Fast Machine Tool Calibration using a single Laser Tracker

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Abstract
Reduced batch sizes and increasing complexity of machined parts with tight tolerances require machine tools to maintain a high volumetric performance. Machine Tool Calibration is both a possibility to quantify the volumetric performance and a basis for controller-based compensation of geometric errors. Current calibration methods often require complex, time-consuming measurement setups and strategies, deteriorating the economic and maintenance benefits of frequent machine tool calibration due to increased downtime.
The authors propose a new calibration method using a single laser tracker in conjunction with a standard or active spherical mounted retroreflector (SMR). In contrast to methods based on multilateration, only one coarsely defined position of the laser tracker on the machine tools table and one measurement cycle are required, significantly reducing the complexity of the process. Measurement strategies are based on homogenously distributed random points combined with a regular grid of stationary points solely limited by the working volume of the machine tool. A novel formulation of the rigid body model for serial manipulators is used to calculate discretized geometric error parameters according to ISO 230-1:2012 from the measured deviations between commended and actual position of the SMR serving as functional point. By using B-splines and matrix notations, linear solving methods and covariance propagation according to the Guide to the Expression of Measurement Uncertainty (GUM) are enabled.
Comparative measurements with a multi-dimensional machine tool calibration interferometer carried out on two machine tools show good agreement within uncertainties for all geometric error parameters of the Cartesian axes. In both cases, the required on-machine time was not longer than 30 minutes for working volumes of 2 m ×1.5 m × 0.5 m and 0.8 m × 0.6 m × 0.3 m respectively.

1 Motivation

The volumetric performance of a machine tool is limited by the remaining deviation between actual and commanded position of the functional point [1]. This deviation is a superposition of effects originating from different influences, such as thermal loads, dynamic forces, limitations of the motion control or degradation of components over time [2]. Insufficient volumetric performance can lead to tolerance mismatches and is hence to be avoided. Machine tool calibration and controller-based compensation are effective methods to survey and maintain the required volumetric performance. Due to the non-constant nature of mentioned error sources, calibration of a machine tool must be regularly repeated. Current direct and indirect methods suffer from drawbacks regarding their applicability, among them the requirement for specific measurement instruments and trained personnel or service providers as well as a substantial downtime inhibiting frequent calibration. While structure-integrated approaches can partially overcome these disadvantages, their development is still in a very early state and the principle cannot applied retroactively to all machine tools [3].

Since their invention in the mid-1980s, laser trackers have become a widely established and available industrial large scale metrology instrument due to their multitude of applications [4,5]. At the same time, research has been focussing on reducing the associated uncertainty, such that the notion of laser trackers having an insufficient performance for machine tool calibration using schemes other than multilateration should be revised [6]. While the uncertainty of an individual calibration measurement performed with a laser tracker may be larger when compared to direct or indirect interferometric methods, the uncertainty in compensation originating from geometric error sources varying over time and infrequent calibration can be dominating. At the same time, especially for large machine tools, reduced volumetric performance requirements can justify the use of less accurate calibration methods if the associated cost is significantly lowered.

As a result, the development of a fast machine tool calibration method using a laser tracker is of high interest and envisaged by the authors maintaining the following key benefits:

- The use of laser trackers available on-site is enabled while relying on as less additional equipment as possible.
- The presence of highly trained operators is avoided by shifting most of the complexity of the method to the mathematical model.
- Downtime is limited to a minimum by using a single setup and measurement run without need for alignment.

2 Related Work

Calibration of and volumetric performance verification of machine tools and CMMs by means of tracking interferometers has been subject to research since the establishment of the laser tracker as industrial metrology instrument. LAU et al. hold a patent for *Volumetric Error Compensation System with Laser Tracker and Active Target* [7]. The procedure requires to measure the same grid of random stationary points within the machine tool's working volume twice, one time with the active target attached to a short and a second time attached to a long mounting adapter, effectively measuring five degrees of freedom at each location for repeatable machine tools. Details on the distribution of measurement points, the underlying mathematical model and the relation to ISO 230-1:2012 remain undisclosed. UMETSU et al. present a calibration method for CMMs using four laser tracking interferometry systems based on multilateration, using the same discretized parametrization of geometric error parameters as the approach presented in section 4.2 of this paper does [8]. In contrast AGUADO et. al. use different continuous regression functions (e.g. polynomial or Legendre type) to approximate the geometric error parameters from measurements with a single laser tracker [9]. SCHWENKE et al. describe the foundations of machine tool calibration using a single tracking interferometer repeatedly at multiple locations, which is today commercially available by Etalon as LaserTRACER™ [10]. Different to hitherto presented methods, MUTILBA et al. use a laser tracker attached to the functional point of the machine tool and fiducial reflectors at fixed locations related to the machine tool table [11]. Their mathematical model only accounts for the volumetric deviation, not for a full calibration.

