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Complex coordinate measurement models for GUM uncertainty framework

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Abstract

The aim of the research is to develop a method for estimating coordinate measurement uncertainty that is understandable and easy to use in industrial conditions. It was decided to develop a method which, according to the classification given in ISO/TS 15530-1, belongs to the "sensitivity analysis" group. This method requires the development of separate models for each measured characteristic. The term "characteristic" includes dimensions, but above all, various geometrical deviations. In many cases, it is necessary to develop several models for a single type of geometrical deviation (e.g. parallelism deviations), such as parallelism of axes, parallelism of planes, etc. In the article, models with uncertainty evaluation for geometric deviations are presented: parallelism and perpendicularity, position of a point, axis and plane with respect to different datum systems.

Keywords: task specific uncertainty, coordinate measurement models, type B evaluation, sensitivity analysis

1. Introduction

The search for a simple, industrially applicable method of estimating the uncertainty of coordinate measurements is in progress. The basic problem is that numerous different characteristics (dimensions, geometric deviations) are measured in a single measurement run, and each characteristic is measured with a different measurement uncertainty.

According to the ISO 15530-3 standard [1], measurement uncertainty can be assessed experimentally. However, this requires the use of a calibrated workpiece [2–5]. Mutilba [2] notes that due to the requirement of similarity between the manufactured part and the reference part, this approach is only useful in mass production. In contrast, Strbac [3] considers the method presented in [1] as a reference method, but too expensive for everyday use. Furthermore, he is aware that MPE directly concerns only point-to-point distances. Therefore, he attempts to apply the GUM methodology and use information about CMM MPE to determine the uncertainty of flatness measurements. Cheng [4] highlights the diverse geometric characteristics encountered in machine part measurements. While acknowledging the strengths of the method described in [1], he also identifies the necessity for more straightforward approaches. To address this, Cheng suggests a simplified methodology that incorporates MPE data along with repeatability test outcomes, even though it may result in overestimated findings. Kortaberria [5] points out the expenses associated with storing and maintaining reference parts when employing the technique outlined in [1]. Considerable expectations were placed on simulation techniques [6-10]. Heisselmann [6] observes the substantial information requirements for implementing such methods. Forbes [7] elaborates on the model utilized in VCMM software, whereas Baldwin [8] discusses the approach adopted in Pundit software. Aggogeri [9] introduces a streamlined simulation method from Politecnico di Torino based on the bootstrap technique. Sładek [10] presents a method developed at CUT that leverages laser tracker test results to assess CMM accuracy. Currently, only VCMM and Pundit software are available commercially, with VCMM being employed in some calibration laboratories.

Between June 2019 and November 2021, the EURAMET project 17NRM03 EUCoM, titled "Standards for the Evaluation of the Uncertainty of Coordinate Measurements in Industry," was conducted. The project coordinator was A. Balsamo from the Italian Metrology Institute (INRIM). The project partners were national metrology institutes: PTB (Germany), NPL (Great Britain), CMI (Czech Republic), GUM (Poland), DTI (Denmark), TUBITAK (Turkey) and AIST (Japan), research institutes IK4-TEKNIKER (Spain) and Metrosert (Estonia) and universities: University of Padua, University of Bielsko-Biała, Cracow University of Technology joined the partnership. As part of the project, several studies were conducted on three approaches to estimating the uncertainty in coordinate measurements:

- multi-position method, which was the subject of draft standard ISO 15530-2,
- method based on a simplified CMM kinematic model and
- sensitivity analysis method (named after classification from ISO/TS 15530-1).

The paper presents results concerning the last method.

The possibility of evaluation of coordinate measurement uncertainty for significant number of geometrical deviations (straightness, flatness, coaxiality, concentricity, position of axis, runout and position of a point, axis and plane related to a datum plane) based on the $E_{L,MPE}$ formula and possibly the CMM verification test results was presented in works [17, 18]. Appropriate measurement models directly used formulae for the point-straight line distance [17, 19] or point-plane distance [18, 19]. All combined standard uncertainty components are calculated by type B evaluation.

