Assessment of a laser-based multilateration system for measurement of low-slope metre-scale surfaces

J. Norman, C. Giusca, X. Tonnellier
Cranfield University, UK

Abstract

The manufacture of metre-scale mirror segments for telescopes requires measurement with low uncertainty to enable accurate form correction. The state-of-the-art system for measurement of these surfaces has a length measurement uncertainty of 1.2 µm over 1 m. A four laser tracker multilateration system has been proposed and tested with the aim to determine whether a system can measure the specified surfaces with measurement uncertainties below 1 µm. Influence parameters that affect the multilateration input parameters have been identified and utilised in Monte Carlo simulation of the multilateration system to estimate the uncertainty associated with the coordinate measurement. Multilateration measurements of a 0.4 m × 0.4 m flat optic and of a 0.2 m × 0.2 m ULE® flat resulted in a measurement solutions with an associated standard deviation of 0.73 µm and 0.26 µm, respectively. The Monte Carlo simulations indicate the independence of measurement area and measurement noise for the experimental setup tested. It is concluded that a laser-based multilateration system can measure a metre-scale optic with z-coordinate measurement uncertainty below 1 µm. A 15.5 h measurement of the larger flat was carried out to determine the effect of time dependent parameters on measurement uncertainty. The measurement solution had a standard deviation of 1.88 µm. Further work is required to compensate for time dependent measurement error.

1 Introduction

Manufacture of metre-scale mirror segments for large telescopes requires measurement of these segments early in the manufacturing process chains whilst the surface is non-specular. Small uncertainty associated with low- slope surface measurement early in the manufacturing process enables more accurate form correction with relatively fast machining processes, such as fixed abrasive grinding, thus reducing total manufacturing time [1]. There are several measurement systems with capability to measure optics with relatively low uncertainty, including deflectometric measurement systems [2]. The state-of-the-art system that meets the requirement for measurement of these surfaces is the Leitz PMM-C coordinate measuring machine (CMM) [3], with length measurement uncertainty (distance between measured coordinates) of 1.2 μm (k = 1) over 1 m; therefore, this research aims to investigate whether a measurement system based on the principle of laser-interferometry multilateration can measure the specified surfaces with a standard measurement uncertainty (σ_z) below 1 μm.

The system utilises four laser trackers to measure the displacement of a reflector as it is moved over a surface to be measured. A large CMM was used to translate the reflector and as a reference measurement.

An assessment of influence factors enabled their propagation through a system model to determine estimates of coordinate measurement uncertainty using Monte Carlo (MC) simulation. Experimental testing involved the z-coordinate measurement of two artefacts. Additionally, the effect of changes to the environment on measurement uncertainty over a 15 h period were investigated to assist with the impact of long-term influence of the environment required for high density measurements of mirror segments.

2 Multilateration measurement method

Multilateration is a method for determining Cartesian coordinates of a set of measured positions based on range measurements of these positions from a number of measuring stations. For this measurement setup, steerable displacement measuring interferometers – the interferometric mode on laser trackers – are used as measuring stations [4, 5]; whilst a spherically mounted reflector (SMR) is used as a measurement target. By moving the SMR between multiple measurement positions, and recording range displacement, it is possible to self-calibrate the measurement system to determine the relative positions of measuring stations and measurement targets; this is calculated via multilateration and is described within [6-8]. Utilising at least four measuring stations, and measuring many positions, allows the calculation of measurement target positions using the following equation set:

$$(X_i - x_j)^2 + (Y_i - y_j)^2 + (Z_i - z_j)^2 = (dp_{1j} + d_{ij})^2, \quad (1)$$

where X, Y, and Z are the coordinates of the measured positions; i = 1, ..., N where N is the total number of positions; x, y, and z are the coordinates of the measuring stations; and j = 1, ..., M where M is the number of stations. dp_{1j} are the interferometric deadpaths between each of the measuring stations and the initial measured position. d_{ij} are the measured displacements of the SMR relative to the measuring station positions. Figure 1 shows the system setup.