The method developed by the authors differs from the mentioned approaches by enabling the calculation of error parameters according to ISO 230-1 in a discretized, linear interpolated form from a measurement with a single laser tracker that usually does not need to be repeated with different setups.

3 Method Description

The fundamental assumption is that the machine tool can be modelled as serial manipulator where the individual axes behave as rigid bodies possessing six degrees of freedom [1,12]. Therewith the volumetric deviation, i.e. the displacement between commanded and actual position of the functional point, can be calculated from the individual geometric error motions of the single axes and their current positions. By measuring the volumetric deviation at different stationary locations of the functional point, contributions of rotational and translational error movements can be separated due to the change of the effective lever arm established by the following axes (figure 1) [6]. Error motions that do not have an actual influence to the volumetric deviation (e.g. roll around the last axis) can only be measured if a specific offset (comparable to a tool offset) is chosen between the functional point of the machine tool and the reflector being attached. However, depending on the use of the machine tool in production, typically there is no substantial benefit from this additional knowledge such that the additional steps can be skipped.

The distribution of points within the measurement strategy must be adequate to ensure that this separation is possible for all error parameters and axis positions, i.e. mathematically a sufficient rank of the system matrix shown in eq. (14) is needed. A straight-forward approach is to measure at equidistant points along the edges of the working volume of the machine tool for linear axes and shift rotary axes back- and forwards in discrete steps multiple times with different positions of the functional point.

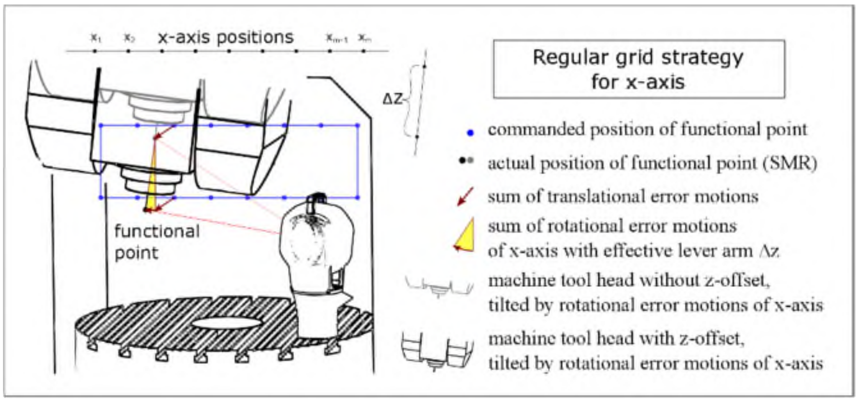

Figure 1: Sketch of the measurement principle separating rotary and linear error movements using different lever arms.

The mathematical model elaborated in section 4 is strategy-agnostic and able to work with sufficiently distributed random points. Hence the strategy can be adapted to specific requirements of the machine tool (e.g. obstructed areas). Measuring all volumetric deviations in the same orthonormal reference coordinate system allows to determine squareness and location errors of the axes by subtracting offsets and slopes from the obtained translational and rotational error parameters with common reference as the former are no physical degrees of freedom but conventions according to ISO 230-1:2012 [1]. While measuring the volumetric deviation in three dimensions, the laser tracker must be placed on the machine tool's table or a frame rigidly related to the former. The target can either be a standard SMR or, if required due to space and view angle constraints, an active or wide-angle reflector and must be attached as functional point of the machine tool. After a possible warm-up phase of the laser tracker, the stationary positions included in the previously defined strategy are captured either manually or automatically by means of standstill-detection. The positions measured by the laser tracker directly correspond to the (erroneous) movement of a tool from the perspective of a work piece. As described in 4.1, the coordinate system registration can directly be performed on the measurement data, hence there are no further on-machine steps required in the procedure.

