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A Reduced-Order Modelling Framework for the Receptance Coupling Method

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Abstract

This study presents a reduced-order modelling technique for the Receptance Coupling Substructure Analysis, RCSA. The approach addresses critical aspects of precision manufacturing: accurately estimating tool-spindle dynamics and the practical limitations and trade-offs of carrying out reduced-order analysis.
The first stage involves identifying the coupled dynamics of the tool holder and spindle system using the Receptance Coupling Method. Numerical methods are employed to model the tool and holder assembly under free-free boundary conditions, while experimental frequency response measurements capture spindle characteristics. Mathematical coupling achieved by the Receptance Coupling Method is used to derive the tool center point frequency response function. The second stage focuses on the resultant dynamic characteristics of the reduced order RCSA and discusses the practical limitations and the supposed benefits of reduced experimental processes and lessened computational effort. The results are compared with impact hammer tests, and the margins of error are argued, and complementary methods to decrease the said errors are discussed.
By integrating dynamic modelling with geometric simulation, this methodology may provide a predictive capability for compensating machining errors before production. The combined use of numerical and experimental techniques reduces reliance on trial and error adjustments, improving efficiency and ensuring compliance with tolerance requirements in high-precision manufacturing operations.

Keywords: Receptance Coupling, Frequency Response Function, Substructure Coupling

1. Introduction

The frequency response function (FRF) is an analytical tool for improving machining process stability, estimating surface quality and dimensional tolerance accuracy, and modelling the dynamic behaviour of the tool-center-point (TCP). There exists several factors that affect the resultant workpiece geometry, such as thermo-dynamic factors, workpiece compliance, and structural vibrations. Among these, structural vibrations can be identified using several different methods, such as hammer or shaker tests of the assembled spindle-tool pairs, the Receptance Coupling Method, the Inverse Receptance Coupling Method, Operational Modal Analysis, as well as methods employing neural networks [1].

Often, these methods require extensive experimentation and are often impractical in industrial environments. Limited accessibility, industry standards, and sensor constraints are detrimental to in-situ measurement of assembled spindle and tool-tip pair FRFs. On the other hand, methods that require operational states of the CNC machine tools prove to be ill-suited to industrial settings. Although FEM can be used to represent spindle assemblies, the practical modelling application and resultant characterization are severely limited. Accurate characterization of the spindle through numerical methods for accuracy is not practically useful, as high-load machinery is prone to wear and tear.

Consequently, modelling approaches that combine experimentally identified substructures with numerical methods have gained significant attention. Receptance Coupling Substructure Analysis (RCSA) is proven to be an effective framework for predicting the dynamic behaviour of the assembled spindle and tool-tool holder pairs.

Several studies have investigated different methods for the formulation of RCSA [1], including tool-holder modelling using beam theory or FEM formulations, and demonstrated the importance of including shear deformation and the importance of rotational degrees of freedom. Timoshenko beam representation has been shown to provide improved agreement with experimental validation compared to Euler-Bernoulli assemblies of tool and tool holder pairs [2].

Despite these advances, many studies assume the availability of complete spindle driving point FRFs or neglect the influence of missing rotational outputs and moment inputs on the receptances. The extent of reduced order of spindle characteristics that can be neglected without detrimentally affecting the coupling solution, therefore, remains an open question, particularly when rotational degrees of freedom can be included in the coupling formulation.

In this study, a receptance coupling framework is presented where we incorporate a *4x4* non-axial translational and rotational representation of the tool and tool-holder pair employing Timoshenko beam theory and an experimentally identified spindle assembly, characterized by a semi-reduced *2x4* driving point FRF matrix, incorporating only translational inputs. The proposed approach is validated through impact hammer experiments performed on the physically assembled model and is compared to the mathematical coupling. The results demonstrate that reliable tool-tip dynamics can be estimated through the semi-reduced modelling of the spindle dynamic identification.

2. Methodology

Formulation of the mathematical coupling of receptances is prone to error due to a number of unmodelled dynamics. The lack of modelled dynamics of the rotational degrees of freedom of the numerically modelled tool and tool holder pairs and experimentally estimated spindle is shown to be detrimental [1].