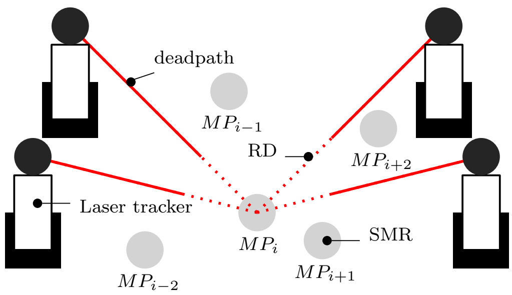

Figure 1: The measuring principle for multilateration self-calibration. *MP* are the measured positions of the SMR and *RD* is the radial displacement measured between each target measurement position.

3 Input error analysis

Identification of the influence factors that affect the self-calibration measurement routine is critical to analysing the multilateration measurement system and estimating measurement output uncertainties. The influence factors are defined as either time independent or time dependent.

3.1 Time independent effects

The time independent influence factors that affect the measurement of positions are: interferometric range displacement measurement uncertainty, SMR sphericity, laser interferometer deadpath measurement uncertainty, arrangement of measuring stations, arrangement and number of measured positions, and SMR viewing angle.

The interferometric range displacement uncertainty is length dependent. SMR sphericity affects the measured range by inducing an uncertainty caused by change in the distance between SMR surface contact point and SMR optical centre.

The arrangement of measuring stations affects the multilateration self-calibration: Takatsuji *et al*. [6] have shown that the position of measuring station four should be outside the plane defined by the positions of measuring stations 1-3. If station four is in or close to the plane, there are an infinite number of solutions to the set of equations (1). Zhang et al. [9] detail a parameter that relates the arrangement of measuring stations and measured positions to the multilateration self-calibration solution uncertainty.

The SMR viewing angle dictates the positioning of measuring stations and measured positions: laser trackers must be positioned within the zone defined by the SMR viewing angle and measurement volume.

3.2 Time dependent environmental effects

The time dependent influence factors are caused by changes in the measurement environment: specifically, local temperature changes; this can result in thermal drift within the measurement setup. Thermal drift can affect, among others: a laser tracker's home position stability and interferometric displacement measurement stability.

4 Experimental setup and method

A laser-based multilateration measurement system that used four Leica AT960LR laser trackers was tested. During the measurement a Leica Super CatEye Reflector (SCE) was used as a measurement target. The SCE was integrated with a compatible CMM probe. The experimental setup is shown in figure 2. Two sample surfaces were used during this investigation: a 0.4 m × 0.4 m flat optic and a 0.2 m × 0.2 m ULE® flat optic; a CMM was used to move the SCE-probe into a 10 × 10 grid and 6 × 6 grid of equally spaced points over the measurand surfaces respectively. Figure 2 shows the setup.

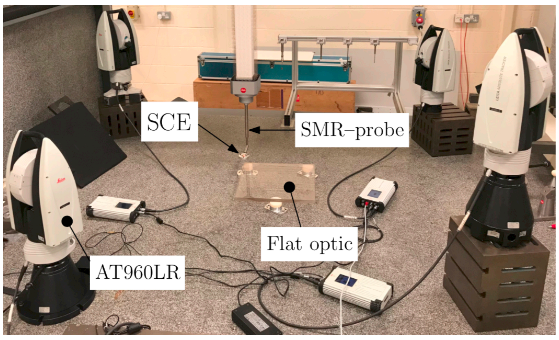

Figure 2: Four AT960LRs installed on a CMM granite base. The CMM is moving an SCE-probe across the surface of a 0.4 m × 0.4 m flat optic.

5 Multilateration results

The multilateration measurement solutions are detailed in this section. The displacement measurements undertaken in the measurement method described in section 4 were utilised in the calculation of Cartesian coordinates with equation (1).