4 Mathematical Modelling

A linear relation between discretized geometric error parameters and measured volumetric deviations is needed in matrix formulation to establish an overdetermined set of linear equations. The machine tool is described as a series of m axes $K_1, \dots, K_m$ (e.g. $K_1 \mathrel{\hat{=}} Y, K_2 \mathrel{\hat{=}} X, K_3 \mathrel{\hat{=}} Z$) for a FYXZ configuration), while $k_1, \dots, k_m$ denote the associated axes values. For the further elaboration of the model throughout this section an existing model of the machine tool which describes the position of the functional point $\boldsymbol{g}(k_1, \dots, k_m, \boldsymbol{f})$ as work piece coordinates in dependency of the current axis readings and the tool vector $\boldsymbol{f}$ is presupposed.

4.1 Coordinate System Registration

The laser tracker possesses its own coordinate system Σ', which initially is not aligned to the coordinate system Σ of the machine tool being calibrated. Coordinate System Registration refers to finding the rotation matrix $\mathbf{R}$ and translation vector $\boldsymbol{T}$ between both systems, such that the relation (1) for two points $\boldsymbol{p}', \boldsymbol{p}$ holds.

$$\boldsymbol{p} = \mathbf{R} \cdot \boldsymbol{p}' + \boldsymbol{T} \tag{1}$$

While one approach would be to determine the transformation parameters beforehand by measuring at least three reference points within the machine tool's working volume, an alternative strategy is to perform a least-squares minimization over all corresponding positions of the measurement strategy described in section 3 and defined in eq. (2).

$$\min_{\mathbf{R},\boldsymbol{T}} \sum_{i=1}^{n} ||\,\boldsymbol{p}_i - \boldsymbol{g}_i||^2 = \min_{\mathbf{R},\boldsymbol{T}} \sum_{i=1}^{n} ||\,\mathbf{R} \cdot \boldsymbol{p}'_i + \boldsymbol{T} - \boldsymbol{g}_i||^2 \tag{2}$$

This approach has two main advantages: No additional registration step is needed during the overall calibration procedure and the registration becomes more robust to the influence of geometric errors at individual points. After registration, the residual at each measured position corresponds to the volumetric deviation. The underlying mathematical method used for the rigid motion estimation between both point sets can be arbitrary, while the authors used the singular value decomposition algorithm as described by EGGERT et al. [13].

4.2 Matrix Formulation of Volumetric Error Calculation

Assuming serial rigid body behaviour as explained in section 3, each axis K has error motions in three degrees of freedom for translation and rotation, denoted as $EXK(k), EYK(k), EZK(k)$ and $EAK(k), EBK(k), ECK(k)$ respectively. Throughout the further modelling, k represents the current axis value and therewith indicates a position dependency of the respective term. Moreover, each axis is assigned an orientation relative to the work piece coordinate system of the machine tool denoted as a rotation matrix $\mathbf{Q}_K(k)$ as well as an effective lever arm for rotary error motions $\boldsymbol{l}_K(k_1, \dots, k_m)$ depending on the current position of the respective axes. The tool vector $\boldsymbol{f}$ also corresponds to the distance between the reference point of the laser tracker target to the functional point. The geometric error parameters are summarized as a vector $\boldsymbol{U}$ as defined in eq. (3).

$$\boldsymbol{U}(k_1, \dots, k_m) = (EXk_1, EYk_1, EZk_1, EAk_1, EBk_1, ECk_1, \dots, EBk_m, ECk_m)^T \tag{3}$$

$$= \left(\boldsymbol{U}_{K_1}^T | \cdots | \boldsymbol{U}_{K_m}^T\right)^T \tag{4}$$

Moreover, the kinematic chain of machine tool is represented in a matrix $\mathbf{M}$ as in eqs. (5-6).

