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Metrological constraints in predicting tool deflection for error compensation

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Abstract
Accurate prediction of tool deflection is essential for compensating force-induced machining errors, yet it depends on the metrological quality of cutting force and stiffness measurements. While cutting force measurement benefits from well-established calibration artefacts and traceability, stiffness remains a derived quantity with inconsistent measurands, limited comparability, and largely unquantified uncertainties. This work investigates the feasibility of tool deflection prediction by linking measurement uncertainties of force and stiffness through a simplified elastic model. Representative benchmark values were defined from the literature, and the resulting uncertainty of tool deflection was analysed for different combinations of cutting force and machine tool stiffness. The results show that cutting force uncertainty plays a lesser role than stiffness uncertainty, which dominates particularly in low-stiffness regimes. For high-stiffness machines, current measurement practice is sufficient to support compensation, whereas for medium stiffness improved accuracy is required, and for low stiffness reliable prediction remains impractical. The study highlights stiffness measurement as the key bottleneck, underscoring the need for standardized measurands, procedures, and validated uncertainty budgets.

tool deflection; cutting force; machine tool stiffness; uncertainty machine tool

1. Introduction

The accuracy of manufactured components underpins the performance and competitiveness of modern industries. Aerospace, automotive, medical, and energy sectors demand dimensional tolerances that are increasingly stringent, since even small deviations can compromise functionality, safety, and cost-effectiveness. Despite major advances in machine design, error compensation, and digital control, deviations from the programmed tool path remain unavoidable due to the interaction between machine structure and cutting process physics [1].

Among the multiple error sources – geometric imperfections, thermal drift, and vibration – tool deflection caused by cutting forces acting on a structure of limited stiffness is particularly critical [2]. Elastic displacement of the tool directly alters the form and size of the workpiece. As tolerances tighten and productivity demands increase, reliable prediction and compensation of tool deflection has become a central challenge in precision engineering [3].

Compensation strategies, whether offline or adaptive, require two measurable inputs: the applied cutting forces and the stiffness/compliance of the machine's structural loop [4]. Together, these define the expected elastic displacement of the tool under load. Although conceptually straightforward, prediction accuracy is fundamentally limited by measurement quality, the lack of interlaboratory validation, and the absence of universally accepted standards [5].

Cutting force measurement benefits from a mature metrological foundation. Piezoelectric dynamometers remain the reference, supported by internationally recognized calibration procedures and traceability to national standards. Research trends include structure-integrated sensors [6] and non-invasive estimation from spindle currents [7] or accelerometers [8]. These approaches enhance industrial applicability but introduce calibration challenges and typically lower accuracy [9].

Machine tool stiffness (or compliance) is a derived quantity defined as the ratio of externally applied force to the resulting displacement. Unlike cutting force, it lacks standardized measurands, and accredited calibration procedures, which limits comparability across studies. ISO 230-1 defines external and internal loading methods [10], but both are limited to simplified evaluations. More advanced techniques include six-degree-of-freedom multi-sensor systems [11], stiffness workspace mapping [12], and loaded double ball bar tests combined with multi-body or compliance modelling [13]. Other methods incorporate indirect joint compliance estimation [14] or triaxial excitation mapping [15].

This contrast highlights a critical metrological imbalance: while cutting force measurement is sufficiently mature for many compensation tasks, stiffness measurement lacks standardized measurands, calibration procedures, and validated uncertainty budgets. As a result, predictive models of tool deflection may rest on inputs of widely differing credibility.

Previous studies have largely advanced either measurement techniques for stiffness or cutting force monitoring, or developed compensation algorithms assuming idealized input data [16]. The distinctive contribution of this work is to address explicitly the metrological feasibility of tool deflection prediction. Rather than proposing new hardware or algorithms, the focus here is on assessing whether current measurement accuracies suffice to justify machining error compensation.

A measurement-oriented framework is introduced in which tool deflection is evaluated from a simplified elastic model, linking the uncertainties of cutting force and stiffness measurements. This approach enables identification of conditions in which compensation meets target uncertainty requirements and those where current metrological limitations preclude its application.