The general formula for distance l between point S and straight-line p defined by point P and unit vector v is (the analysed distance is equal to the length of the vector which is the vector product of two vectors):

$$l(S,p) = |(\boldsymbol{P} - \boldsymbol{S}) \times \boldsymbol{v}| = |\boldsymbol{SP} \times \boldsymbol{v}| \quad (1)$$

The general formula for distance *l* between point *S* and plane *p* defined by point *P* and unit normal vector *v* is (the analysed distance is equal to the absolute value of the dot product of two vectors):

$$l(S,p) = |(\boldsymbol{P} - \boldsymbol{S}) \cdot \boldsymbol{v}| = |\boldsymbol{SP} \cdot \boldsymbol{v}| \quad (2)$$

The important assumption of the presented methodology is that the models used express the measured quantities as a function of distance, and more precisely as a function of the coordinate differences of pairs of points. Another element of the methodology is the ISO 14253-2 guideline [20] which states how „MPE values can be used to derive the related uncertainty components”.

In the equations (1) and (2) the vectors' coordinates are the differences of the coordinates of the characteristic points, which means that the output quantity *l* is a function of six inputs and thus the uncertainty budget will consist of six components. In individual uncertainty budget rows there will be following sensitivity coefficients $\frac{\partial l}{\partial sp_1}, \frac{\partial l}{\partial sp_2}, \frac{\partial l}{\partial sp_3}, \frac{\partial l}{\partial v_1}, \frac{\partial l}{\partial v_2}, \frac{\partial l}{\partial v_3}$.

Thus, the combined standard uncertainty of the distance *l*

$$u_l = \sqrt{\sum\nolimits_{i=1}^{n} \left(\frac{\partial l}{\partial x_i} u_{xi}\right)^2} \quad (3)$$

in which

$$u_{xi} = b \cdot E_{L,MPE}(x_i) \quad (4)$$

According to the classification of techniques for uncertainty estimation given in [21], the presented technique belongs to the "sensitivity analysis" approach. According to the classification given in [22], the developed technique belongs to the "GUM uncertainty framework". The assumption which was crucial for development of measurement models is that the coordinate measurement is treated as an indirect measurement, relying on the smallest possible number of points mathematically. The differences between the coordinates of these points serve as the input values for the measurement model. These points can be points of integral feature (surface points) as well as derived feature (points of axes and planes of symmetry, centre of the sphere) and are called “characteristic points”. In actual measurements, the number of sampling points is much greater than the minimum mathematical number. Substituting the probing points with characteristic points makes a lot easier developing the measurement models which become clear to understand even for a regular CMM user. Earlier publications [17, 18] showed that the calculated measurement uncertainty does not depend on the position and orientation of the object in the CMM space. This allows to perform calculations in the coordinate system of the workpiece.

It is worth recalling that in many measurement models, a vector normal to the plane defined by ABC points can be determined using the vector product in three ways as: *AB* × *AC*, *BA* × *BC* or *CA* × *CB*. Then it is necessary to consider all three options and use the one that gives the lowest uncertainty value.

2. Models

2.1. Distance between the point and the straight line defined by given point and parallel to the straight line defined by two points

The straight line *p* in the formula (1) is defined by the point *K* and is parallel to the straight line defined by points *A* and *B* (Figure 1).

The measurement model is

$$l(AB, KS) = \left|\boldsymbol{KS} \times \frac{\boldsymbol{AB}}{|\boldsymbol{AB}|}\right| \quad (5)$$

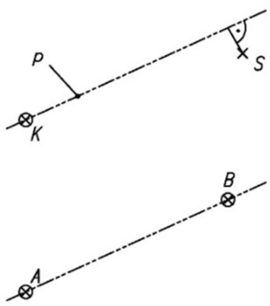

Figure 1. Distance between the point *S* and the straight-line *p* defined by point *K* and parallel to the straight line defined by points *A* and *B*

It should be understood as follows: parallelism is a function of two vectors i.e. six coordinates of these vectors

$$l(ab_1, ab_2, ab_3, ks_1, ks_2, ks_3) = \left|\boldsymbol{KS} \times \frac{\boldsymbol{AB}}{|\boldsymbol{AB}|}\right| \quad (6)$$

The vector coordinate is the difference of the coordinates of the characteristic points, which means that the output quantity *l* is a function of six inputs and thus the uncertainty budget will consist of six components. In individual uncertainty budget rows there will be following sensitivity coefficients $\frac{\partial l}{\partial ab_1}, \frac{\partial l}{\partial ab_2}, \frac{\partial l}{\partial ab_3}, \frac{\partial l}{\partial ks_1}, \frac{\partial l}{\partial ks_2}, \frac{\partial l}{\partial ks_3}$.

