurements over each linear axis can have any

mathematical function with respect to the position regardless of the shape of the guideways, geometry of ballscrews and accuracy of the linear/rotary encoders.
For generating positioning deviations, the concept shown in Figure 2 is used. Since the positioning tolerance for the axes upto 500 mm is 0.010 mm (10 micrometre) and generally the reading of the measuring instrument is set to zero for the first target position.

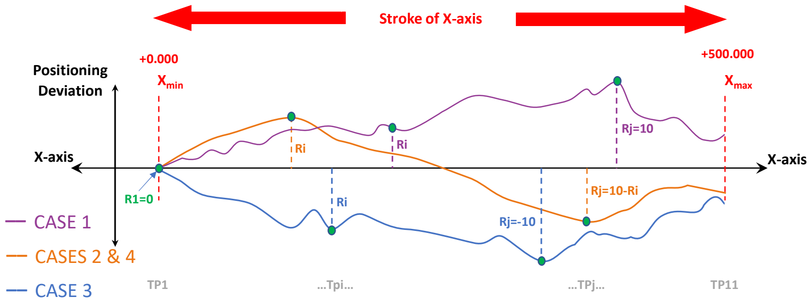

Figure 2: Concept of the algorithm for generating simulated positioning deviations

A computer code was developed in MATLAB to generate random values for the positioning deviations. By setting the value of the first target position to zero (X= 0, R_0 = 0), the second target position (i) is randomly selected and a random value (Ri) between -10 and +10 is allocated to that. After randomly choosing the third target position (j), the corresponding value for the other limit satisfying 10 micrometre is determined based on the rule that we need to have the curve either on both sides of the zero line or on the same side of it. The R values for the remaining 8 target positions are randomly selected between Ri and Rj. Following this algorithm results in a set of values whose variation range is equal to 10 micrometres. Depending on the intention to have values distributed only above the zero line, or, only below the zero line, or, both above and below the zero line, different sets of values can be randomly generated.
For randomly generating a set of straightness deviations, end-points method as the reference straight line is used based on ISO 230-1. Since total tolerance of 10 and local tolerance of 7 micrometres must be satisfied at the same time, a polynomial function with degree of up to 7 as shown in equation (2) is used.

$$R(x) = a_7x^7+a_6x^6+ a_5x^5+a_4x^4+ a_3x^3+a_2x^2+ a_1x+ a_0 \quad (2)$$

Polynomial coefficients, a_7 to a_0 are randomly selected so that they simultaneously meet those specified total and local tolerances on the full stroke of the axis. Since end-points fit is used, a_0 must set to zero to satisfy zero value at the start point (X=0). Figure 3 shows an example of generated profile of straightness of a linear axis with 500 mm stroke and the total tolerance of 10 micrometres which occurs between X=200 and X=400 mm and the local tolerance of 7 micrometres between X=200 and X=500 mm in MATLAB with an algorithm according to the explained concept. In this figure, blue dashed line shows the maximum local straightness error over a length of 300 mm which is equal to the local tolerance of 7 micrometres.

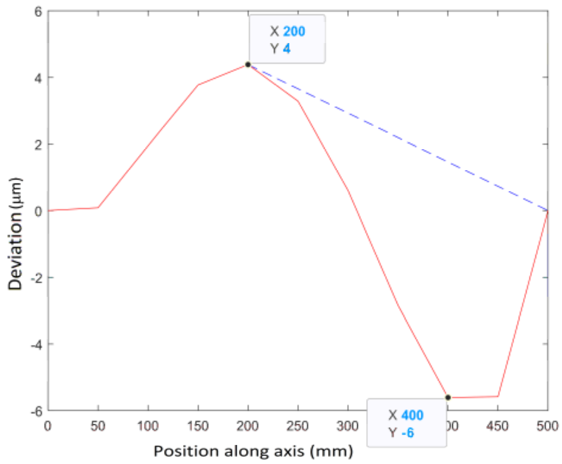

Figure 3: An example of generated straightness profile for a 500 mm axis

As seen in Table 1 the total tolerance and local tolerance for angular errors are 60 and 16 µm/1000 mm (except roll of Z-axis) respectively. Figure 4 shows that if we assume the smoothest variation of angular deviations for a linear axis of 500 mm for both total and local tolerances, they cannot be satisfied simultaneously. In other words, if we set the total tolerance to 60 µm/1000 mm over total stroke of 500 mm (the smoothest shape), the local tolerance becomes 36 µm/1000 mm over 300 mm which is far above the specified tolerance in ISO/FDIS 10791-2. From the other side, if we set the local tolerance to 16 µm/1000 mm, the total tolerance cannot be greater than 26.7 µm/1000 mm over any length of 300 mm.