$$\mathbf{M}_K = \mathbf{Q}_K \cdot \begin{pmatrix} 1 & 0 & 0 & 0 & l_{K,z} + f_z & -l_{K,y} - f_y \\ 0 & 1 & 0 & -l_{K,z} - f_z & 0 & l_{K,x} + f_x \\ 0 & 0 & 1 & l_{K,y} + f_y & -l_{K,x} - f_x & 0 \end{pmatrix} \tag{5}$$

$$\mathbf{M} = (\mathbf{M}_{K_1} | \cdots | \mathbf{M}_{K_m}) \tag{6}$$

With these prerequisites, the volumetric deviation $\boldsymbol{V}$ at one specific axis configuration can be calculated according to eq. (7) as matrix multiplication.

$$\boldsymbol{V}(k_1, \dots, k_m) = \mathbf{M}(k_1, \dots, k_m, \boldsymbol{f}) \cdot \boldsymbol{U}(k_1, \dots, k_m) \tag{7}$$

A B-Spline approach [8] as shown in eq. (8) is used to represent a linear interpolation between discrete values as continuous function $e(k)$, which is a sum of o base functions $b_j(k)$ multiplied with coefficients s_j. They correspond to the function values at the interpolation points k_j', while b_j are triangular functions as defined in eq. (9).

$$e(k) = \sum_{j=1}^{o_K} b_j(k) \cdot s_j \tag{8}$$

$$b_j(k) = \begin{cases} 1 - \dfrac{k - k_{j-1}'}{k' - k_{j-1}'} & k'_{j-1} < k < k_j' \\ 1 & k = k_j' \\ 1 - \dfrac{k_{j+1}' - k}{k_{j+1}' - k'} & k'_j < k < k_{j+1}' \\ 0 & \text{else} \end{cases} \tag{9}$$

Equation (8) can be rewritten as matrix-vector multiplication as in eq. (10), such that the dependency on the current axis position is shifted to a separate matrix **B**.

$$\boldsymbol{U}_K = \mathbf{B}_K \cdot \boldsymbol{U}_K^* = \begin{pmatrix} b_1 & b_2 & \cdots & b_{o_K} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & b_1 & \cdots & b_{o_K} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & b_{o_K} \end{pmatrix} \cdot \begin{pmatrix} s_1[EXK] \\ s_2[EXK] \\ \vdots \\ s_{o_K}[EXK] \\ s_1[EYK] \\ \vdots \\ s_{o_K}[EYK] \\ \vdots \\ s_{o_K}[ECK] \end{pmatrix} \tag{10}$$

In total, eq. (7) can be extended to represent the volumetric deviation at a single location in dependency of geometric errors in discretized, location independent form $\boldsymbol{U}^*$and the current axis positions $k_1, \dots k_m$ as shown in eqs. (11-12).

$$\boldsymbol{V}(k_1, \dots, k_m, \boldsymbol{f}) = \left(\mathbf{M}_{K_1} | \cdots | \mathbf{M}_{K_m}\right) \cdot \begin{pmatrix} \mathbf{B}_{K_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{B}_{K_m} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{U}_{K_1}^* \\ \vdots \\ \boldsymbol{U}_{K_m}^* \end{pmatrix} \tag{11}$$

$$= \mathbf{M}(k_1, \dots, k_m, \boldsymbol{f}) \cdot \mathbf{B}(k_1, \dots, k_m) \cdot \boldsymbol{U}^* \tag{12}$$

4.3 Linear System of Equations

After initially determining the coordinate system transformation as described in 4.1, the volumetric error at a commanded machine tool position $k_1, \dots k_m$ can be extracted from the position $\boldsymbol{p}_i$ measured by the laser tracker as in eq. (13).

$$\boldsymbol{V}_i^{measured} = \mathbf{R} \cdot \boldsymbol{p}_i + \boldsymbol{T} - \boldsymbol{g}_i(k_{1,i}, \dots k_{m,i}, \boldsymbol{f}) \tag{13}$$

At the same time, the volumetric deviation can be modelled as in eq. (12). By stacking the individual linear equations, a complete system can be set up to relate the measurements $i = 1 \dots n$ to the sought geometric error parameters.