5.1 Measurement of 0.4 m × 0.4 m flat optic

Figure 3 shows the z-coordinates of the multilateration solution and the equivalent CMM measurement, as well as the difference between the multilateration solution and the CMM measurement. The standard deviation of Δz is $\sigma_{\Delta z} = 0.73$ µm. The peak-to-valley of $\Delta z \approx 3.3$ µm.

The large optic was measured within the multilateration system to test the effect of measured area size on the measurement uncertainty in z-coordinates. The Δz map has a saddle form that indicates the presence of a systematic error source. When the saddle is removed with a second-order polynomial fit, $\sigma_{\Delta z} = 0.28$ µm. This gives an indication of the noise-level for the multilateration setup tested with a measurement on an optic of this size. This result is clearly correlated between the uncertainty of the CMM measurement and the multilateration solution uncertainty.

The square root mean variance of 10 repeated multilateration measurements was σ_z = 0.43 µm; however, this result is correlated between the multilateration solution uncertainty and the CMM positioning repeatability: repeated measured positions vary with x and y due to SMR-mover repeatability, this results in a difference in z caused by the relatively high form of the sample under investigation.

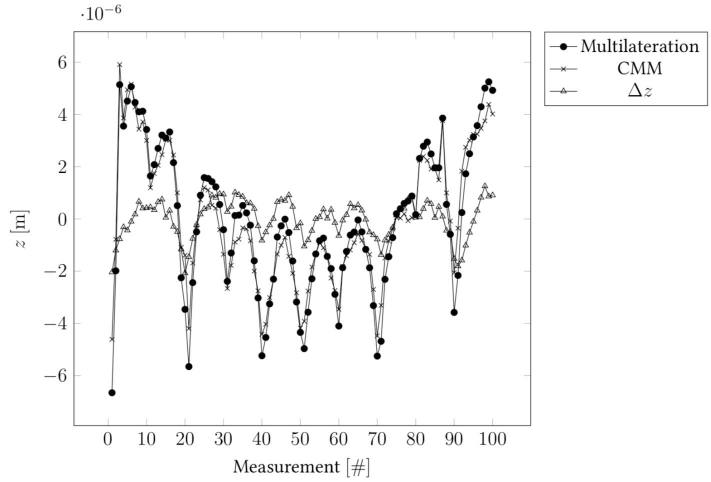

Figure 3: The z-coordinate measurements of a 10 × 10 grid of measured positions on the surface of a 0.4 m × 0.4 m flat optic, as determined via: four-tracker multilateration and a CMM measurement. The difference, Δz, between the z-coordinates of the multilateration and CMM measurements is plotted.

5.2 Measurement of 0.2 m × 0.2 m ULE® flat optic

Figure 4 shows the z-coordinates of the multilateration solution and the equivalent CMM measurement, as well as the multilateration solution with a second-order fit removed. The standard deviation of z-coordinates of the multilateration solution is σ_z = 0.26 µm and a peak-to-valley of ~1 µm. The standard deviation of z-coordinates of the CMM measurement is σ_z = 0.19 µm and a peak-to-valley of approximately 0.6 µm. The standard deviation of z-coordinates of the multilateration solution, with a second-order polynomial fit removed, is σ_z = 0.19 µm. The most significant difference the multilateration z-coordinates, with and without the fit removed, occurs at the first and last points of the data sets.

The solution for z-coordinate measurements of the 0.2 m × 0.2 m ULE® flat optic have relatively high noise. Despite this, there is a form in the z-coordinates that has a similar saddle shape to the solution for the measurement of the larger optic. This further indicates the presence of a systematic error source within the multilateration solution. When this saddle is removed, the multilateration solution σ_z is equivalent CMM σ_z result.

