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Robust Parameter Estimation for Optical Surface Roughness Measurement Using a Physics-Based Model

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Abstract
Optical-based surface roughness measurement is widely used in precision manufacturing; however, measured signals are strongly influenced by surface-induced angular scattering and optical system geometry, limiting the generalizability of empirical regression models. To address this issue, this study proposes a physics-based $FR2$ model that represents a machined surface as a statistical ensemble of micro-facets with a Gaussian slope angle distribution. The slope distribution is parameterized as a logarithmic function of the arithmetic mean roughness R_a, and the optical response is defined as the probability that reflected light falls within the finite acceptance angles of the receiving optics. Robust parameter estimation under strong nonlinearity and numerical integration is achieved using a multi-start optimization framework combined with the Nelder–Mead algorithm, resulting in improved fitting accuracy compared to conventional grid-based approaches.

Keywords: Surface Roughness, Optical-based, Flux Ratio, Global Optimization

1. Introduction

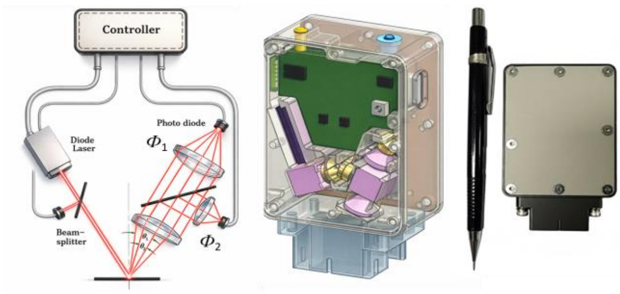
Optical-based surface roughness measurement is widely used in precision manufacturing due to its non-contact nature. However, measured signals depend not only on surface height but also strongly on the angular distribution of reflected light, imposing a fundamental limitation on simple empirical regression models. Physics-based modeling approaches that interpret surface roughness as a statistical ensemble of micro-facets and probabilistically describe the collected light within the effective angular range of the optical system are therefore required to overcome these limitations [1,2].

In this study, an $FR2$ model is proposed to describe the relationship between the surface roughness parameter R_a and the optical measurement responses. The standard deviation of the micro-slope angle distribution is parameterized as a function of R_a. Considering the nonlinear characteristics of the model, including numerical integration, this study aims to establish a coefficient estimation framework with low sensitivity to initial values and high reproducibility.

2. Formulation of the Surface Roughness Model

2.1. Concept of the Surface Roughness Model

Figure 1 illustrates the conceptual configuration of an optical-based surface roughness measurement sensor. A diode laser is used as the light source, and the incident light flux is measured after beam splitting and focusing. In the receiving unit for reflected light measurement, scattered light is collimated using an objective lens, and the reflected light flux is measured through an additional beam splitter and collecting lenses with different apertures. The effective receiving angles of the lenses are defined as θ_1=9.46° and θ_2 =3.58°, respectively as shown in Figure 1. Based on the relationship between the incident light flux and the two reflected light fluxes, a light flux ratio is defined. The ratio of Φ_2 to Φ_1 is defined as $FR2$ and is used as the primary measurement indicator because it reduces the number of input variables and is advantageous for optimization [3].



(a) (b) (c)

Figure 1. (a) concept (b) conceptual design (c) product of optical based surface roughness measurement sensor

2.2. Micro-Slope Angle Distribution Model

A machined surface can be microscopically interpreted as a structure composed of numerous micro-facets, and the distribution of their normal directions reflects the statistical characteristics of surface roughness. Previous studies have commonly approximated the micro-slope angle distribution using a Gaussian distribution [3].

In this study, the probability density function of the micro-slope angle θ is assumed to follow a Gaussian distribution with zero mean and standard deviation σ_s.

$$f(\theta) = \left(\frac{1}{\sqrt{(2\pi)}\sigma_s}\right) exp\left\{-\frac{1}{2}\left(\frac{\theta_s}{\sigma_s}\right)^2\right\} \quad (1)$$

The parameter σ_s represents the statistical magnitude of surface roughness; as σ_s increases, reflected light is distributed over a wider angular range.

Since it is difficult to directly measure the relationship between the surface roughness parameter R_a and angular dispersion, σs is modelled as a function of R_a.

$$\sigma_s = a + b\, log_{10}(R_a) \quad (2)$$

A logarithmic linear model is adopted to represent the gradual increase in angular dispersion with increasing roughness, while ensuring model stability and identifiability with a minimal number of parameters.

2.3. Definition of the Finite-Angle Probability Function

The optical signal detected by the system is determined not by all reflected light but by the portion contained within the effective angular range of the optical system. To reflect this, a probability function Φ_i is defined by integrating the Gaussian probability density function over a finite angular range.

$$\Phi_i = \int_{-0.5\theta_i}^{0.5\theta_i} f(\theta_s)\, d\theta_s \tag{3}$$

Here, θ_i represents the effective angular range of the receiving optics. Φ_i corresponds to the probability that reflected light falls within the specified angular range due to surface roughness and is directly related to the geometric collection efficiency of the optical system.