The structure of the paper is as follows. Section 2 reviews the state of the art in stiffness and cutting force measurements and establishes representative values for benchmarking. Section 3 introduces the simplified measurement model and the uncertainty evaluation. Section 4 presents and discusses the results of the parametric analysis. Section 5 concludes by highlighting the implications for machining accuracy and the need for validated stiffness measurement methods as a prerequisite for reliable tool deflection compensation in both machine tools and robots.

2. Uncertainties in stiffness and cutting force measurements

Accurate prediction of tool deflection requires dependable information on both machine tool stiffness and cutting forces. Both quantities, however, are affected by metrological limitations that constrain their reliability in practical use. Reviewing these limitations allows identification of representative benchmark values that form the basis for subsequent analysis.

2.1. Cutting force measurements

Cutting force is one of the few quantities in machining that can be confirmed with well-established calibration and traceability. Piezoelectric dynamometers remain the reference, supported by accredited calibration procedures and direct links to national standards. Under controlled conditions, expanded uncertainties as low as 0.1-0.2 % of full scale have been reported [17,18] confirming their suitability as reference instruments.

In machining applications, however, the dominant contributions to uncertainty arise not from the sensors themselves but from process-related effects such as workpiece heterogeneity, fixturing, dynamic excitation, and thermal drift. Consequently, while calibration studies define the achievable metrological performance of dynamometers, cutting experiments consistently reveal higher variability.

Axinte et al. [19] reported an expanded uncertainty of 24 N (3.2 %) for turning forces up to 740 N, with about half of the error attributed to process variations. Ciecieląg et al. [20] observed 9-12 % uncertainty (3.5-5 N) for forces up to 40 N, driven by repeatability, reproducibility, and sensor error. Under analogous tests on aluminium alloys, Zawada-Michałowska et al. [21] reported expanded uncertainties of 7-11 N (9-14 %) for forces up to 80 N.

As an alternative approach, *feeling machines* enable in-process force monitoring and tool deflection prediction through in-situ calibration. This is typically achieved using controlled tool-workpiece collision cycles, which simultaneously calibrate the force response and determine static tool stiffness. Reported accuracy, however, remains limited: Denkena et al. [22] observed mean-square errors of 25 N in X/Y and 100 N in Z, reduced to 25 N after low-pass filtering. Errors closely matched bias relative to dynamometer readings up to 300 N, decreasing at higher loads. Process-based and offline calibration achieved maximum bias of about 10 % for forces up to 500 N, with a resolution of 10 N in X and Y [23].

In summary, although cutting force measurement benefits from established calibration methods, its practical application remains constrained by process-related variability that significantly increases measurement uncertainty.

2.2. Stiffness measurements

Stiffness, in contrast, is a derived quantity defined as the ratio of applied force to measured displacement. Unlike force, it lacks standardized measurands, reference artefacts, and accredited calibration procedures. Reported values vary widely depending on machine configuration, loading method, and measurement setup, which complicates comparison between studies. Consequently, while stiffness data provide essential insight into machine tool behaviour, their metrological credibility remains limited by the absence of traceability and consistent uncertainty evaluation.

Several approaches have been reported in the literature. Archenti [13] introduced an elastically linked system using a loaded double ball bar combined with multi-body and finite element modelling, reporting stiffness in the range of 7-12 N/µm with repeatability of 0.2-0.6 N/µm. Beglarzadeh et al. [14] applied indirect joint compliance estimation with a telescopic double ball bar, obtaining stiffness around 17 N/µm with repeatability of 0.1 N/µm. Majda and Jastrzębska [11] proposed a generalized 6-DoF method, reporting translational stiffness of 2-30 N/µm and rotational stiffness of 0.2-8 Nm/µrad, with axis-dependent standard uncertainties ranging from 0.03 to 0.34 N/µm. Pawełko et al. [24] developed a hydraulic multi-axis loading system to map workspace stiffness, reporting directional values of 8-18 N/µm with expanded uncertainties of about 1.4-1.8 N/µm depending on axis.

In summary, although stiffness measurement provides valuable insight into machine tool behaviour, its practical application remains constrained by the lack of standardized procedures and traceable uncertainty evaluation.

