Simultaneous Multi-type Feature Separation for Complex Structured Surface Analysis

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Abstract

Manufactured parts with complex structured surfaces have been widely used in automobile, bio-engineering, medical and consumer electronics etc. The content of a complex structured surface is often complex. The organization of surface features, the feature types and sizes all have fundamental effects on the resulting function of the surface. There is not a single transform which is optimal to represent all the contained features. In this research, a novel method called Morphological Component Analysis (MCA) is investigated to characterise different types of features on complex structured surfaces. Due to the fact that different types of features on the structured surface present different morphological aspects, the MCA method employs a set of dictionaries/basis functions with various morphological characteristics to represent various types of features, such as random noise, directional texture, steps, edges and others. By using MCA one can separate different types of features simultaneously. It is recommended that the wavelet type functions, especially DT-CWT, can be selected for modelling the geometrical features (such as peaks, valley, holes, steps and edges); the Fourier type basis functions such as basis function of DCT can be used for mapping of the surface texture. Other advantages of the MCA include: it can separate multiple types of features simultaneously and have better feature preservation properties compared with the convolution based filters.

1 Introduction

Surfaces provide the functional interface through which all advanced and emerging products operate at the most fundamental level. Surface geometry and its measurement is thus one of the most important functionality indicators in the performance of high precision and nanometer scale devices and components. In recent years, many applications for “structured surface” related products have
been found in the diverse automotive, biotech, medical and consumer applications and are one of the current drivers of future economic growth. Surfaces with a dominant deterministic feature pattern are termed “structured surfaces” and the dominant deterministic feature pattern can be repeated patterns, designed structures, or randomly generated functional topography [1-6]. Different from the traditional “stochastic surface”, all structured surfaces are specifically designed to meet a highly defined functional requirement. A key common feature of structured surfaces is that they are normally high aspect ratio surfaces where the surface geometry is the critical determination of component function. The scale of structured surfaces ranges from the macro scale down to the nano scale. Specific examples include MEMS/NEMS devices, micro moulding, micro fluidic systems (lab on a chip), defined geometry abrasives, structured coatings, micro-lenses, Fresnel lenses, bio integration coatings, self-cleaning coatings and sheet metal products [6]. The ability to adequately characterise these structured surface geometry features is crucial in the optimisation and control of such functional devices/components.

Filtration has always been important in surface metrology as it is the means by which the surface geometrical features of interest are extracted from the measured data for further analysis [5, 6]. In traditional surface creation processes, such as grinding, milling, polishing, honing etc., the surface features and the organisation of the surface features are stochastic in nature. Even in turned surfaces that have a fundamental periodic component there is still a large stochastic element in the surface due to the material shear mechanisms, ploughing and smearing at the tool workpiece interface. Filtration techniques based on Gaussian, spline and wavelet methods for stochastic surfaces have been comprehensively investigated in last decade. The most widely used filter for surface characterization is the Gaussian filter and it has been recommended as the standard filtering technique for both profile and areal surface analysis due to its effective finite impulse response (FIR) and linear/zero phase characteristics [7]. To address the boundary effect and form removal issues with ordinary Gaussian filtering, a Gaussian Regression (GR) filtering technique is proposed [7]. Furthermore, by introducing a reweighted iteration procedure, the Robust Gaussian Regression (RGR) filter can obtain robust results against outliers [7,8]. Another widely used filtration technique is the spline filter, which has the advantage that it is natural and has no boundary or form following problems compared with the standard Gaussian filter [9]. A robust spline filter based on the M-estimation method has been proposed to obtain outlier-resistant results [8]. Wavelet filters including the biorthogonal wavelet filters, the lifting wavelet filters and the complex wavelet filters are usually used for morphological feature extraction and multi-scale analysis [11].

Due to the fact that the geometrical features and the feature pattern, which cannot be described by a stochastic process, are more important than surface texture in most cases for the structured surface analysis from the function point of view, the filtration techniques designed for stochastic surface are not suitable for structured surfaces. When traditional filtration methods are applied, the extracted features will be distorted, edges will be smoothed and the datum
extracted will be distorted. For example, when using a Gaussian filter for MEMs surfaces, the boundary of line and step features are smoothed and the mean line or the datum will be distorted.

In recent years, several techniques have been investigated for structured surface analysis. An approach combines segmentation algorithms with a novel implementation of the angular radial transform to implement shape descriptors and associated similarity metrics for tessellated surface is investigated in [2]. A Sobel edge operator has been used to separate the surface into different regions and then the fitting algorithm is used to find the datum of the measurement data [12]. A pattern analysis and wolf pruning based method has also been investigated for feature extraction [13]. A Nonlinear diffusion filter, which has the properties of direction sensitive and edge sensitive, has been proposed for used for geometrical feature extraction, especially for surface with edges and steps [14]. Furthermore, to improve the performance of the proposed filters, various diffusivity functions as well as regularization functions are also introduced into the PDE filtration process [14, 15].