The $FR2$ indicator is finally defined as the ratio of two probability functions.

$$FR2 = \frac{\Phi_2}{\Phi_1} \tag{4}$$

This definition quantitatively reflects the physical phenomenon in which increasing surface roughness causes the micro-slope angle distribution to broaden, thereby reducing the proportion of light concentrated within a specific angular range. This study models surface roughness as a probabilistic slope distribution, consistent with physics-based modeling requirements in precision engineering.

3. Parameter Optimization

3.1. Initial Value Optimization Strategy

The coefficient estimation problem of the proposed $FR2$-based roughness model constitutes a nonlinear least-squares problem with high sensitivity to initial values. This sensitivity arises from the combination of multiple nonlinear transformations and numerical integration processes linking surface roughness and optical response, rather than from nonlinearity alone.

In regions where the denominator term becomes small, the gradient of the objective function changes rapidly, resulting in a smooth but generally non-convex objective function. Under such conditions, local optimization methods initiated from a single initial value are prone to converge to local minima rather than the global minimum, particularly when analytical gradients are unavailable and numerical integration is involved.

To mitigate initial value dependence and ensure reliable parameter estimation, a multi-start optimization strategy is adopted. This strategy defines a physically meaningful parameter space and performs independent local optimizations from multiple initial points within that space. The solution yielding the minimum objective function value is selected as the final estimate.

In this study, the parameter ranges are restricted to $a \in [0.01, 1.0]$ and $b \in [-1.0, 1.0]$, which satisfy the condition $\sigma_s > 0$ while sufficiently covering roughness–angular dispersion relationships observed in practical machining processes.

3.2. Parameter Optimization Method

For local optimization at each initial point, the Nelder–Mead simplex algorithm is employed. This derivative-free direct search method iteratively transforms a simplex in the parameter space to reduce the objective function value. In the two-dimensional case, the simplex consists of three vertices forming a triangle, which is updated through reflection, expansion, contraction, and shrink operations.

The dataset used for parameter estimation consists of five surfaces with nominal Ra values of 0.2, 0.4, 0.8, 1.6, and 3.2 μm, prepared by surface grinding. Ra was measured using a contact profilometer traceable to national standards (ISO 4287). $FR2$ measurements were repeated 10 times per surface under identical conditions, and the mean values were used for model fitting and are shown as data points in Figure 3. The 2σ repeatability of $FR2$ ranged from 0.011 to 0.024 across the five surfaces, corresponding to a coefficient of variation below 1.8 %.

As a result, the multi-start Nelder–Mead optimization framework adopted in this study enables robust parameter estimation without simplifying the numerical integration-based physical model. Compared to the conventional grid-based parameter search, the proposed approach reduced the sum of squared errors (SSE) from 0.00311 to 0.00216, corresponding to an improvement of approximately 30.5 % (Figure 3).

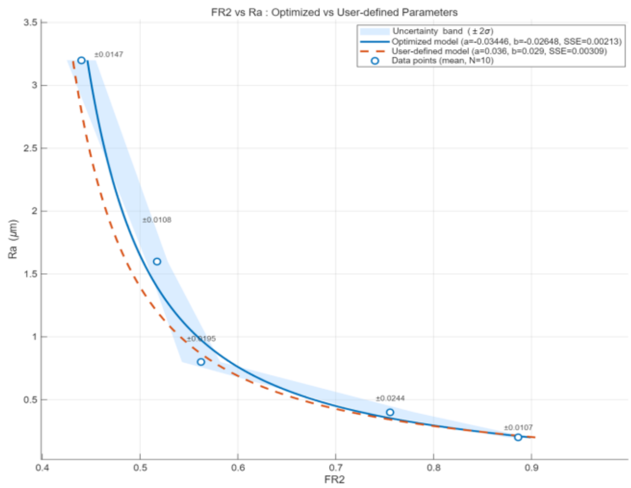


Figure 3. Comparison of optimization results: optimized vs. previously selected variables

4. Conclusion

This study aimed to establish a physics-based $FR2$ model and a robust parameter estimation framework for optical surface roughness measurement, overcoming the generalization limitations of conventional empirical regression approaches. By representing machined surfaces as a statistical ensemble of micro-facets and physically linking surface roughness to optical response through a Gaussian slope distribution, the proposed model maintains full physical interpretability of the model parameters. A multi-start optimization strategy combined with the Nelder–Mead algorithm was employed to mitigate strong initial value sensitivity, reducing the SSE by approximately 30.5 % compared to conventional grid-based parameter search, thereby demonstrating improved fitting accuracy and reproducibility across diverse machining processes and optical configurations.

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