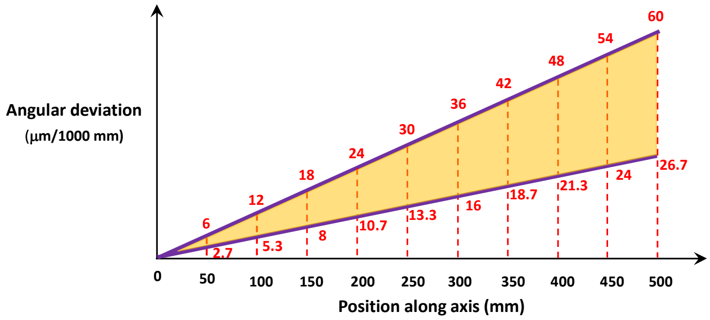

Figure 4: Discrepancy between total and local tolerances of angular deviation in ISO/FDIS 10791-2

Because of this observed discrepancy in ISO standards and due to this fact that most people pay more attention to total tolerances, especially during contracting and acceptance tests of machining centres, only two smoothest cases (positive and negative values) of angular errors are considered for this study. The same concept applies for roll of Z-axis and similar stepwise total tolerance of 60 µm/1000 mm as displayed in Figure 4 is used for it too.

Based on Table 1, squareness of two mutual linear axes can have two extreme values which are -40 μm/1000 mm and +40 μm/1000 mm.

For this work, we generated 20 simulated files for positioning error and 17 simulated files for straightness error by MATLAB. Since we assume two sets of data for angular errors and two extreme values for squareness ones, the total number of unique combination of these error files is calculated more than 790 trillions as shown in Table 2.

Table 2: Calculation of possible combinations of simulated error files

Errors (number of error in a 3-axis VMC)	Number of generated simulated error files	Number of different conditions	Total
Positioning (3)	20	20^3	8,000
Straightness (6)	17	17^6	24,137,569
Angularity (9)	2	2^9	512
Squareness (3)	2	2^3	8
Total			7.9×10^{13}

5 Analysis of volumetric deviations on a typical VMC

Since we have 11 target positions along any linear axes, the total number of coordinate points at which we can calculate the volumetric deviations is 11×11×11=1331. Therefore, we need to compute separately three volumetric deviations at each coordinate point along X, Y and Z-axes. Figure 5 shows the error vectors at all coordinate points for a VMC with randomly selected error motions from those possible combinations of simulated error files explained in Table 2.

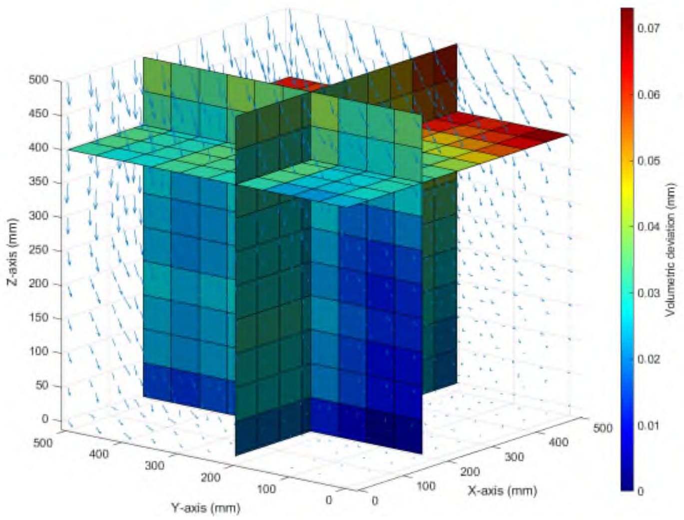

Figure 5: Volumetric deviation vectors derived by HTM for the typical machine with simulated data when all deviations are set to zero at axis positions X/Y/Z = 0/0/0 (colour scale shows the size of the volumetric deviation vector obtained from eq. 3)

All results presented in this paper are based on the assumption that the deviation values are set to zero at the first target position for all axes, X/Y/Z = 0. The practical reasoning for this assumption is that when performing geometric tests on a VMC with 500 mm stroke for each axis from 0 to 500 mm, it is common to set laser system to zero at X/Y/Z = 0 as the starting point. Depending on machine use, this datum can be numerically shifted to the centre point of a rotary table or workpiece reference point.
Histogram of the volumetric deviations at all 1331 coordinate points demonstrates the distribution of these differences along any linear axes. Figure 6 shows histograms for obtained volumetric deviations along X, Y and Z-axes.