The content of a structured surface is often complex. The organization of surface features, the feature types and sizes all have fundamental effects on the resulting function of the surface. The ability to characterize these features adequately is crucial. There is not a single transform which is optimal to represent all the contained features. For example, the Fourier transform better represents some textures, while the wavelet transform better represents singularities. Even if we limit our class of transforms to the wavelet class, decisions must be taken between an isotropic wavelet transform which produces good results for isotropic objects, or an orthogonal wavelet transform, which is better for images with edges.

In this paper, a new method called Morphological Component Analysis (MCA) is proposed to characterise various types of features on structured surfaces. MCA allows us to separate features contained on the surfaces when these features present different morphological aspects.

2 MCA for structured surfaces

The employment of MCA allows us to separate features contained in the measured data when these features present different morphological characteristics. In general, morphological characteristic is the form and structure (normally in the time/spatial domain) of the signal and its components. It mainly refers to the outward appearance (shape, form, structure and pattern) of a signal and its component parts. In our present application, morphological characteristic refers to not only the outward appearance of a signal and its component parts, but also the time/frequency distribution of the signal and its component parts. The MCA relies on the assumption that for every signal to be separated, there exists a dictionary that enables its construction using a sparse representation. The other assumption is that the different dictionaries are highly inefficient in representing the other behaviours in the mixture. Here, the
2.1 The MCA concept

Assume that the data $S$ is a linear combination of $K$ parts, $s = \sum_{i} s_i$, where each $s_i$ represents a different type of signal to be separated. The MCA model assumes that [16-18]:

(1) For every possible signal $s_i$ there exists a dictionary, $\Phi_i \in M^{N \times L_i}$ (where typically $L_i = N$) such that solving:

$$\alpha_i^{\text{sp}} = \text{Arg} \min_{\alpha} \| \alpha \|_0 \text{ subject to: } s_i = \Phi_i \alpha$$  \hspace{1cm} (1)

leads to a very sparse solution i.e. $\| \alpha_i^{\text{sp}} \|_0$ is very small. The zero-norm $\| \alpha_i^{\text{sp}} \|_0$ is simply the number of non-zero elements in $\alpha_i^{\text{sp}}$. The sparse solution is one in which most of the elements in the transformation domain are zero. The sparse solution is a highly effective and efficient description of the signal in the transformation domain, which means that use very few elements can describe the signal. The definition in the above equation is essentially the overcomplete transform of $s_i$, yielding a representation $\alpha_i$.

(2) For every possible signal $s_j$, solving for $k \neq l$

$$\alpha_j^{\text{np}} = \text{Arg} \min_{\alpha} \| \alpha \|_0 \text{ subject to: } s_j = \Phi_j \alpha$$  \hspace{1cm} (2)

leads to a very non-sparse solution. The non-sparse solution is the one in which most of the elements in the transformation domain are non-zero. Non-sparse solutions are an ineffective and inefficient description of the signal in the transformation, which means that most of the elements are needed to describe the signal. This requirement suggests that the dictionary $\Phi_j$ is distinguishing between the different types of signals to be separated.

Thus, the dictionary $\Phi_j$ plays the role of discriminant between different content types. For an arbitrary signal $s$ containing $K$ layers as a linear combination, in order to seek the sparsest of all representations over the augmented dictionary containing all $\Phi_i$, we need to solve

$$[\alpha_1^{\text{sp}}, ..., \alpha_K^{\text{sp}}] = \text{Arg} \min_{\alpha_1, ..., \alpha_K} \sum_k \| \alpha_k \|_0 \text{ subject to: } s = \sum_k \Phi_k \alpha_k$$  \hspace{1cm} (3)

This optimization task is likely to lead to a successful separation of the signal content, based on the assumption that $\Phi_j$ is very efficient in representing one phenomenon and highly non-effective in representing the other signal types.

