# Robust coordinate system alignment using high density point clouds from laser line probe 

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#### Abstract

Coordinate system alignment is required for improved comparisons from scenes to computer generated models. Oftentimes, the scanning coordinate system does not pre align with the encompassed modelled geometry and additional alignment is necessary. In this study, we investigate the effect of different surfaces of an Invar 36 cube using a laser line probe when undertaking coordinate system alignment using planes. Random sample consensus (RANSAC), least squares (LS) are statistical algorithms that will be compared and combined to determine the underlining optimal transformations of the world coordinate system. In addition, 3D cube models are 3D printed to be contrasted with the fitting algorithms performance to the Invar 36 cube. The surface finish of these models produce more deviations and outliers when being laser scanned with visible defects.


## 1 Introduction

Alignment is a crucial initial step of reverse engineering [1] and comparisons of as-built to as-designed models. Correct coordinate alignment enables more fluid dimensioning and easier comparisons from the design intent. Likewise, in the case of scanners where the origin is usually located at the base of the machine, the areas of interest are commonly found a distance away [2]. Moreover, closer coordinate alignment to the scene reduces the file size as the coordinates are less extensively spread out. Numerous computer aided design (CAD) packages support different modes of alignment in relation to user defined specifications, with this study we are focusing on the three plane alignment. This process aims
to investigate any shortcomings of the coordinate alignment when determining the cube sizes and surface finishing used with a non-contact laser line probe (LLP) scanner.

## 2 Alignment Process Setup

Four different sized cubes have been 3D filament deposit modelled (FDM) printed to ascertain the effects of printing quality and scanning coverage on the fitting of the subsequent planes. The cubes were printed on a Prusa Mini with a Prusa PLA filament of 0.2 mm seen in Figure 1. There are visible layering occurring with the prints which induces surface variations to the cubes (in addition to other 3D manufacturing defects such as shrinking and warping of surfaces). The differences in printed volume size affect the waviness of the surface strongly, as this study aims to mimic a wide range of high density points with noise surface finishes captured by the scanner.


Figure 1: 3D printed cubes
The " 50 mm " Invar 36 cube has three orthogonal sides with peripheral milling up-cut milled, and the opposite three orthongonal sides with a face milling surface (creating the six faces of the cube). A relative dimension of 50 mm for the Invar 36 cube was chosen due to the higher reflection of the surface finish corresponding to a lower point density obtained when scanning (requiring a larger surface area of scan). All cubes have been scanned by a LLP with a compensation error of 0.0194 mm .


Figure 2: Invar 36 " 50 mm " cube, face milling (left), peripheral milling (right).

## 3 Results

### 3.1 Laser Line Probe Scanning

When scanned, the largest 3D printed cube ("100 mm") has a point cloud size of 15 million points, the " 50 mm " cube has a size of 4 million points, the " 25 mm " cube has a size of 1.7 million points, and the " 10 mm " cube has a size of 500 thousand points. The final dimensions of the printed cubes are summarised in Table 1 where measurements are taken with three different micrometres and multiple measurements taken across the surface of the cube.

Table 1: FDM 3D printed cubes dimensions

| 3D FDM Cube | Width | Length | Height | Note Equipment |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{m m}$ (uncertainty k=2) |  |  |  |
| $" 10 \times 10 \times 10 "$ | 9.90 | 9.76 | 9.83 |  |  | $\pm 20 \mu \mathrm{~m}$ | Micrometre |
| $" 25 \times 25 \times 25 "$ | 24.927 | 24.857 | 24.739 | $\pm 1 \mu \mathrm{~m}$ | Micrometre |
| $" 50 \times 50 \times 50 "$ | 49.858 | 49.807 | 49.567 | $\pm 1 \mu \mathrm{~m}$ | Micrometre |
| $" 100 \times 100 \times 100 "$ | 99.72 | 99.83 | 99.54 | $\pm 20 \mu \mathrm{~m}$ | Micrometre |

In contrast, the " 50 mm " Invar 36 cube point cloud density was observed to be influenced by the direction of the scan relative to the surface type that was scanned. Lowest point cloud density was seen to be 200 thousand points, and peaking at 300 thousand points when scanning the peripheral milling surface finish (Figure 3). In each case only one pass of the plane surface was conducted. The scanner settings were adjusted for no filtration of points occurring whilst scanning. The dimensions of the machined Invar 36 cube are seen in Table 2.


Figure 3: Invar 36 peripheral milling surface finish 3D LLP scan.
Table 2: Invar 36 cube dimensions

| Invar 36 Cube | Width | Length | Height | Note Equipment |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{m m}$ (uncertainty $\mathrm{k}=2$ ) |  |  |  |  |
| "50x50x50" | 50.025 | 50.002 | 50.005 | $\pm 1 \mu \mathrm{~m}$ | Micrometre |

### 3.2 Plane Fitting

The unit normals of the xyz points are calculated on a hybrid approach taking the nearest and radius based search of the points. These points are then oriented to correspond to the correct orientation to the cubes surfaces. This encompasses a portion of the surface deviations present from the manufacturing.