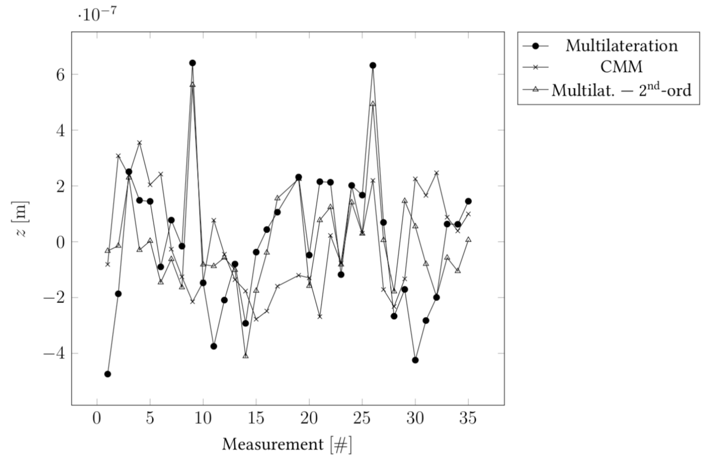

Figure 4: The z-coordinate measurements of a 6 × 6 grid of measured positions on the surface of a 0.2 m × 0.2 m ULE® flat optic, as determined via: four-tracker multilateration, a CMM measurement, and the multilateration solution with a second-order polynomial fit removed.

5.3 Long-term measurement of 0.4 m × 0.4 m flat optic

The long--term measurement of a 0.4 m × 0.4 m flat optic was conducted with an equivalent measurement setup to that shown in figure 2; however, 34 positions were measured across the surface of the sample. The total measurement time was 15.5 h. The time between the measurement of each position was 28 minutes.

Figure 5 shows a comparison in the z-coordinates measured by the long-term measurement of the large flat optic and both the equivalent long-term CMM and short-term multilateration measurements. This has been achieved by using the equivalent short-term CMM measurement as a reference measurement. The difference, in z, between the three measurements — long-term multilateration and CMM measurements, and short-term multilateration measurement — and the short-term CMM measurement are shown.

The difference between long-term multilateration and short-term CMM measurements is significantly larger than either of the other measurements — long-term CMM or short-term multilateration. The standard deviation of the difference between long-term multilateration and short-term CMM measurements is $\sigma_{\Delta z}$ = 1.88 µm, with a peak-to-valley of ~6 µm.

The long-term measurement of the large flat optic aimed to determine the effect of temperature change, ΔT, on the measurement uncertainty of the four-tracker multilateration system. Figure 5 shows how Δz varies as a function of measured position; and therefore, as the time difference between measuring each point was ~ 0.5 h, this shows how Δz varies as a function of time.

The differences in z for the long-term measurement of the large flat, relative to the short-term measurements, follows a U-shape as a function of time; with the greatest magnitude of difference at the beginning, middle, and end of the measurement period.

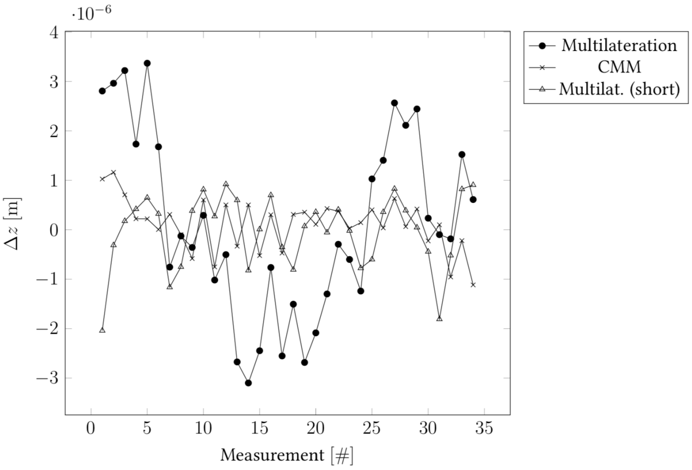

Figure 5: Measurement of 34 measured positions on the surface of a 0.4 m × 0.4 m flat optic was conducted with both short- and long-term four-tracker multilateration and CMM measurements. The difference in z-coordinates of the long-term measurements and short-term CMM measurement, and the difference in z-coordinates the short-term multilateration solution and short-term CMM measurement are plotted.