2.1. Experimental identification of the spindle FRFs

An SK40 ER16 collect chuck holder (HOLEX, A=100 mm) is axially shortened by wire EDM in order to create a well-defined coupling plane for the receptance coupling procedure. Figure 2 shows the resultant components of the wire EDM operation. The shortened tool holder is then installed on the CNC plant—DMU 50 machining center, where all experiments are conducted. The spindle position in the machine coordinate system is saved and ensured to remain consistent through the experimentations.

The base of the tool holder is excited using an instrumented impact hammer (PCB Piezotronics, Model 086C03). The vibration responses are measured utilizing a set of triaxial accelerometers (PCB Piezotronics, Model 356A17). In total, four distinct combinations of sensor placement and axial excitation sets are recorded. The axial excitations consist of *X* and *Y* axes of the DMU 50 machining center, while the accelerometer positions consist of the cut tool holder's outermost edges on the axes, lying on the axis line, aimed to measure the vertical displacements.

Figure 2. Axially shortened tool holder, HOLEX SK40 ER16.

The vertical displacement results of the two sensors are later utilized for the estimation of the rotational frequency response functions. The force and acceleration signals were acquired using a CompactDAQ data acquisition chassis (NI, cDAQ).

$$H_{22} = \begin{bmatrix} H_{xx}(\omega) & H_{xy}(\omega) & H_{xM_x}(\omega) & H_{xM_y}(\omega) \\ H_{yx}(\omega) & H_{yy}(\omega) & H_{yM_x}(\omega) & H_{yM_y}(\omega) \\ H_{R_xx}(\omega) & H_{R_xy}(\omega) & H_{R_xM_x}(\omega) & H_{R_yM_y}(\omega) \\ H_{R_yx}(\omega) & H_{R_yy}(\omega) & H_{R_yM_x}(\omega) & H_{R_yM_y}(\omega) \end{bmatrix} \quad \text{Eq. (1)}$$

From each set of unique impact locations and sensor placements, five impacts are selected. Equation 1 depicts the FRF matrix of the spindle that is aimed to be formed. From the selected data sets, the first two columns of the said FRF matrix are estimated using the H_1 estimator. The coherence function is computed as the normalized squared cross-spectrum, and the results are accepted as valid where the coherence function stays above a value of 0.6 without the interference of the anti-resonance frequencies.

2.2. Numerical Modelling of Tool Holder

The tool holder and tool pair are modelled as a Timoshenko beam to account for the transverse shear and rotary inertia. Introduction of the said parameters is significant for short and thick structures in the mid-to-high range operating frequencies.

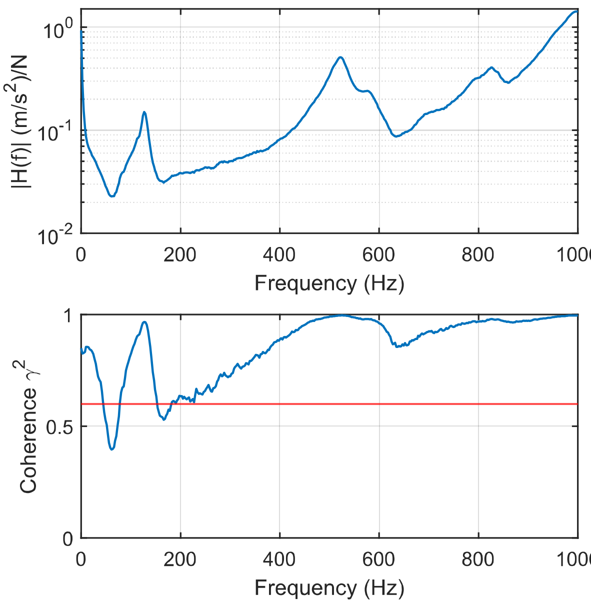

Figure 3. Frequency response function and resulting coherence of the collocated impact hammer experiments of the spindle. First element, $H_{xx}(\omega)$, of the four-by-four FRF matrix given in Equation 1.

Timoshenko beam theory for modelling of tool and tool-holder pairs is particularly beneficial for the purposes of dynamic compliance estimation due to the Euler-Bernoulli formulation's assumption on planer section remaining normal to the perpendicular axis of the tool. Timoshenko beam theory relaxes the said constraint, resulting in estimations that carry higher fidelity [3].

