Machine tool thermal state representation using modal analysis

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Abstract

Thermally induced deformations degrade the performance of machine tools leading to dimensional errors in manufactured products. Therefore, models are often used to map related observed data such as temperature of key points of the structure to the resultant thermal errors. Predictions from these models are then factored in to the controller commands to offset the errors. However, these data driven models can only learn from the experiences recorded in their training data. Therefore, being able to quantify the state of the machine tool from the data can lead to better modelling results.

This work proposes a novel approach for representing the thermal state of a machine tool. Modal analysis and K-Means clustering are used to extract the descriptor Proper Orthogonal Decomposition (POD) modes in the temeprature data which encode the thermal state of the machine tool. These descriptor POD modes identify the different conditions experienced during machining. These features are then used in determining whether any future observed data contains thermal states in the training process. The results obtained show that the appraoch is able to quantify the differences in the machine's thermal state. These finding will be used to improve thermal error modelling in machine tools.

1 Introduction

Thermally induced errors contribute up to 75% of the overall geometric errors in machine tools[1]. Compensating for these errors using empirical models offers the advantage of being both computationally efficient and applicable after the design and manufacture of the machine tool. Various empirical models have been presented for this use including Neural Network models, Adaptive Neuro-Fuzzy Inference System (ANFIS) models, and linear regression models. In most cases these models are trained off-line and then used online under machining conditions that may differ from those experienced in the training data. This leads to an increase in uncertainties in the mode's predictions. Approaches that have been proposed to reduce this uncertainty include online adaptive schemes that tune the model's parameters using process-intermittent probing. Different approaches have been presented for determining when to probe and update the model. This include using a high sampling rate when training the model and a lower sampling rate when interrupting the machining process to update the trained model [2]. In [3] an action control limit is placed on the model's predictions which determines when to trigger the probing procedure needed to tune the model. A challenge that may be faced with these approaches is having to retrain a model on data that the model has previously experienced either in the training or update phase. This may lead to unnecessarily high number of probing cycles resulting in reduced throughput in machining. This work proposes the use of modal analysis in identifying machining conditions that have been experienced in the modelling data which can then be used in determining when to gather more data to update the thermal error model. In the instances mentioned above, a compact representation of the thermal state of the machine tool would improve the modelling approaches.

High dimensional data is observed to exist in low-dimensional subspaces in many fields such as image processing, computer vision, pattern recognition and data compression [4]. This can be compared to data in a three dimensional space being closely scattered about a vector or a plane. These low-dimensional subspaces can be extracted through subspace clustring approaches using Principal Component Analysis (PCA) and its variants [5]. PCA is a useful tool in applications such as dimensionality reduction, data visualisation, regression, classification and clustering [6]. An equivalent approach, Proper Orthogonal Decomposition (POD), is used in the analysis of fluid flows, partial differential equations among others from discreete snapshots of data [7], [8]. This approach of snapshots enables the use of POD analysis nonlinear systems even though it is designed for use with linear systems.

This work makes use POD to extract modal data (POD modes) from discreet snapshots of machining data. Sampling and K-Means clustering are then used to extract modal descriptors which encode the thermal state of the machine. These descriptors are then used in determining how similar to the training data is any other data from the machine tool.

2 Methodology

The process of exracting POD modal descriptors of the training data is summarised in *Figure 1*. This process begins with collecting the training data from from the experiment. Snapshots of the input (temperature) data are then obtained by slidding a window of fixed length across the data. A moving average filter is applied to the temperature data to reduce noise in the signal. POD modes are then computed for each snapshot. Random Sample and Consensus (RANSAC) [9] is then used to extract modal descriptors of the training data. To do this, random consecutive modes are then selected and their cluster centroids obtained using K-Means clustering. This sampling and clustering process is repeated for a fixed number of times with the cluster centroids being saved each time. The distance of each POD mode in the training data from each of the saved cluster centroids is calculated. The number of distances whose value is lower than a set threshold is obtained. The cluster centroid with the highest number of distances below the threshold is saved as one of the modal descriptors of the data. The POD modes whose distance fall below the threshold of that modal descriptor are then removed from the POD modes data under evaluation and the process repeated. This is done until the stop criterion is met. All the saved modal descriptors are then used to identify the training data.