$$\underbrace{\begin{pmatrix} \boldsymbol{V}_1^{measured} \\ \boldsymbol{V}_2^{measured} \\ \vdots \\ \boldsymbol{V}_n^{measured} \end{pmatrix}}_{\hat{\boldsymbol{V}}} = \underbrace{\begin{pmatrix} \mathbf{M}(k_{1,1}, \dots, k_{m,1}, \boldsymbol{f}) \\ \mathbf{M}(k_{1,2}, \dots, k_{m,2}, \boldsymbol{f}) \\ \vdots \\ \mathbf{M}(k_{1,n}, \dots, k_{m,n}, \boldsymbol{f}) \end{pmatrix}}_{\hat{\mathbf{M}}} \cdot \underbrace{\begin{pmatrix} \mathbf{B}(k_{1,1}, \dots, k_{m,1}) \\ \mathbf{B}(k_{1,2}, \dots, k_{m,2}) \\ \vdots \\ \mathbf{B}(k_{1,n}, \dots, k_{m,n}) \end{pmatrix}}_{\hat{\mathbf{B}}} \cdot \boldsymbol{U}^* \tag{14}$$

The rank of the effective system matrix $\widehat{\mathbf{M}} \cdot \widehat{\mathbf{B}}$ depends on the actual distribution of measurement points and the machine tool's error motions, while the dimension of $\boldsymbol{U}^*$ depends on the interpolation points chosen for the representation of geometric error parameters. To allow the use of the Moore-Penrose-pseudoinverse (†) as a least-square optimization with respect to $\boldsymbol{U}^*$, the condition in eq. (15) needs to hold.

$$\dim(\widehat{\boldsymbol{V}}) = 3 \cdot n \geq \operatorname{rank}(\widehat{\mathbf{M}} \cdot \widehat{\mathbf{B}}) \geq \dim(\boldsymbol{U}^*) = 6 \cdot \sum_{j=1}^{m} o_{K_m} \tag{15}$$

$$\Rightarrow \boldsymbol{U}^*_{opt} = \left(\widehat{\mathbf{M}} \cdot \widehat{\mathbf{B}}\right)^{\dagger} \cdot \widehat{\boldsymbol{V}} \tag{16}$$

At last, the entries of $\boldsymbol{U}^*_{opt}$ can be split into the individual error parameters according to ISO 230-1:2012 while applying additional calculations to adhere to conventions as described in section 3.

4.4 Uncertainty Propagation

Let $\mathbf{C}'$ be the covariance matrix associated with a laser tracker measurement. Assuming that there is no statistical uncertainty on the commanded positions of the machine tool, the covariance $\mathbf{C}_i^{measured}$ of a measured volumetric deviation is obtained by applying the same coordinate transformation, as noted eqs. (17-18).

$$\mathbf{C}_i^{measured} = \mathbf{R} \cdot \mathbf{C}_i' \cdot \mathbf{R}^T \tag{17}$$

$$\widehat{\mathbf{C}} = \begin{pmatrix} \mathbf{C}_1^{measured} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{C}_n^{measured} \end{pmatrix} \tag{18}$$

By performing a Cholesky decomposition, a weight matrix $\mathbf{W}_i$ can be defined per point as in eq. (19), from which the global weight matrix $\widehat{\mathbf{W}}$ is obtained by building a block diagonal matrix.

$$\mathbf{C}_i^{measured} = \mathbf{L}_i \cdot \mathbf{L}_i^T \Rightarrow \mathbf{W}_i = \mathbf{L}_i^{-1} \tag{19}$$

$$\widehat{\mathbf{W}} = \begin{pmatrix} \mathbf{W}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{W}_n \end{pmatrix} \tag{20}$$

Equation (21) represents the weighted, rewritten form of eq. (12). By representing the least-squares solution using the Moore-Penrose-pseudoinverse (†), the sensitivity coefficients of the relation between $\boldsymbol{U}^*$ and $\widehat{\boldsymbol{V}}$ for uncertainty propagation according to the *Guide to the Expression of Measurement Uncertainty* are directly obtained.