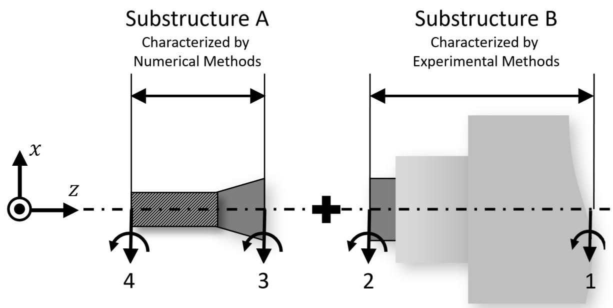

Figure 4. Substructures of the coupling. Substructure (A): the tool and tool holder pair where point 4 lies on the TCP and point 3 lies on the coupling plane. Substructure (B): the spindle of the machinery, where point 2 lies on the coupling plane and point 1 is accepted as grounded with infinite stiffness.

The assembled mass and stiffness matrices are used for the computation of the frequency-domain acceleration and receptances by solving the dynamic equilibrium equation. The driving point translational and rotational sub-matrices are extracted to form *4x4* blocks that are needed for the coupling. The formulation enables compatibility between the sample frequency of the experimental data and the numerical method, resulting in a physically consistent estimation of the coupled TCP.

Figure 4 shows the coupling structure of the tool and tool holder pair and the spindle. Substructure B contains points 1 and 2, where they are defined as the grounding and the coupling plane of the spindle assembly, respectively. $H_{22}(\omega)$ is the FRF of the said coupling plane, only containing translational inputs. Accordingly, four sets of matrices are acquired to formulate Substructure A. The matrices relate to;

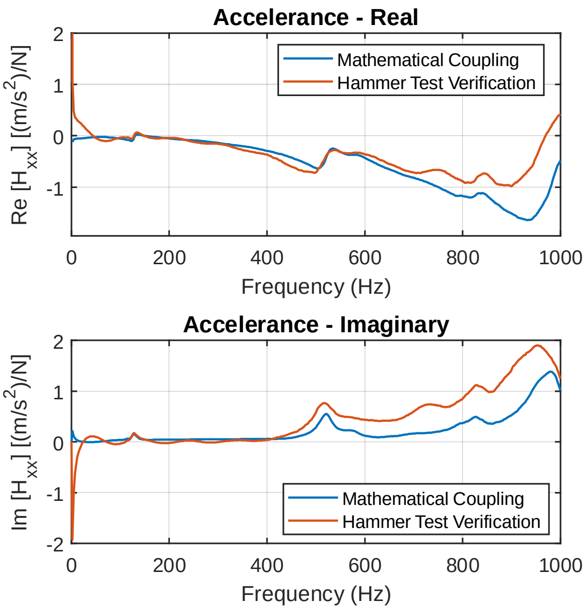

Figure 5. Real and Imaginary parts of the mathematical coupling and the hammer test verification results for the accelerance of the TCP.

(i) Uncoupled TCP FRF, $H_{44}(\omega)$,
(ii) Cross FRF between the TCP and coupling plane, $H_{43}(\omega)$,
(iii) Cross FRF between the coupling plane and TCP, $H_{34}(\omega)$,
(iv) Uncoupled coupling plane FRF, $H_{33}(\omega)$,
where each matrix follows the generalized form that is defined in Eq. 1, formed through translational and rotational degrees of freedom.

2.3. Formulation of the Coupling

The resultant FRFs of the machine tool's spindle and the tool and tool holder pair are used for the mathematical coupling of the physical assembly. The mathematical coupling is estimated as;

$$H^{c}{}_{44} = H_{44} - H_{43}\, H^{-1}{}_{T}\, H_{34}, \quad \text{Eq.(2)}$$

where,

$$H_T = H_{22} + H_{33}. \quad \text{Eq.(3)}$$

The elements H_{44}, H_{43}, H_{34} and H_{33} are the resultant FRFs of the numerical formulation explained in Subsection 2.2, while H_{22} is the FRF of the machine tool assembly explained in Subsection 2.1.

The product of Equation 2, the dynamic behaviour of the TCP where the substuctures are formulated to be mathematically coupled, is denoted as, $H^{c}{}_{44}$.