Replacing the L-0 norm with the L-1 norm in equation (3), leads to a solvable optimization problem of the form [16-18]:

$$[\alpha_1^{\text{sp}}, ..., \alpha_K^{\text{sp}}] = \text{Arg} \min_{\alpha_1, ..., \alpha_K} \sum_k \| \alpha_k \|_1 \text{ subject to: } s = \sum_k \Phi_k \alpha_k$$  \hspace{1cm} (4)

by relaxing the constraint, an approximate solution can be obtained:
\[
\{\alpha_1^{\text{opt}}, \ldots, \alpha_k^{\text{opt}}\} = \text{Arg} \min_{\{\alpha_1, \ldots, \alpha_k\}} \sum_{i=1}^{k} \|z_i\| + \lambda \|\sum_{i=1}^{k} \Phi_i \alpha_i\|_1
\]  

(5)

The choice of L-2 norm as the error norm is related to the assumption that the residual behaves like white zero-mean Gaussian white noise. Other norms can be similarly introduced to account for different noise models, such as Laplacian (L-1), uniformly distributed noise (L∞) and others.

2.2 MCA for structured surface analysis

Let \( z \) represent the measured noisy data, \( t \) represent the geometrical feature (normally it includes edges, steps, and thus can be modelled as the transient events) to be extracted, \( s \) represent the surface texture (in most time it includes periodic machining marks, thus it can be modelled as stationary signal) and \( n \) represents the random measurement noise. When assuming that \( t, s, n \) are uncorrelated, there exists a linear combination:

\[
z = t + s + n
\]

(6)

Let \( T \) represent the forward transformation, \( R \) represent the reconstruction transform, \( \alpha_t \) and \( \alpha_s \) are the overcomplete representation of \( t \) and \( s \) in the transform domain. \( \Phi_t \) and \( \Phi_s \) are dictionaries used for overcomplete transformation of \( s \) and \( t \) respectively. The MCA analysis for the separation of geometrical features \( t \), surface texture \( s \), and measurement noise \( n \) on the measured structured surface can be modelled as:

\[
\{\alpha_t^{\text{opt}}, \alpha_s^{\text{opt}}\} = \text{Arg} \min_{\{\alpha_t, \alpha_s\}} \left( \|z_t\| + \|z_s\| + \lambda \| - R(\Phi_t) \alpha_t - R(\Phi_s) \alpha_s \|_1 \right)
\]

(7)

where \( \alpha_t^{\text{opt}} \) and \( \alpha_s^{\text{opt}} \) are the sparsest solutions of geometrical features \( t \) and surface texture \( s \) respectively. By using the reconstruction transform the transient events and the real signal can be reconstructed by:

\[
t = R(\Phi_t) \alpha_t^{\text{opt}} \quad s = R(\Phi_s) \alpha_s^{\text{opt}}
\]

(8)

and the random noise components are the residuals: \( n = z - t - s \).

The algorithm used to solve the above optimization problem is based on the block coordinate relaxation (BCR) method. The algorithm is given below[14-16]: (1) Initialize the number of iterations \( L \) and the threshold factor \( \delta = \lambda \cdot L \); Initialize \( \alpha_t \) and \( \alpha_s \); (2) Perform N times: Update \( \alpha_t \) with \( \alpha_s \) fixed; Update \( \alpha_s \) with \( \alpha_t \) fixed; Update the threshold \( \delta = \delta - \lambda / N \); (3) If \( \delta > \lambda \), return to step (2). Else, Finish.

This numerical scheme presents two advantages: (1) it does not need to keep all the transformations in memory. This is particularly important when redundant transformations are used; (2) different constraints can be added on the components.

3 Selected dictionary
As aforementioned, the MCA allows us to separate features contained in the measured data if these features present different morphological characteristics. Subsequently, the effectiveness of MCA relies strongly upon the choice of the dictionary (or basis function, or kernel function) and how these dictionaries can represent the feature or texture efficiently and sparsely.

Different types of basis functions can be selected as the dictionary. Theoretical analysis of the properties of various basis functions, provides the following recommendations: (1) Wavelet type functions (including orthogonal wavelet, biorthogonal wavelet and complex wavelet) are selected for the one dimensional geometrical features(such as steps, edges, blocks etc. which are normally transient events in signal processing) and for the point type features (holes, peaks, etc, call point singularities in signal processing) detection in two dimensional data; (2) Ridgelets and Curvelets basis functions are selected for line/curl like feature detection of two or higher dimensional data; (3) Fourier type functions can be selected for surface texture (roughness, waviness, which are frequency/wavelength related).

### 3.1. Dual tree complex wavelet transforms for geometry feature

By nature, the wavelet transforms have the ability to detect transient events. Its basis function can be used as a dictionary in MCA to extract geometrical features. Real discrete wavelet transforms (DWT) include orthogonal and biorthogonal can provide multi-scalar analysis, which has been widely used for engineering surface filtering [9]. However, the real DWT remains hampered for use in many applications due to the lack of shift invariance, which means small shifts in the input data can cause major variations in the distribution of the energy between DWT coefficients at different scales, and the characterization results are highly dependent on the measurement frame [17, 18].