Example. The parallelism of axes with cylindrical tolerance zone *PRLAX* equals the distance *l* between point *S* (nominally laying on the toleranced axis) and straight line passing through the point *K* and parallel to the datum axis *AB* (*PRLAX* = *l*) (Figure 2).

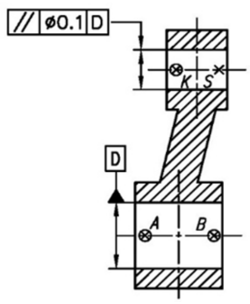

Figure 2. Example of specification with characteristic points of parallelism of axes with cylindrical tolerance zone

The uncertainty of measurement of the parallelism of axes u_{PRLAX} equals the uncertainty u_l of measurement of the distance *l* ($u_{PRLAX} = u_l$).

The table presents the uncertainty budget for following input data (coordinates of four characteristic points): *A*(50, 50, 10), *B*(350, 50, 10), *C*(200, 350, 10), *S*(200, 150, 10.1). The CMM used for measurement has maximum permissible error $E_{0,MPE}$ = 2 + *L*/250. Because a rectangular distribution of errors was assumed, the coefficient *b* was set to 0.58 (see formula 4).

Table 1 Example uncertainty budget for parallelism of axes

Name	x_i	$\partial l/\partial x_i$	$E_{0,MPE}(x_i)$	u_{xi}	$\partial l/\partial x_i \cdot u_{xi}$
xAB	20	0	2.1	1.21	0
yAB	0	0	2	1.16	0
zAB	0	-1	2	1.16	-1.16
xKS	20	0	2.1	1.21	0
yKS	0	0	2	1.16	0
zKS	0,01	1	2	1.16	1.16
				u_{PRLAX}	1.64

If the length of the element and the length of the datum are the same the uncertainty does not depend on the distances of the lines or on the distances between the points defining the lines (i.e. on the lengths of the lines). Therefore, only the constant component (A) from the $E_{0,MPE}$ formula is important. There are two identical, non-zero components in the budget, both with a weight of 1. Therefore, the measurement uncertainty of the straight-line parallelism deviation can be calculated according to the formula

$$u = \sqrt{2} \cdot A \cdot b \tag{7}$$

2.2. Distance between the point and the plane defined by the given point and parallel to the plane defined by three points

In case the plane p is represented by point K and is parallel to the plane defined by points A, B and C (Figure 3) the unit normal vector $\boldsymbol{v}$ can be defined in three ways and as the point P in the formula (2) the point K is assumed. Thus, there are three variants of the model:

$$l_1(AB, AC, KS) = \left| \boldsymbol{KS} \cdot \frac{\boldsymbol{AB} \times \boldsymbol{AC}}{|\boldsymbol{AB} \times \boldsymbol{AC}|} \right| \tag{8}$$

$$l_2(BA, BC, KS) = \left| \boldsymbol{KS} \cdot \frac{\boldsymbol{BA} \times \boldsymbol{BC}}{|\boldsymbol{BA} \times \boldsymbol{BC}|} \right| \tag{9}$$

$$l_3(CA, CB, KS) = \left| \boldsymbol{KS} \cdot \frac{\boldsymbol{CA} \times \boldsymbol{CB}}{|\boldsymbol{CA} \times \boldsymbol{CB}|} \right| \tag{10}$$

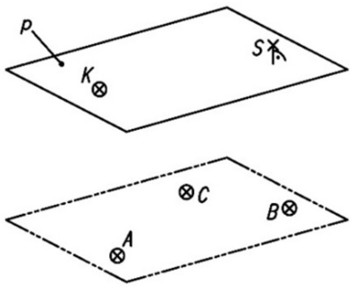

Figure 3. Distance between the point S and the plane p defined by the point K and parallel to the plane defined by points A, B and C.