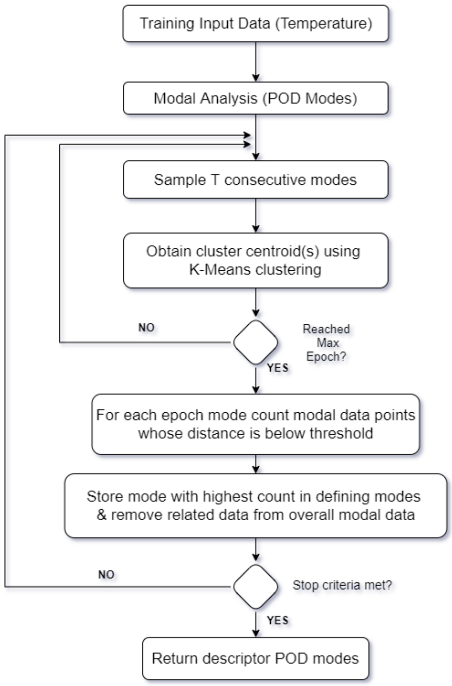

Figure 1: Process of extracting descriptor POD modes from training data

2.1.1 Proper Orthogonal Decomposition (POD)

Each snapshot, $X \in R^{m \times k}$, consists of k discrete samples from m temperature sensors. The mean of each snapshot is subtracted then Singular Value Decomposition (SVD) is used to extract the Proper Orthogonal Decomposition (POD) modes of the data. This was done using MATLAB R2020a svd command.

$$X = U\Sigma V^* \tag{1}$$

The result of SVD are left singular vectors, $U \in \mathbb{C}^{m \times k}$, which define the subspaces (k vectors) in which the snapshot data is spread. These are the POD modes of X. The singular values, $\Sigma \in \mathbb{C}^{k \times k}$, is a diagonal matrix whose values are in decreasing order. Each of the k singular values captures the scaling of the data along the corresponding left singular vectors. A sharp decrease in the singular values indicates that most of the data is spread in the leading left singular vectors which forms the basis for subspace clustering and data compression approaches. Finaly, the right singular vectors, $V \in \mathbb{C}^{m \times k}$, encode the temporal dynamics in the snapshots which is not made use of in POD analysis.

2.1.2 K-Means Clustering

T consecutive POD modes are sampled from a random point in the training data. K-Means algorithm is used to group each sampled POD modes, $\Phi \in R^{m \times T}$, into two clusters. Clustering is done using the cosine distance metric which quantifies the angular distance between vectors. The sampling window is made small enough such that it can be assumed to contain modes associated with a heating or cooling cycle or both. Clustering is done by randomly selecting two modes in that act as the cluster centroids, $\{\varphi_{c_i}\}_{i=1}^{2}$. The distance of each mode from the centroids is calculated.

$$d(\varphi_j, \varphi_{c_i}) = 1 - \langle \varphi_j, \varphi_{c_i} \rangle \tag{2}$$

Where $\langle \varphi_j, \varphi_{c_i} \rangle$ signifies the inner product. These distances are used in determining the cluster that each j^{th} POD mode belongs to.

$$c_j = \arg\min_{i=1,2} d(\varphi_j, \varphi_{c_i}) \tag{3}$$

The centroids are then updated by averaging the POD modes in their cluster according to:

$$\varphi_{c_i} = \frac{\sum_{j=1}^{T} 1_{c_j=i}\, \varphi_j}{\sum_{j=1}^{T} 1_{c_j=i}} \tag{4}$$

Where $j = 1,2$ and $1_{c_j=i} = 1$ if j^{th} POD mode belongs to the i^{th} cluster and zero otherwise. Steps represented by equations (2) to (4) are repeated to convergence. The distance between the cluster centroids is then evaluated. If this distance is above a set threshold then the sampled POD modes are assumed to have contained POD modes from two distinct heating or cooling cycles. Conversely, if this distance is below the threshold then the sampled POD modes are assumed to have been from the same unique heating or cooling cycle and a single centroid is obtained and returned by another K-Means clustering process.