2.3. Benchmark values

For the analysis, simplified benchmark ranges were adopted to represent typical and extreme operating conditions. Two cutting force levels were selected: 50 N, corresponding to finishing operations, and 800 N, representative of roughing or high-performance machining. To capture the extremes of structural rigidity, two stiffness levels were considered: 2 N/µm, typical of low-stiffness systems such as robots or slender machines, and 30 N/µm, characteristic of high-stiffness machine tools. These consolidated benchmark values, summarized in Table 1, form the basis for the uncertainty analysis presented in Section 4.

Table 1. Representative benchmark values of cutting force and stiffness adopted for uncertainty analysis.

Measurand	Reference value	Representative system
machine tool stiffness	2 N/µm	robots, low-stiffness machine tools, machine tools with long overhang tools or workpieces
	30 N/µm	high-stiffness machine tools
cutting force	50 N	finishing operations
	800 N	roughing/high-performance machining

Based on the literature, achievable values of expanded uncertainty were also defined, ranging from 8-40 N for force and 0.5-2.0 N/µm for stiffness. These benchmarks provide a consistent basis for evaluating the metrological feasibility of tool deflection prediction.

3. Uncertainty of Tool Deflection Measurement

The starting point is a measurement model for tool deflection δ consistent with the measurement objective, *i.e.* determining tool deflection with an uncertainty that represents a specified fraction of the tolerance of the relevant feature of size.

In the simplest one-dimensional case, δ is expressed as:

$$\delta = \frac{F}{K} + \sum \Delta_i, \tag{1}$$

where F is the constant component of the cutting force, K the equivalent stiffness of the structural loop, and Δ_i correction reflecting additional effects (tool/workpiece geometry, machine properties, dynamics, modelling corrections).

Since uncertainties of the corrections Δ_i of are practically unattainable, the model is simplified to:

$$\delta = \frac{F}{K}. \tag{2}$$

A justification for this approximation, including the relative order of magnitude of neglected terms, is given in [25], where the simplified model is supported by a theoretical and metrological analysis with a quantitative evaluation of systematic error and uncertainty.

This retains the dominant elastic effect while allowing uncertainty evaluation based only on inputs with quantifiable data. Assuming independent inputs, the combined standard uncertainty is:

$$u_\delta = \sqrt{\sum (c_i \cdot u_i)^2} = \sqrt{\left(\frac{1}{K} \cdot u_F\right)^2 + \left(-\frac{F}{K^2} \cdot u_K\right)^2}, \tag{3}$$

where: u_i – standard uncertainties, c_i – sensitivity coefficient and the expanded uncertainty is:

$$U_\delta = k \cdot u_\delta. \tag{4}$$

where k – coverage factor.

This simplified model provides a consistent basis for assessing the impact of force and stiffness uncertainties on tool deflection.

4. Results and discussion

The results presented in this section follow directly from the benchmark values established in Section 2.3 and the simplified measurement model introduced in Section 3. Tool deflection uncertainties were evaluated as a function of the input uncertainties in cutting force and machine tool stiffness, with representative scenarios defined by two force levels (finishing and roughing) and two stiffness levels (low- and high-rigidity systems).

Figure 1 illustrates the uncertainty of tool deflection as a function of the uncertainties in cutting force and stiffness. The results present the expanded uncertainty corresponding to a coverage factor $k = 2$. Each subplot represents a specific combination of cutting force and stiffness, allowing direct comparison of how input variability propagates into the variability of tool-deflection prediction. In the contour plots, fully transparent regions indicate ranges of measured quantities for which current metrological methods provide sufficiently accurate determination, whereas the opaque regions correspond to values beyond the presently achievable measurement capability and thus lie outside the range of reliable metrological assessment.

The results in Figure 1 reflect how uncertainties in cutting force and stiffness propagate into overall dispersion of tool deflection prediction through the simplified elastic model. The subplots illustrate that both parameters contribute significantly, but their influence depends strongly on the absolute values of force and stiffness.

At low stiffness values (≈2 N/μm), the uncertainty of tool deflection increases rapidly even with modest rises in stiffness uncertainty, particularly under high cutting forces. For example (see Fig. 1), at a cutting force of F = 800 N and stiffness K = 2 N/μm, the resulting tool deflection is approximately 400 μm, while the associated uncertainty ranges from 200 μm to 750 μm, corresponding to a relative uncertainty of about 50% to 190% of the deflection value. This indicates that for low-rigidity systems such as robots or slender machine tools, reliable prediction of tool deflection is strongly constrained by the present state of stiffness metrology. For finishing forces (≈50 N), the predicted uncertainties remain moderate, but at roughing levels (≈800 N) they increase sharply, suggesting that error compensation in this regime is unlikely to be justified. When compared with currently achievable accuracies, expanded uncertainties of 8-40 N for force and 0.5-2.0 N/μm for stiffness, as identified from the literature, it becomes clear that such low-stiffness scenarios lie beyond present metrological feasibility.