6 Monte Carlo simulation

Estimates for the output parameter uncertainties — target coordinate uncertainties and measuring station coordinate uncertainties — are determined by quantifying the uncertainty of input parameters and propagating them through a mathematical model of the system. These input parameters are as detailed in section 3.1.

Of the time independent influence factors, some are inherent in the multilateration measurement system setup arrangement, and some are not. Those that are not, such as interferometric range displacement measurement uncertainty, SMR sphericity, and laser interferometer deadpath measurement uncertainty require probability density functions (pdf) to be assigned; they can then be propagated through the mathematical model of the measurement system to predict their effect on the z-coordinate measurement uncertainty of the multilateration solution. The error sources that are inherent in the multilateration measurement system setup are tested by defining a nominal measurement system arrangement.

6.1 Measurement simulation

Simulated displacements are calculated using the input uncertainty distributions of influence factors, detailed in section 3.1, and are modelled:

$$d_{i,j\,\mathrm{sim}} = d_{i,j\,\mathrm{nom}} + (u_0 + R_{i,j\,\mathrm{nom}} \cdot u_\mathrm{L}) \cdot P_\mathrm{int} + u_\mathrm{sph} \cdot P_\mathrm{sph} \ , \quad (2)$$

where $d_{i,j\,\mathrm{nom}}$ and $R_{i,j\,\mathrm{nom}}$ are nominal displacement and range respectively. P_int and P_sph are pseudo-random numbers generated with a normal distribution. u_sph is the standard uncertainty of the SMR sphericity error. To produce a large sample (n_samples) of simulated displacement measurements for Monte Carlo analysis, equation (2) is calculated 50000. In this way, a combined pdf of input parameters is estimated. This is used to estimate pdf of the sample surface coordinates.

7 MC simulation results

Monte Carlo simulations for the measurement of the two artefacts was implemented to determine estimates of the setup measurement uncertainty.

7.1 Simulation of 0.4 m × 0.4 m flat optic

To determine an estimate of uncertainty of the measurement described in section 5.1, a set of nominal parameter values for equation (2) were selected. The value of SMR sphericity were defined based on θ = 1°: where θ is an estimate of the SMR-probes tilt angle repeatability, when the CMM moves the SMR into the surface to be measured. The interferometric displacement uncertainty is the manufacturer determined values for the AT960LR, as shown in table 1. Other system setup parameters are defined as equivalent to the setup of the measurement under analysis. This is a time independent evaluation of the system z-coordinate measurement uncertainty.

Table 1: MC simulation parameters and results for the large flat measurement.

Parameter	**Setting/result**
n_samples	50000
m	4
n	100
u_0	0.2 µm
u_L	0.15 µm
θ	1°
ΔT	0 K
σ_z	1.13 µm

The square root mean variance of $\sigma_{z,i}$, the standard deviations of 50000 predicted measurement per measured position, of the measured positions on the surface of the sample is σ_z = 1.13 µm. This prediction is 1.5 times greater than the difference in the multilateration solution and the CMM measurement of the setup under investigation, $\sigma_{\Delta z} = 0.73$ µm.

7.2 Simulation of 0.2 m × 0.2 m ULE® flat optic

To determine an estimate of uncertainty of the measurement described in section 5.2, a set of nominal parameter values for equation (2) were selected, as shown in table 2. All other parameters remained the same as in the simulation of the measurement of the larger flat optic. No time dependent parameters were considered due to the short time of the real measurement.

The square root mean variance of $\sigma_{z,i}$, the standard deviations of 50000 predicted measurement per measured position, of the measured positions on the surface of the sample is $\sigma_z = 1.07$ µm. This prediction is a factor of 4.3 greater than the actual measured value of $\sigma_z = 0.25$ µm for this measurement setup.

Table 2: MC simulation parameters and results for the small flat measurement.