2.1.3 Random Sample Consensus (RANSAC)

Sampling and K-Means clustering is performed for a set number of times, P, which returns a maximum of P centroids. RANSAC algorithm is used in determining which centroids returned from the sampling and clustering run are most significant in defining the data. First, the cosine distance between each centroid and the sum of the number of POD modes whose distance lies below set threshold obtained.

$$s_p = \sum_{j=1}^{K} \left(d\left(\varphi_j, \varphi_{c_p} \right) \leq Threshold \right) \tag{5}$$

Where φ_j is j^{th} POD mode of all the K POD modes in the training data and φ_{c_p} is the p^{th} centroid. The most significant centroid that defines the data is chosen as the one that maximizes the number of POD modes that lie below the set threshold:

$$\varphi_d = \arg \max_{p=1\ldots P} s_p \tag{6}$$

2.2 Experiment setup

To test the performance of the proposed approach, experiments were carried out on a small vertical machining centre (VMC). The machine was subjected to two cycles, a random duty cycle heating of the spindle and another random duty cycle heating of the spindle and Z axis. The thermal state of the machine was captured using twenty seven temperature sensors placed in strips at the carrier, spindle boss, axes motors and axes ballscrew nuts. The setup is summarised in *Figure 2*. A sampling rate of 0.1 Hz was used.

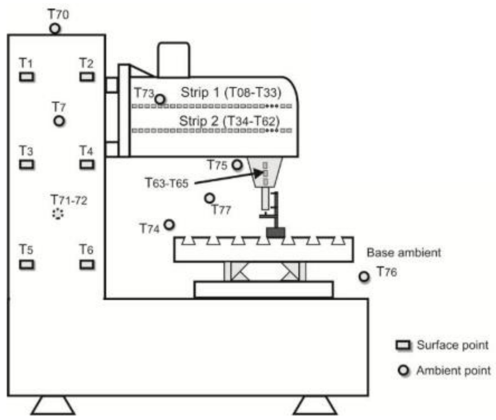

Figure 2: Experiment setup showing location of temperature sensors

3 Results and Discussion

The temperature data collected from the twenty-seven temperature sensors in the training phase is summarised in the first plot of *Figure 3*. This data was collected under random duty cycle heating of the spindle conditions. POD modes were then calculated at each time step from the recent 120 samples (window size) of data resulting in a 27-dimensional vector. These vectors were plotted as a surface plot in the second plot of *Figure 3*. This plot consists of coherent patterns which can be used in identifying the thermal state of the machine. The extracted POD modes show a strong alignment along the 16th dimension. This signifies a relatively high dispersion in the 16th temperature sensor of the data which was attached to the spindle motor bolt. Though there seems to be a direct mapping from temperature sensor to the dimension of alignment of the POD mode, this is not always the case. For example, after the 300th minute, the rate of decrease in temperature readings reduces to a rate matching other temperature sensors. This results in a shift in the POD mode alignment. Thus, the POD modes can encode the dispersions in the data observed over a period to a vector representation.