The element, H_T, computes the compliances of the series spindle assembly and the tool and tool holder coupling planes, and is prone to error for being ill-conditioned near anti-resonances and therefore is inverted with the Moore-Penrose pseudo-inverse.

2.4. Verification of the mathematical coupling

An uncut version of the SK40 ER16 collect chuck holder, arqued in Subsection 2.1, is installed on the DMU 50 machining center. Impact hammer experiments on the uncut tool holder are carried out with the same methodology, and the results are saved. The measured FRFs are subsequently used for the validation of the mathematical coupling approach.

3. Results

This section presents the estimated TCP dynamic response obtained using the Receptance Coupling formulation against the hammer measurements performed on the uncut tool holder. The mathematical coupling is carried out numerically with each corresponding data point frequency pair. Agreement is evaluated in terms of frequency-dependent compliance, providing a quantitative validation.

Figure 5 shows the accelerance comparison between the mathematical coupling and the impact hammer verification test. It is observed that the real part of the accelerance FRF shows good tracking characteristics to the verification tests; however, the Imaginary part shows a divergence. The error is thought to have occurred because of the unmodelled damping and energy dissipation characteristics. Accurate system identification of damping in tool-holder-spindle assemblies remains a challenge due to its high dependence on coupling conditions and preload, and therefore requires experimental frequency response identification rather than numerical modelling [4].

Figure 6 shows the receptance comparison between the mathematical coupling and the impact hammer verification test. It is observed that receptance results show similar characteristics to the accelerance. The peak behaviour near zero frequency is assumed to be an estimation error due to the conversion of accelerance to receptance. This effect is thought to be amplified by the limited low-frequency response prediction characteristics of piezoelectric accelerometers.

Figure 7 compares the magnitude of the tool-tip receptance obtained from the receptance coupling method with the corresponding impact hammer measurements. The coupled model accurately reproduces the compliance level and the dominant characteristics across the investigated frequency range. Above 200 Hz, a good agreement is observed between the measured and estimated response on the basis of amplitude and trend. Discrepancies are still observed at low frequencies, which are thought to be a result of accelerance to receptance conversion and limited near quasi-static capabilities of the sensors, as argued beforehand.

The results are then compared on a quantitative basis. Table 1 depicts the mean absolute error, median absolute error, RMS error, correlation coefficient, and the frequency band of the tool-tip frequency response estimation and the corresponding impact hammer measurements in the 250-1000 Hz frequency range.

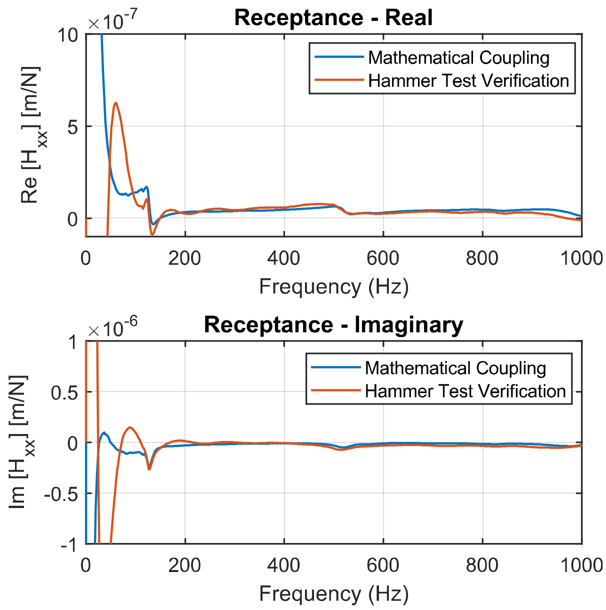

Figure 6. Real and Imaginary parts of the mathematical coupling and the hammer test verification results for the receptance of the TCP.

Within the concerned range, the coupled model shows a mean absolute error of 13.03% while the median error is 8.85%. In addition correlation coefficient of 0.986 depicts a good agreement in the response shape. Therefore, the quantitative results support the agreement observed through the frequency response comparisons and validate the receptance coupling method for the estimation of TCP dynamics.