Rough RANSAC [3] is calculated for planes and segmented out in to the five individual planes (three planes for Invar 36 cube). These planes contain the manufacturing artefacts, including the waviness of the surface and defects. A fine normal filtration is then performed where the median unit normals of each subset of the rough planes is calculated and the angle of each point is filtered (where a is the median unit normal and n is the unit normal):

$$
a \cdot n=|a| \times|n| \cos \alpha
$$

This produces five filtered planar surfaces from the four printed cubes and 6 filtered planar surfaces (three per surface type) for the Invar 36 cube. Next LS linear regression was utilised on these filtered planes to obtain the corresponding coefficients of the plane equation (2), where $p_{i}$ are the xyz points, $n$ is the unit normal, and c is the centroid. With the following 3D Cartesian plane equation used for the plane fitting of the linear regression (the coefficients $a, b, c$ are the normal of the 3D plane):

$$
\begin{gather*}
\sum_{i=1}^{n}\left(\left(p_{i}-c\right)^{T} n\right)^{2}  \tag{2}\\
a x+b y+c z+d=0 \tag{3}
\end{gather*}
$$

An example of the output of Cartesian coefficients when undertaking the above steps is seen in Table 3.

Table 3: Plane Cartesian coefficients 10 mm cube

| Cube | Plane | $\tilde{\mathrm{a}}$ | $\tilde{\mathrm{b}}$ | $\tilde{\mathrm{c}}$ | $\tilde{\mathrm{d}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | I | 0.00 | -1.00 | 0.00 | -8.20 |
|  | II | 0.02 | 0.00 | 1.00 | -740.54 |
| "10×10×10 | III | 0.02 | -0.00 | 1.00 | -731.59 |
|  | IV | -1.00 | -0.01 | 0.03 | -174.26 |
|  | V | -1.00 | -0.01 | 0.03 | -164.91 |

### 3.3 Distance Computation

In the Figures 4 and 5 below the planes are arranged in order of largest point cloud to the smallest point cloud density plane (plane one being most dense and plane five least dense). RANSAC (blue) plane fitting was plotted next to normal
filtration (red) step plane fitting. Using the following formula the distance from a point projected onto the plane is calculated:

$$
\text { Distance }=\frac{\tilde{a} x_{i}+\tilde{b} y_{i}+\tilde{c} z_{i}+\tilde{d}}{\sqrt{\tilde{a}+\tilde{b}+\tilde{c}}}
$$


(b) Cube " $25 \times 25 \times 25$ "



Figure 4: Distance calculations from points to plane comparing normal filteration (red) with RANSAC filtration (blue), (a) " 10 mm " cube (b) " 25 mm " cube (c) " $50 \mathrm{~mm} "$ cube (d) " $100 \mathrm{~mm} "$ cube.


Figure 5: Invar 36 cube where orange and purple represent perihperal milled, green and yellow patterend colour depict face milled surfaces.

### 3.4 Coordinate Alingment

To obtain the origin, the intersection of three planes is calculated from the three highest density point cloud planes, accordingly the following equations are set up from the previous Cartesian equation:

$$
\begin{aligned}
& \tilde{a} x_{1}+\tilde{b} y_{1}+\tilde{c} z_{1}+\tilde{d}_{1}=0 \\
& \tilde{a} x_{2}+\tilde{b} y_{2}+\tilde{c} z_{2}+\tilde{d}_{2}=0 \\
& \tilde{a} x_{3}+\tilde{b} y_{3}+\tilde{c} z_{3}+\tilde{d}_{3}=0
\end{aligned}
$$

The coordinates of the origin is used in the transformation of the coordinate system to the centre at this point. All four corners of intersections by planes are depicted by the pink spheres within the figures below using the same radius sphere to showcase scale. The Cartesian planes from the normal fitted points are drawn and bounded by the points (light blue, green, and brown in Figures 6 and 7).



Figure 6: Filtered normal plane points top left " 10 mm " cube, top right " 25 mm" cube, bottom left " 50 mm " cube, bottom right " 100 mm " cube.

The same procedure is followed for the Invar 36 cube (Figure 7), however, instead of four spheres from the five planes, there is only one intersection point ( 3 planes per each surface finish). The same radius sphere is plotted in these figures as in Figure 6.


Figure 7: Invar 36 cube filtered normal plane points, left peripheral milled, right face milled.

Obtaining the origin, the z axis is set to correspond between the intersection of the most dense point cloud planes one and two (Figure 4 and 5) creating a vector. The other axis are oriented to best represent their respective vector directions (from the intersection of their corresponding planes), whilst being orthogonal to the z axis vector. The final orientation creates a coordinate system orthogonal to each axis. These transformations are transferred for a coordinate alignment seen below, where the blue arrow depicts the z axis, green arrow the y axis, and red arrow the x axis. The colour gradient shows the direction of the +z axis, closer being a darker shade of blue and further being red.


Figure 7: Final coordiante alignment on (a) " $10 \mathrm{~mm} "$ (b) " 25 mm " (c) " 100 mm " (d) and (e) " $50 \mathrm{~mm} "$ cubes.



Figure 8: Final coordiante alignment on Invar 36 cube (a) and (b) peripheral milled, (c) and (d) face milled.

## 4 Conclusion

During this study a range of different surface finishes and cube sizes were 3D laser line scanned. Adverse conditions were created from reflective to high deviation surface finishes to trial coordinate system alignment. From the observations, it was concluded that the 3D printed " 25 mm " cube was sufficient in obtaining the required geometry for alignment when using a non-reflective surface for a high density point cloud coverage. Peripheral milling surface finish on the Invar 36 cube was chosen as a more optimal surface to laser line scan when compared to the face milling finish. However, both surfaces were prone to high reflectivity from the scanner seen in Figure 8. The increased surface area of the " 100 mm " printed cube was not observed to increase the accuracy of the coordinate alignment due to the inherent manufacturing defects introduced which were not offset by the larger point cloud.

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## References

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