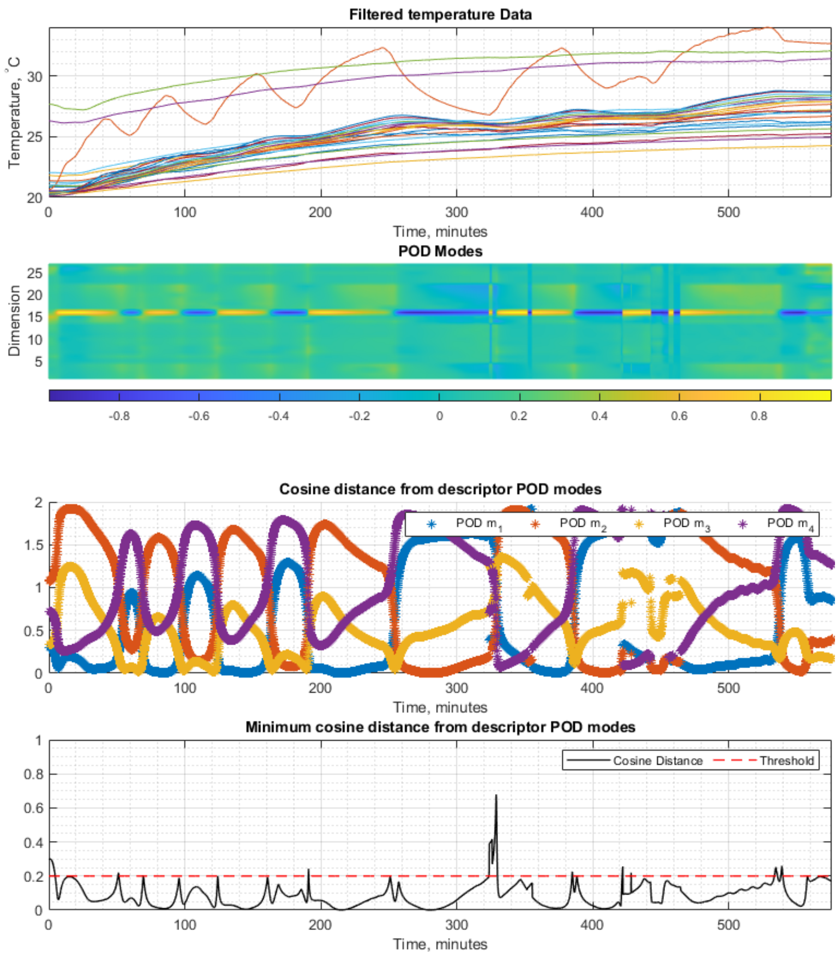

Figure 3: Analysis of temperature data in training dataset

Descriptor POD modes were extracted from the POD modes of the training data using the methodology presentent in section 2. This resulted in four descriptor POD modes as shown in *Figure 4*. A centroid returned by the K-Means clustering algorithm was considered a descriptor POD mode it had a cosine distance lower than 0.2 from more than 200 samples of POD modes. The cosine distances from these descriptor POD modes to all other POD modes of the training data are shown in the third plot of *Figure 3*. The minimum cosine distance from the descriptor POD modes to the training data POD modes is shown in the fourth plot of *Figure 3*.

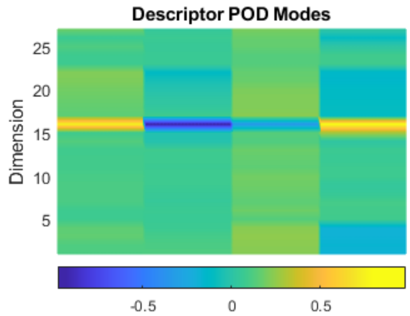

Figure 4: POD mode descriptors extracted from the training dataset

The temperature data and related POD modes of the first test dataset, dataset A, are shown in the first and second plots of *Figure 5* repsectively. Dataset A was collected under random duty cycle heating of the spindle and the Z axis conditions. Coherent patterns in the POD modes show that the data is spread within a low dimensional space as did the training data POD modes. The cosine distance from the descriptor POD modes of the training data to POD modes of this test dataset is shown in the third plot of *Figure 5*. The minimum value of the calculated distances is shown in the fourth plot of *Figure 5*. The distances obtained have values significantly higher than the evaluation threshold value of 0.2. Thus, the approach is able to give a measure of difference between thermal states observed in the data. This disperity results from the fact that cycles run on the machine tool resulting in test dataset B were different to those run in to obtain the training dataset.

The second test dataset, dataset B, was collected under machining cycles similar to those of the training dataset. The resultant temperature data and related POD modes are shown in the first and second plots of *Figure 6* repsectively. The observed coherent patterns in the POD modes resemble those obtained from the training data. Evaluating the POD modes of this dataset using the descriptor POD of the training dataset results in the third and fourth plots of *Figure 6*. The minimum cosine distance values obtained indicate a close resemblance of the machine tool’s thermal state to that experienced under the