The uncertainty of measurement of the distance l equals the minimum of uncertainties u_{li} of measurement of the distance l_i ($u_l = \min\{u_{li}\}$).

Example. The parallelism of planes *PRLPL* (Figure 4a) as well as parallelism of straight line in regard to plane *PRLSTRL* (Figure 4b) equals the distance l between point S (laying on the toleranced element) and plane passing through the point K and parallel to the plane ABC (*PRLSTRL* = *PRLPL* = l).

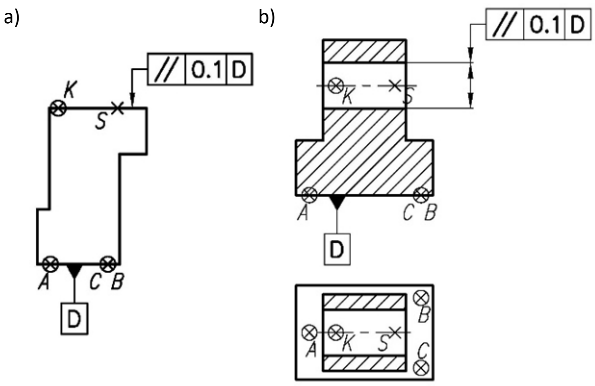

Figure 4. Example of specification with characteristic points: a) parallelism of planes b) parallelism of axis in regard to plane

Thus, the uncertainty of measurement of the parallelism u_{PRLPL} and $u_{PRLSTRL}$ equals the uncertainty u_l of measurement of the distance l ($u_{PRLPL} = u_{PRLSTRL} = u_l$).

2.2. Distance between the point and the plane defined by two points and perpendicular to the plane defined by three points

In case the plane is represented by two points D and E and is perpendicular to plane defined by points A, B and C (Figure 5) as the point P in the formula (**2**) the point D or E can be assumed and the unit normal vector $\boldsymbol{v}$ can be defined as a cross product of the normal vector of the plane ABC and vector DE.

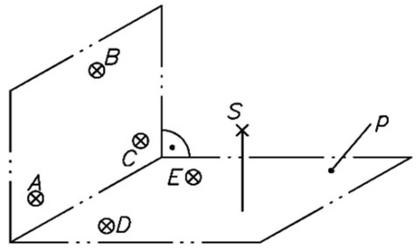

Figure 5. Distance between the point S and the plane p defined by points D, E and perpendicular to the plane ABC

Since the normal vector of plane ABC can be determined by three distinct methods (for example, using different combinations of points or vector products), and for each method, either point D or E can be chosen as point P in formula (2), this results in a total of six possible measurement model variants:

$$l_1(AB, AC, DE, DS) = \left| \boldsymbol{DS} \cdot \frac{(\boldsymbol{AB} \times \boldsymbol{AC}) \times \boldsymbol{DE}}{|(\boldsymbol{AB} \times \boldsymbol{AC}) \times \boldsymbol{DE}|} \right| \tag{11}$$

$$l_2(BA, BC, DE, DS) = \left| \boldsymbol{DS} \cdot \frac{(\boldsymbol{BA} \times \boldsymbol{BC}) \times \boldsymbol{DE}}{|(\boldsymbol{BA} \times \boldsymbol{BC}) \times \boldsymbol{DE}|} \right| \tag{12}$$

$$l_3(CA, CB, DE, DS) = \left| \boldsymbol{DS} \cdot \frac{(\boldsymbol{CA} \times \boldsymbol{CB}) \times \boldsymbol{DE}}{|(\boldsymbol{CA} \times \boldsymbol{CB}) \times \boldsymbol{DE}|} \right| \tag{13}$$

$$l_4(AB, AC, DE, ES) = \left| \boldsymbol{ES} \cdot \frac{(\boldsymbol{AB} \times \boldsymbol{AC}) \times \boldsymbol{DE}}{|(\boldsymbol{AB} \times \boldsymbol{AC}) \times \boldsymbol{DE}|} \right| \tag{14}$$