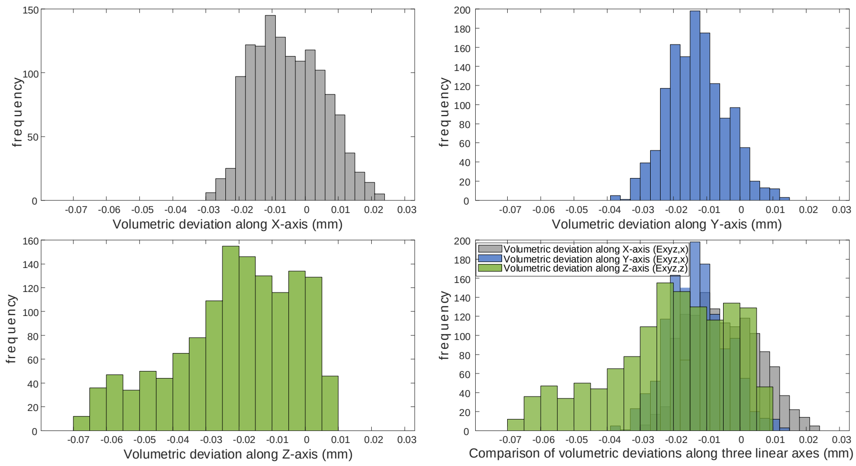

Figure 6: Histogram of volumetric deviations and accuracies along X, Y and Z-axes according to ISO 230-1 for a typical VMC

Finding minimum and maximum deviations along any linear axis of the machine can give the volumetric accuracy according to the definistion given in ISO 230-1:2012 [1]. For above-shown histograms,

Table **3** summarises all extreme values of volumetric deviations and corresponding accuracies along X, Y and Z-axes for a typical machine with randomly selected error files.
At any coordinate point, the magnitude of deviation vector is calculated by equation (3) where d_{Xi} , d_{Yj} and d_{Zk} are deviation values along X, Y and Z-axes respectively which are obtained by applying HTM equations.

$$d_{(Xi,Yj,Zk)} = \sqrt{d_{Xi}^2 + d_{Yj}^2 + d_{Zk}^2} \qquad (3)$$

Table 3: HTM Calculated values of volumetric deviations and accuracies along X, Y and Z-axes according to ISO 230-1:2012 for a typical VMC with randomly selected error motions

Mean volumetric deviation at all 1331 coordinate points along X, Y and Z-axes	Minimum volumetric deviation at all 1331 coordinate points along X, Y and Z-axes	Maximum volumetric deviation at all 1331 coordinate points along X, Y and Z-axes	Volumetric accuracy along X, Y and Z-axes (according to ISO 230-1)
$d_{XYZ,Xmean}$ = -0.005	$d_{XYZ,Xmin}$ = -0.029	$d_{XYZ,Xmax}$ = 0.023	$V_{XYZ,X}$ = 0.053
$d_{XYZ,Ymean}$ = -0.013	$d_{XYZ,Ymin}$ = -0.039	$d_{XYZ,Ymax}$ = 0.013	$V_{XYZ,Y}$ = 0.051
$d_{XYZ,Zmean}$ = -0.021	$d_{XYZ,Zmin}$ = -0.070	$d_{XYZ,Zmax}$ = 0.010	$V_{XYZ,Z}$ = 0.080

Applying this equation for all 1331 coordinate points in the working volume of the VMC under investigation and showing them in a histogram plot results Figure 7 in which the maximum magnitude of the volumetric deviation vector is computed equal to 0.073 mm.

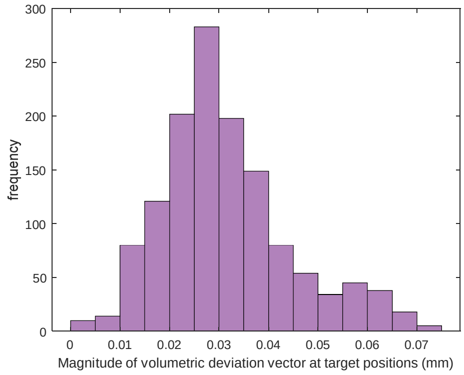

Figure 7: Histogram of magnitudes of volumetric deviation vectors for a typical VMC with randomly selected error motions as per Figure 6

6 Statistical analysis of volumetric deviations on a batch of VMCs

All plots and calculations addressed in section 0 of this paper was for a typical VMC. It would be more informative for purchasers of the VMC with mentioned kinematic chain to have an evaluation of a population of machines with statistical analysis. We applied the same procedure for 5000 simulations with different geometric error files according to the extreme values of the tolerances specified in ISO 10791 series. We picked the computed volumetric accuracy (last column in

Table **3**) from each simulation along X, Y and Z-axes as indices for volumetric performance capability of each machine. The histogram of those 5000 sets of 3 values is depicted in Figure 8.