Table 1 Quantitative error metrics for tool-tip FRF estimation

Metric	Accelerance $[(m/s^2)/N]$
Mean absolute error	13.03%
Median absolute error	8.85%
RMS error	14.2%
Correlation coefficient	0.986
Frequency band	250-1000 Hz

4. Discussion

The results demonstrate receptance coupling approach, combined with the Timeshenko beam representation of the tool and tool holder dynamics, is capable of predicting the tool-tip frequency response with good accuracy compared to impact hammer tests for each unique tool and tool holder assembly and spindle pairs. In particular, the coupled model captures the response magnitude in the frequency range of 250-1000 Hz, which is the most relevant for milling fidelity estimation.

The agreement observed in both real and imaginary components of the FRFs indicates that the proposed formulation not only reproduces the response magnitude but also captures the phase behaviour of the system. The correlation coefficient of 0.968 also confirms this behaviour, despite the presence of unavoidable experimental and numerical uncertainties.

The discrepancies in the imaginary magnitude are thought to be due to the non-ideal modelling of the damping in the tool and tool holder pair. It is shown in the literature that accurate identification of damping ratios in tooling systems is inherently difficult and detrimental to the accuracy of FRF estimation. In particular, it has been found that identified damping ratios for tool holders show significant variance and cannot be reliably predicted using simplified numerical models, and experimental identification instead shows higher accuracy [5].

It should also be noted that the experimental measurements of the spindle assembly to be employed in the coupling method are carried out with the employment of piezoelectric accelerometers. Those show an incapable characteristic of measuring static, near-DC signals, and as a consequence, low-frequency components of the measurement are affected. When converting accelerance to receptance through uncausal methods, these effects are significantly amplified, leading to an apparent divergence and spurious peaks and zeros.

Alternative sensing technologies, such as capacitive displacement probes or laser-Doppler vibrometers, offer improved low-frequency and quasi-static response estimations and can provide a more reliable receptance measurement near zero frequencies. However, many times those instruments may require careful alignment and calibration, which may also prove unsuitable for industrial settings.

Overall, the proposed methodology provides a practical balance between the modelling fidelity and experimental effort.

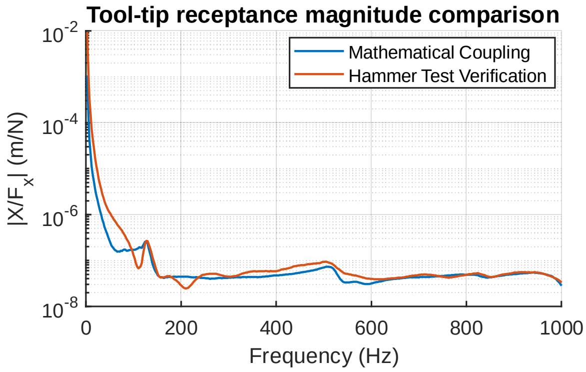

Figure 7. Absolute magnitude of the TCP receptance.

Experimentally measured spindle dynamics, which incorporate three-axis accelerometers to form the rotational responses of the spindle, can be utilized together with Timoshenko beam theory to estimate the coupled response with-in an acceptable margin of error.

5. Conclusion and Future Work

This study presented a receptance coupling framework which incorporates the full DOF of moments and rotations, a *4x4* matrix, of the nonaxial displacement and rotational input and output pairs of the tool and tool holder, with displacement and rotational outputs only, totaling to a *2x4* matrix, of the spindle assembly.

Nevertheless, the results are deemed acceptable for the employment of the methodology and FRF extraction of TCP, and its effect on resultant workpiece geometry.

The future work consists of identifying a common Laplace-domain realization of the coupled dynamics and calculating the full rank spindle FRF matrix through a series of reduced-order modals that preserve the symmetric consistency. The moment inputs are to be obtained from the fitted model by enforcing the generalized modal shapes upon the diagonal elements of the spindle coupling plane.

Another planned development is to incorporate capacitive sensors and Laser-Doppler sensors to measure both the spindle assembly FRF and the tool and tool holder pair FRF, and the extraction of the tool-tool holder pair's damping characteristics. These developments have the potential of further enhancing the predictive capability of the receptance coupling approach for tool-tip dynamics in practical applications.

Overall, the presented framework demonstrates that a reliable tool-tip dynamics estimation can be acquired through reduced representation. The proposed method provides a practical and experimentally feasible mathematical-based modelling framework, while offering a pathway for future studies that require future refinement through model-based methods and more capable sensing techniques.

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