$$l_5(BA, BC, DE, ES) = \left| \boldsymbol{ES} \cdot \frac{(\boldsymbol{BA} \times \boldsymbol{BC}) \times \boldsymbol{DE}}{|(\boldsymbol{BA} \times \boldsymbol{BC}) \times \boldsymbol{DE}|} \right| \tag{15}$$

$$l_6(CA, CB, DE, ES) = \left| \boldsymbol{ES} \cdot \frac{(\boldsymbol{CA} \times \boldsymbol{CB}) \times \boldsymbol{DE}}{|(\boldsymbol{CA} \times \boldsymbol{CB}) \times \boldsymbol{DE}|} \right| \tag{16}$$

The uncertainty of measurement of the distance l equals the minimum of uncertainties u_{li} of measurement of the distance l_i ($u_l = \min\{u_{li}\}$).

Example. The position of the point (centre of the ball) *POS* relative to secondary datum of the datum system (Figure 6) is a doubled value of the distance l of the point S from the plane positioned at the theoretically exact position defined by *TED* (*POS* = 2 (l – *TED*)).

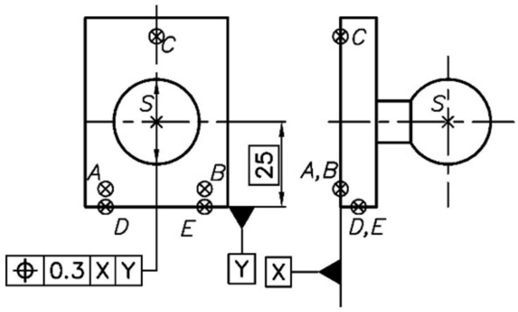

Figure 6. Example of specification with characteristic points of position of the point (centre of the ball) in regard to the secondary datum of datum system defined by two perpendicular planes

Therefore, the uncertainty of measurement of the position u_{POS} equals twice the uncertainty u_l of the distance of the point from the plane being the secondary datum (*TED* =25) ($u_{POS} = 2u_l$).

Example. The position of the axis of a hole *POS* relative to secondary datum of the datum system (Figure 7) is a doubled value of the distance *l* of the point *S* from the plane positioned at the theoretically exact position defined by *TED* (*POS* = 2 (*l* – *TED*)).

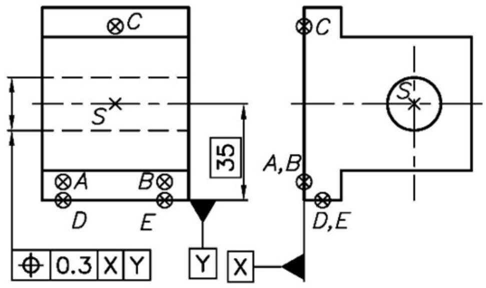

Figure 7. Example of specification with characteristic points of position of the axis of a hole related to the datum system

Therefore, the uncertainty of measurement of the position u_{POS} equals twice the uncertainty u_l of the distance of the point from the plane being the secondary datum (*TED*=35) ($u_{POS} = 2u_l$). The mid-point of the axis can be considered as point *S* in the calculation.

Example. The perpendicularity *PER* of the plane in regard to datum plane (Figure 8) is a value of the distance *l* of the point *S* from the plane defined by points *K* and *L* and perpendicular to plane defined by point *A*, *B* and *C* (*PER* = *l*).

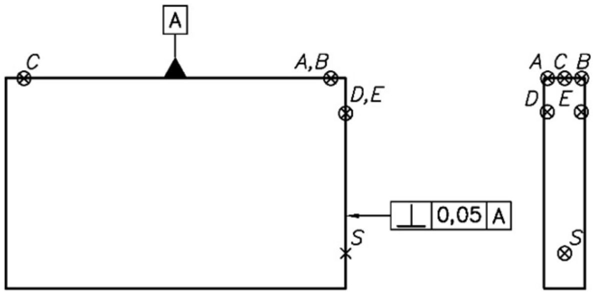

Figure 8. Example of specification with characteristic points of perpendicularity of two planes

Therefore, the uncertainty of measurement of the perpendicularity u_{PER} equals the uncertainty u_l of the distance of the point from the plane ($u_{PER} = u_l$).