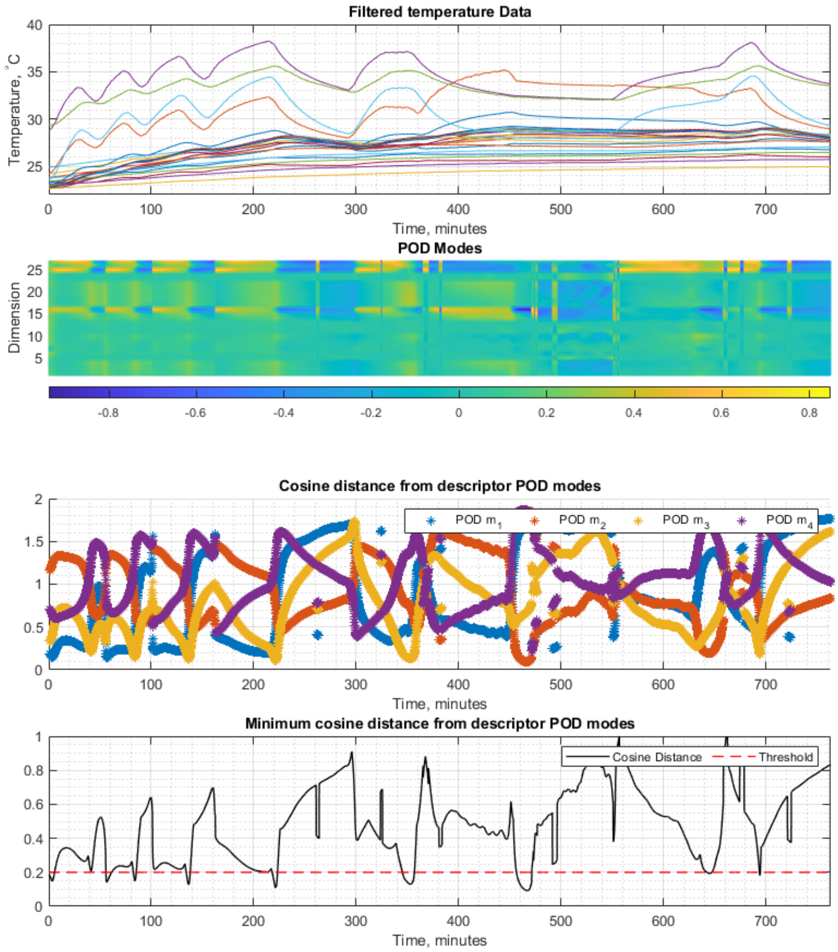

Figure 5: Analysis of temperature data in test dataset A

training data machine cycles. With the exception of values about the 300th, 410th and 550th minutes, most of the cosine values fall bellow the threshold value. The pattern observed in the POD modes at those time stamps are similar and also match the pattern observed in the POD modes of the training data about the 320th minute of the second plot in *Figure 3*. Thus, these exceptions in performance may be resolved by further tuning of the algorithm that extracts the descriptor POD modes.