$$\widehat{\mathbf{W}} \cdot \widehat{\boldsymbol{V}} = \widehat{\mathbf{W}} \cdot \widehat{\mathbf{M}} \cdot \widehat{\mathbf{B}} \cdot \boldsymbol{U}^* \quad \Rightarrow \boldsymbol{U}^*_{opt} = \left(\widehat{\mathbf{W}} \cdot \widehat{\mathbf{M}} \cdot \widehat{\mathbf{B}}\right)^{\dagger} \cdot \ \widehat{\mathbf{W}} \cdot \widehat{\boldsymbol{V}} \tag{21}$$

$$\mathbf{Cov}(\boldsymbol{U}^*_{opt}) = \left(\widehat{\mathbf{W}} \cdot \widehat{\mathbf{M}} \cdot \widehat{\mathbf{B}}\right)^{\dagger} \cdot \widehat{\mathbf{W}} \cdot \widehat{\mathbf{C}} \cdot \widehat{\mathbf{W}}^T \cdot \left[\left(\widehat{\mathbf{W}} \cdot \widehat{\mathbf{M}} \cdot \widehat{\mathbf{B}}\right)^{\dagger}\right]^{\boldsymbol{T}} \tag{22}$$

4.5 Summary

The derived mathematical procedure is applicable to any machine tool that can be described as serial manipulator with a rigid body error model. Only $\mathbf{M}(k_1, \ldots, k_m, \boldsymbol{f})$, $\mathbf{B}(k_1, \ldots, k_m)$ and $\boldsymbol{U}^*$ have to be adapted to the kinematic chain of the machine tool in question and the measurement strategy respectively discretization spacing chosen. This includes rotary axes, in which case special care has to be taken correctly defining $\mathbf{Q}_K(k)$.

5 Evaluation on two Machine Tools

The developed method was used to calibrate the linear axes of two machine tools with an API Radian™ laser tracker. The results are compared to direct calibration measurements using an API XD-Laser™ Precision after correcting for Abbe-Errors. Uncertainties are determined by sample variance in spherical coordinates for the laser tracker and deduced from the manufacturer's specifications compared to the sensors' individual variance in case of the XD-Laser™ Precision.

5.1 BZT PFU-S-2 2015-G FYXZ Machine

The first machine was used for validation within a working volume of 1.8 m×1.5 m×0.4 m with an FYXZ kinematic chain and no built-in glass scales. For the experimental setup, the laser tracker was placed on a stiff aluminium profile connected to the machine table allowing the use of a standard SMR instead of the active target. The geometric error parameters were sought with an interpolation spacing of 100 mm for X- and Y-axis and 50 mm for the Z-axis, effectively leading to $\dim(\boldsymbol{U}^*) = 276$. The chosen measurement strategy consisted of 170 points on the edges of the working volume and 150 random points on the inside with a machine dwell time of 3 seconds, leading to an execution time of 30 minutes. In comparison with the calibration parameters obtained with the API XD-Laser™ Precision (excerpt shown in figure 2), a coarse agreement with regard to the uncertainties can be observed with punctual systematic effects. As the developed method does not account separately for back- and forward directions, backlash effects limit comparability. Moreover, during other experiments the authors observed partial non-rigid behaviour of this specific machine tool, which are a potential source of additional systematic errors.

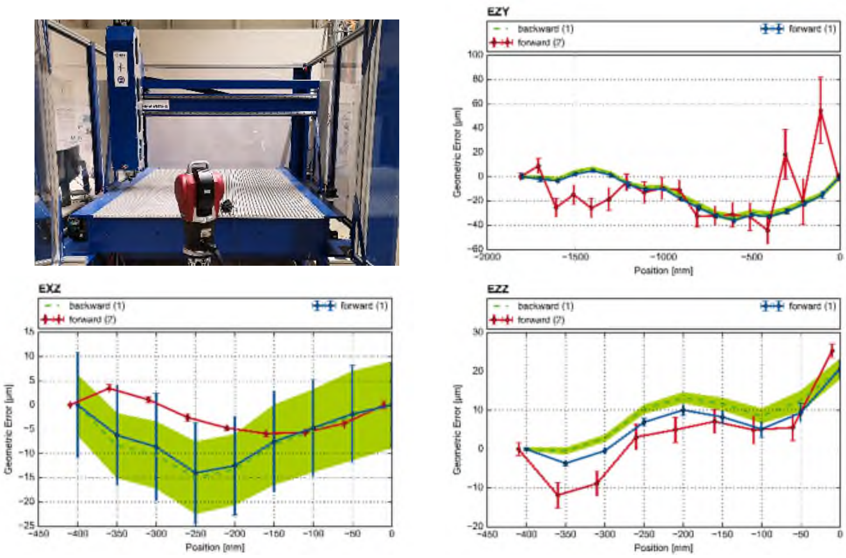

Figure 2: Measurement setup and comparison of EZY, EXZ and EZZ between laser tracker (red) and API XD-Laser™ Precision (blue/green). The error bars do deliberately not include systematic errors.