Table 4 gives a summary of variation of volumetric accuracies of all 5000 simulations along X, Y and Z-axes in which the minimum values show that the best VMC made according to the extreme limits of ISO tolerances is statistically very unlikely to have a tighter volumetric accuracy than this. In other words, this type of VMC is volumetrically not likely to be better than 49 µm, 45 µm and 30 µm along X, Y and Z-axes respectively. Corresponding maximum values also determine that this type of VMC is volumetrically not likely to be worse than 61 µm, 87 µm and 85 µm along X, Y and Z-axes respectively. As a summary of this statistical analysis, the volumetric accuracy terms of a VMC with [w X' Y' b Z (C) t] kinematic chain mostly lie between 49 and 61 µm for X-axis, 45 and 87 µm for Y-axis and 30 and 85 µm for Z-axis.

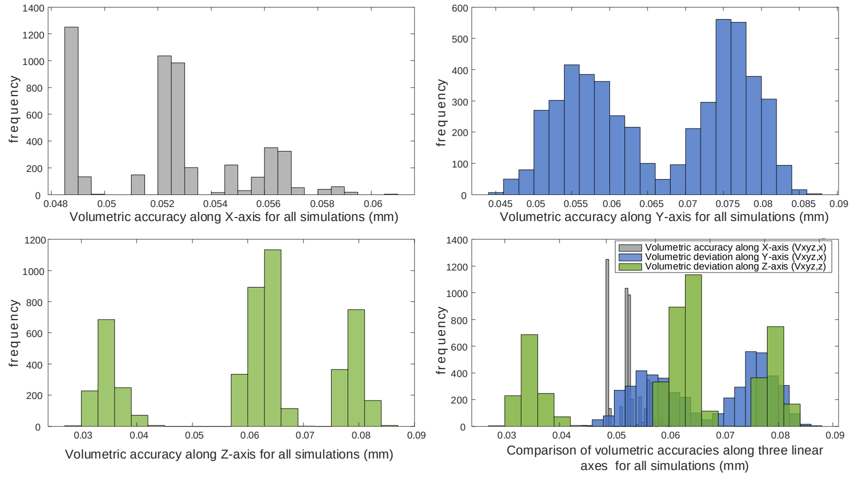

Figure 8: Histogram of volumetric accuracy along X, Y and Z-axes according to ISO 230-1 for a batch of 5000 VMCs with randomly selected error motions

Table 4: HTM Calculated limit ranges of volumetric accuracy along X, Y and Z-axes according to ISO 230-1:2012 for 5000 VMCs with randomly selected error motions

$V_{XYZ,Xmin,5000} = 0.049$	$V_{XYZ,Xmax,5000} = 0.061$
$V_{XYZ,Ymin,5000} = 0.045$	$V_{XYZ,Ymax,5000} = 0.087$
$V_{XYZ,Zmin,5000} = 0.030$	$V_{XYZ,Zmax,5000} = 0.085$

Calculations show that the minimum and maximum of the largest volumetric deviation vectors for all 5000 simulations are 0.036 and 0.111 mm respectively which means that at the best combinations of having a 3-axis VMC with above-mentioned kinematic chain when we meet the limits of ISO 10791 series, the reachable volumetric deviation vector cannot be less than 36 µm whereas the worst combination of the limits results this vector value not more than 111 µm. It should be taken into consideration that we did neither assume any uncertainty in the measurement (generating simulated data sets) nor in the error modelling (HTM calculations). This should be taken into account for manufacturing of accurate workpieces with tight dimensional and geometrical tolerances and is part of the future work for this research.

7 Conclusions

Purchasing a machining centre conforming the geometric tolerances specified in ISO 10791 series does not fully assure required dimensional and geometrical tolerances of a workpiece. A thorough analysis needs to be implemented to check whether a certain machine tool can fulfil volumetric accuracy demands of an accurate workpiece. This aseesment would be a significant tool for the users of machines during contracting and commissioning of a production asset. This statistical research work demonstrated that the volumetric accuracy according to ISO 2301-1 for VMCs with all axes stroke of 500 mm and [w X' Y' b Z (C) t] kinematic chain made in conformance to ISO 10791 series mostly lie between 49 and 61 μm for X-axis, 45 and 87 μm for Y-axis and 30 and 85 μm for Z-axis. Moreover, the magnitude of the largest deviation vector was estimated between 36 μm and 111 μm for the best and worst combination of the error motions respectively. Although this study shows extreme conditions of volumetric accuracy for a specific configuration of VMCs but this approach can be generalised for different types of machine tools.

Acknowledgement

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