![Dual tree Complex Wavelet Transform](image)

**Figure 1: Dual tree Complex Wavelet Transform**

Dual-Tree Complex Wavelet Transform (DT-CWT) representation was originally proposed [17] for image processing and the authors have applied it for the separation and extraction of morphological features on engineering surfaces [18]. It has been proved that the DT-CWT can provide approximate shift-
invariance properties and improved directional selectivity. By using two parallel fully-decimated trees, which are subsampled differently and are both real wavelet transform respectively, the DT-CWT achieves perfect reconstruction and near shift invariance. The main approach of DT-CWT is to use two-channel filter banks in the real and imaginary parts as shown in Figure 1. The scale coefficients and wavelet coefficients (or approximations and details) for the two trees are denoted respectively.

3.2. Discrete cosine transform

A discrete cosine transform (DCT) expresses a signal in terms of a sum of cosine functions oscillating at different frequencies. In particular, a DCT is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. The DCT can be defined as:

\[ X_k = \sum_{n=0}^{N-1} x_n \cos \left( \frac{n + \frac{1}{2}}{N} \pi k \right) \quad k = 0, ..., N - 1 \quad (8) \]

Similar to the definition of surface texture which normally has the components of waviness, roughness that are modelled by using the combinations of different frequency sine or cosine waves, it is natural that the cosine basis function can be selected as one of the dictionary in MCA for the separation of surface texture components. In our application, it is \( \Phi_0 \) in Eq.(7).

4 Simulations and experimental results

In order to verify the performance of the proposed MCA method, some simulated surface features are examined.

Figure 2: MCA analysis for a simulated synthetized data (Up: from left to right are sig1, sig2, and sig1+sig2+noise; down: from left to right are recovered sig1, sig2 and denoised data)
In Figure 2, simulated surface data is synthesized by using three different components: (1) sig1 is three patches of 2D cosine data with different frequencies which are used to simulate the surface texture; (2) sig2 is a combination of three different exponential functions with different index and positions to simulate the morphological features on the surface (holes or peaks) (3) Gaussian noise to simulate the measurement noise. In this case, sig1 and sig2 and their combinations are needed to be recovered from the noisy data. We have chosen the DCT basis function as the dictionary with which to match the surface texture sig1, and DT-CWT basis function as the dictionary to match the morphological feature. One can clearly see, even at very a significant noise level (from up right figure in figure 1, sig1+sig merged in the noise and hardly to distinguish), the surface texture of sig1 and the morphological feature sig2 and their combinations sig1+sig2 can be recovered ideally by using the described MCA method.

Figure 3: Analysis of micro lens surface (Original surface; MCA; Gaussian filter; DT-CWT only; profile from surface data)
A group of typical manufactured structured surfaces have also been selected to demonstrate the ability of MCA. Figure 3 is an example, with top left figure showing a measured micro lens mould surface where one can see there are one aspheric and turning marks as well as noise. To accurately evaluate the geometry of the aspheric, one must first separate it from the measured data. Figure 3 compares the proposed MCA method with the most widely used filtration method, Gaussian filtering, and the DT-CWT filter alone. It is shown that these three methods can separate the aspheric geometry from the surface texture and noise acceptably. However, the results from both the Gaussian filter and DT-CWT filter have very smoothed boundary of the aspheric and which cannot reflect the real irregularity of the boundary whereas MCA not only separates the aspheric but also keeps its real boundary shape. From the bottom figure of figure 3 one can also find that the MCA can follow the real shape of the aspheric very well and while both the Gaussian and DT-CWT experience some departure at the edge of the aspheric.

5. Summary and Conclusions

This study demonstrates a new method of filtering called Morphological Component Analysis (MCA) to characterise different types of features on structured surfaces. Due to the fact that different types of features on structured surfaces present different morphological aspects, the MCA method uses a set of dictionaries/basis functions with various morphological characteristics to represent various types of features, such as random noise, directional texture, steps and edges, peaks, valleys, and others. It is also recommended that: the wavelet type functions, especially DT-CWT, can be selected for modelling the geometrical features (such as peaks, valley, holes, steps and edges); the Fourier type basis functions such as basis function of DCT can be used for mapping of the surface texture. Other advantages of the MCA also include: it can separate multiple types of features simultaneously and have better feature preservation properties compared with the convolution based filters. Our further work will focus on: (1) build a series of dictionaries and mapping them to different types of morphological features; (2) develop fast algorithm for the proposed method.

References