Parameter	Setting/result
n	36
σ_z	1.07 µm

The predicted measurement uncertainties for both measurement sets seem to be an overestimate based on the results of the real measurements, detailed in section 5. The predictions for σ_z have similar magnitude for both measurements of the samples, this indicates that the random z-coordinate measurement error is independent of the size of the sample surface: at least within the range of measurement area tested.

8 Conclusions

This research aimed to determine whether a laser-based multilateration system can be used to measure metre-scale optics with uncertainty $\sigma_z < 1$ µm. A short-term multilateration measurement of a 0.4 m × 0.4 m optic resulted in a z-coordinate measurement standard deviation of the difference between CMM measurement and laser tracker multilateration, $\sigma_{\Delta z} = 0.73$ µm. Δz has a clear saddle form. When the saddle is removed, $\sigma_{\Delta z} = 0.28$ µm. The square root mean variance of 10 repeated measurements was $\sigma_z = 0.43$ µm. The same setup was used for the multilateration measurement of a 0.2 m × 0.2 m ULE® flat and resulted in $\sigma_z = 0.26$ µm. With the saddle form removed, is $\sigma_z = 0.19$ µm, equivalent to the CMM measurement. MC simulation of the short-term measurements of the two samples shows how the random measurement error of the multilateration solutions is predicted to be independent of sample size, if all other measurement parameters remain equal. The measurement results and MC simulations indicate that σ_z for measurement area up to 0.4 m × 0.4 m is significantly < 1 µm. The measurement setup utilised in this investigation (see figure 2) had relatively large interferometric deadpaths and range measurements, and the arrangement of the laser trackers was not optimised. The measurement setup had the area required to measure a 1 m × 1 m optic; it is, therefore, concluded that a laser-based multilateration system can measure a metre-scale optic with z-coordinate measurement uncertainty, $\sigma_z < 1$ µm.

A 15.5 h measurement of the larger flat was carried out. The standard deviation of the difference between long-term multilateration and short-term CMM measurements is $\sigma_{\Delta z} = 1.88$ µm, illustrating the effect of time dependent influence factors, which require further work to compensate.

Acknowledgements

We would like to thank the Centre for Doctoral Training in Ultra Precision Engineering (EP/K503241/1) and Hexagon Manufacturing Intelligence for funding this research and supporting the authors.

References

[1] Rees P and Gray C. Metrology requirements for the serial production of ELT primary mirror segments. In: Proc. SPIE 9575, Optical Manufacturing and Testing XI. vol. 9575; 2015. p. 957508--957508--8.

[2] Ehret G *et al* 2013 Optical measurement of absolute flatness with the deflectometric measurement systems at PTB, J. Phys.: Conf. Ser. **425** 152016

[3] Hexagon MI. Leitz CMM - Overview: Coordinate Measuring Machines and Gear Inspection Centers Version 2015-7; 2015

[4] Greenleaf A H 1983 Self-calibrating surface measuring machine. Opt. Eng. **22**(2) 222276

[5] Hughes E B, Wilson A, and Peggs G N 2000 Design of a High-Accuracy CMM Based on Multi-Lateration Techniques, CIRP Annals - Manufacturing Technology. **49**(1) 391-394

[6] Takatsuji T *et al* 1998 Restriction on the arrangement of laser trackers in laser trilateration Meas. Sci. Technol. **9** 1357-1359

[7] Camboulives M, Lartigue C, Bourdet P, Salgado J 2016 Calibration of a 3D working space by multilateration Precision Engineering. **44** 163-170

[8] Wendt K, Franke M, Härtig F 2012 Measuring large 3D structures using four portable tracking laser interferometers Measurement. **45**(10) 2339-2345

[9] Zhang G X et al 2003 A Study on the Optimal Design of Laser-based Multi-lateration Systems CIRP Annals - Manufacturing Technology. 52(1) 427-430

Laser Metrology and Machine Performance XIII