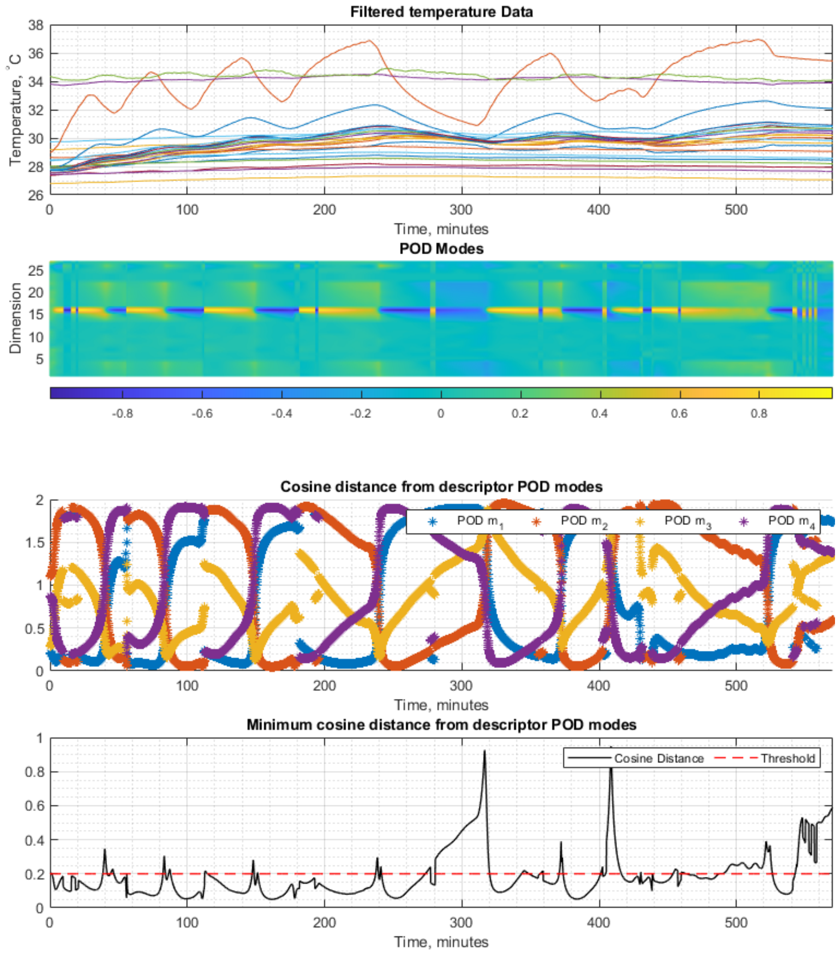

Figure 6: Analysis of temperature data in test dataset B

4 Conclusion

A novel approach for quantifying the thermal state of a machine tool from temperature data has been presented in this paper. Descriptor POD modes that encode the dispersion of temperature readings within snapshots of the data were obtained from a training dataset of machining data. The results indicate that the approach does quantify how similar a test dataset is to the training dataset. Similar thermal states have distance values below the evaluation threshold. The number of sensors to use and their placement should adequately detect the thermal gradients that significantly affect the thermal error of the machine tool. The approach can then be used to inform the experiments to be carried out rather than carrying out a lot of tests that induce heat in a way that would not be seen during normal operation.

The uncertainty of decisions using the first POD mode is captured in the singular values. These values measure the variability of the data represented in each of the POD modes. Further study is being carried out to determine the uncertainty introduced by other factors such as window size selection, resolution of the sensors used, signal to noise ratio and hysterysis.

5 Acknowledgement

The authors gratefully acknowledge the UK's Engineering and Physical Sciences Research Council (EPSRC) funding of the Future Metrology Hub (Grant Ref: EP/P006930/1).

References

[1] J. Mayr *et al.*, "Thermal issues in machine tools," *CIRP Ann. - Manuf. Technol.*, vol. 61, no. 2, pp. 771–791, 2012, doi: 10.1016/j.cirp.2012.05.008.

[2] H. Yang and J. Ni, "Adaptive model estimation of machine-tool thermal errors based on recursive dynamic modeling strategy," *Int. J. Mach. Tools Manuf.*, vol. 45, no. 1, pp. 1–11, 2005, doi: 10.1016/j.ijmachtools.2004.06.023.

[3] J. Mayr, P. Blaser, A. Ryser, and P. Hernandez-Becerro, "An adaptive self-learning compensation approach for thermal errors on 5-axis machine tools handling an arbitrary set of sample rates," *CIRP Ann.*, vol. 67, no. 1, pp. 551–554, 2018, doi: 10.1016/j.cirp.2018.04.001.

[4] E. Elhamifar and R. Vidal, "Sparse subspace clustering: Algorithm, theory, and applications," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 35, no. 11, pp. 2765–2781, 2013, doi: 10.1109/TPAMI.2013.57.

[5] F. W. Townes, "Generalized Principal Component Analysis," *arXiv*, vol. 27, no. 12, pp. 1945–1959, 2019, doi: 10.1007/978-981-10-2915-8_7.

[6] I. T. Jollife and J. Cadima, "Principal component analysis: A review and recent developments," *Philos. Trans. R. Soc. A Math. Phys. Eng. Sci.*, vol. 374, no. 2065, 2016, doi: 10.1098/rsta.2015.0202.

[7] K. Lu *et al.*, "Review for order reduction based on proper orthogonal decomposition and outlooks of applications in mechanical systems," *Mech. Syst. Signal Process.*, vol. 123, pp. 264–297, 2019, doi: 10.1016/j.ymssp.2019.01.018.

[8] S. Sahyoun and S. Djouadi, "Local Proper Orthogonal Decomposition based on space vectors clustering," *2013 3rd Int. Conf. Syst. Control. ICSC 2013*, pp. 665–670, 2013, doi: 10.1109/ICoSC.2013.6750930.

[9] M. A. Fischler and R. C. Bolles, "Random sample consensus," *Commun. ACM*, vol. 24, no. 6, pp. 381–395, Jun. 1981, doi: 10.1145/358669.358692.