2. Conclusions

Individual models have the form of distance of a point from a straight line or a plane, for particular geometrical deviations equal to this distance or twice the value of this distance. The second case concerns particularly position deviations.

Developed method allows for the estimation of the measurement uncertainty of all geometrical deviations.

Models for several cases of parallelism, perpendicularity and position were described. In the examples, there are tolerance zones defined by a pair of planes and a cylinder.

Performing the necessary calculations requires the use of software that performs operations on vectors (e.g. Maple, Matlab).

Described method may take the form of a standard and be applicable in industrial conditions.

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References

[1] ISO 15530-3:2011 Geometrical product specifications (GPS). Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement. Part 3: Use of calibrated workpieces or measurement standards

[2] Mutilba U et al. Traceability of on-machine tool measurement: Uncertainty budget assessment on shop floor conditions *Measurement* **135** (2019) 180–188

[3] Štrbac B et al. The difference between GUM and ISO/TC 15530-3 method to evaluate the measurement uncertainty of flatness by a CMM *MAPAN* **32** (4) (2017) 251–257

[4] Cheng Y et al. Evaluation and optimization of task-oriented measurement uncertainty for coordinate measuring machines based on geometrical product specifications *Applied Sciences* **9** (1) (2019) 1-22

[5] Kortaberria G. et al. Three-dimensional point cloud task-specific uncertainty assessment based on ISO 15530-3 and ISO 15530-4 technical specifications and model-based definition strategy, Metrology 2022, 2, 394–413. https://doi.org/10.3390/metrology2040024

[6] Heisselmann D et al. Determination of measurement uncertainty by simulation *Proceedings of Advanced Mathematical and Computational Tools in Metrology and Testing* **XI**, 2017

[7] Forbes A et al. Simulated instruments and uncertainty estimation *NPL Report CMSC* **01/00**, Teddington, 2000

[8] Baldwin J Application of simulation software to coordinate measurement uncertainty evaluations *Measure* **2** (2007) 40–52

[9] Aggogeri F et al. Measurement uncertainty assessment of Coordinate Measuring Machines by simulation and planned experimentation *CIRP Journal of Manufacturing Science and Technology* **4** (2011) 51–56

[10] Sładek J. et al. Virtual coordinate measuring machine built using lasertracer system and spherical standard, Metrol. Meas. Syst., Vol. XX (2013), No. 1, pp. 77–86

[11] ISO 1101:2017 Geometrical product specifications (GPS). Geometrical tolerancing. Tolerances of form, orientation, location and run-out

[12] ISO 5459:2011 Geometrical product specifications (GPS). Geometrical tolerancing. Datums and datum systems

[13] Humienny Z State of art in standardization in the geometrical product specification area a decade later *CIRP Journal of Manufacturing Science and Technology* **33** (2021) 42–51

[14] Wojtyła M et al. Verification of sensitivity analysis method of measurement uncertainty evaluation *Measurement: Sensors* **18** (2021) 100274

[15] Rosner P et al. Uncertainty evaluation for complex GPS characteristics *Measurement: Sensors* **18** (2021) 100323

[16] Płowucha W et al. Proposal for changes in the ISO 15530 series of standards *Calitatea* **13** (5) (2012) 237–240

[17] Płowucha W Point-straight line distance as model for uncertainty evaluation of coordinate measurement *Measurement* **135** (2019) 83–95

[18] Płowucha W Point-plane distance as model for uncertainty evaluation of coordinate measurement *Metrol. Meas. Syst.* **27** (2020) 625–639

[19] ISO 17450-1:2011 Geometrical product specifications (GPS). General concepts. Part 1: Model for geometrical specification and verification

[20] ISO 14253-2:2011 Geometrical product specifications (GPS). Inspection by measurement of workpieces and measuring equipment. Part 2: Guidance for the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification

[21] ISO/TS 15530-1:2013 Geometrical product specifications (GPS). Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement. Part 1: Overview and metrological characteristics

[22] JCGM 104:2009 Evaluation of measurement data. An introduction to the “Guide to the Expression of Uncertainty in Measurement” and related documents