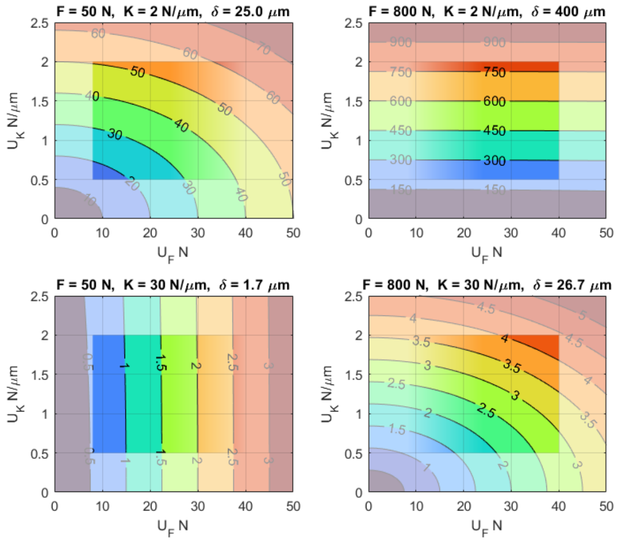

Figure 1. Tool deflection uncertainty as a function of force and stiffness inputs.

At higher stiffness (≈30 N/μm), the effect of stiffness uncertainty is strongly attenuated, as stiffness enters the model in the denominator. Here, force uncertainty becomes the dominant factor, but even with conservative input values the resulting tool deflection uncertainty remains relatively small. Under the same benchmark ranges, these high-stiffness cases fall within practical feasibility, confirming that current metrology already supports reliable compensation strategies.

These results indicate a general trend: the feasibility of tool deflection prediction improves markedly with increasing structural rigidity, whereas low-stiffness systems remain constrained by the absence of stiffness measurement methods capable of achieving the target uncertainty defined in this study. In practical terms, this means that error compensation is realistic for conventional high-stiffness machine tools but remains highly challenging for flexible systems, underscoring the need for standardized stiffness metrology.

From a practical standpoint, the presented results can be used to establish measurement targets for both laboratory and industrial setups. For example, stiffness uncertainty below 0.5 N/μm should be considered a realistic benchmark for validating compensation strategies in high-stiffness machines, whereas values above 1.5-2.0 N/μm make compensation questionable. For robotic or lightweight systems, current stiffness measurement methods do not yet meet this requirement, emphasizing the need for improved calibration standards and in-process identification techniques.

5. Conclusions

This study examined the uncertainty of tool deflection prediction based on cutting force and stiffness measurements. The analysis confirmed that while cutting force measurement is well established and supported by calibration artefacts, stiffness measurement remains inconsistent, poorly standardized, and without confirmation of interlaboratory comparisons.

The results show that in high-stiffness machines, current levels of measurement accuracy are sufficient to experimentally verify reliable compensation of force-induced errors. In medium-stiffness systems, compensation becomes feasible only if stiffness uncertainty can be reduced, while in low-stiffness configurations, particularly in robotic and lightweight machining systems, it remains impractical with present metrology.

From an application perspective, the proposed framework enables assessing the feasibility of tool deflection compensation under real measurement conditions. It identifies scenarios where compensation is justified (in the context of achieving target uncertainty) or limited by metrological constraints, supporting process planning and measurement system design.

Overall, stiffness measurement emerges as the bottleneck for uncertainty-based tool deflection prediction. Future progress will therefore depend not on new force sensors but on validated stiffness measurement methods, including standardized measurands, procedures, and interlaboratory comparisons.

Future work should extend the present uncertainty-based framework to dynamic conditions, where tool-workpiece interaction and thermal effects amplify compliance variability. Integrating stiffness identification into adaptive control or digital twins could bridge the gap between calibration and in-process compensation, especially in flexible or robotic systems.

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