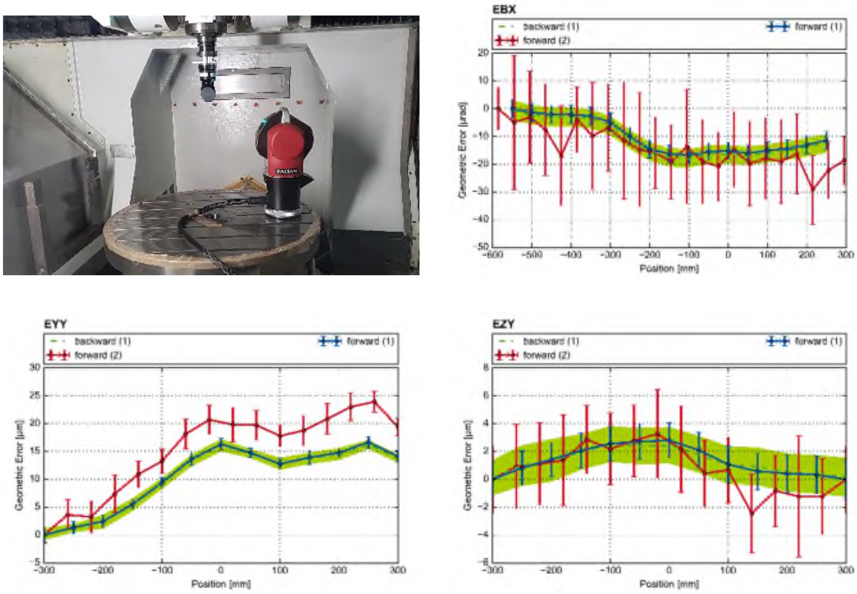

Figure 3: Photo of measurement setup and comparison of EBX, EYY and EZY between laser tracker (red) and API XD-Laser™ Precision (blue/green).

5.2 DMC 75V Linear CXFYZB Machine

As second test carrier, the linear axes of a CXFYZB machine with working volume of 0.8 m × 0.6 m × 0.3 m, internal glass scales and temperature compensation was used. Due to the X-axis being a moving table, the laser tracker was placed on the latter in conjunction with the active target attaches to the spindle. Because of the limited space in the machine (figure 3), only the edges of the machine tool were used for the measurement strategy with a total of 300 points. Interpolation spacings were 40 mm for X- and Y-axis and 50 mm for the Z-axis, leading to $\dim(\boldsymbol{U}^{*}) = 276$. With a 2 second dwell time the total time of the procedure is 20 minutes. While systematic effects due to limited repeatability of the machine tool can be neglected, additional systematic errors up to order of 20 µm are expected to be introduced by the active target. Again, with regard to the uncertainties, the obtained error parameters (excerpt in figure 2) are in good agreement with measurements performed with the API XD-Laser™ Precision. The deviation in EYY is attributed to remaining systematic effects affecting its reference point. As anticipated, the use of the laser tracker leads to a higher statistical uncertainty compared to the former.

6 Conclusion

The newly proposed method for calibration of machine tools using a single laser tracker was successfully implemented and carried out for two machine tools while maintaining the benefits envisaged in section 1. Future work will investigate whether the remaining systematic deviations in comparative measurements of geometric error parameters can certainly be attributed to non-rigid machine tool behaviour and effects introduced by the use of the active target and if other error sources must be taken into account or statistical uncertainties reworked. From the perspective of the authors, the presented method is of special interest to the calibration of very large, repeatable, rigid-body like machine tools with accuracy requirements that justify a higher uncertainty if a complexity and time of the measurement process are significantly reduced.

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