euspen's 26th International Conference & Exhibition, Krakow, PL, June 2026
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# Digital twin of a reference rotary table for traceable angular metrology

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**Abstract**
This work presents a digital twin framework for angular error compensation in a high-precision rotary table. The approach exploits continuous interaction between the physical rotary table and its virtual counterpart, where the physical system provides real-time measurements from a dual encoder, thermal probes, and capacitive sensors representative of the actual operating state. These measurements are fused in a particle-filter-based data assimilation module. The virtual rotary table is continuously updated and incorporates three complementary models: a geometrical error model, a thermal drift model, and sensor error models. By synchronizing the virtual entity with the physical system, the framework estimates the true angular position and delivers a corrected angle in real time. This corrected angle is used for metrological evaluation or fed back into the control loop.

Digital twin, particle filter, rotary table, angular metrology, uncertainty evaluation

## 1. Introduction

The physical system (Figure 1) is the primary angular calibration standard developed at LNE [1], its performance is affected by encoder microinterpolation errors, geometric errors and thermal instabilities, leading to expanded uncertainties of about ±0.01". The proposed digital twin exploits continuous interaction between the physical rotary table and its virtual entity, integrating geometrical and thermal models with multisensor observations from a dual encoder, an autocollimator, capacitive probes, and thermal sensors to represent the actual operating state.

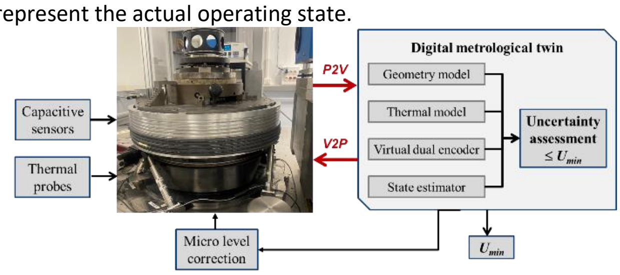


*Figure 1: Reference rotary table digital twin framework*

The framework is structured in three main phases: a virtual experiment that models the measurement process, an uncertainty assessment, and a control phase for real-time correction using a 6-DoF hexapod.

## 2. Geometrical error model

### *2.1. Projection process of SDT (Small Displacement Torsor)*

For each of the eight operational capacitive sensors (Figure 2), the sensor position and measuring direction are defined within the configuration matrix $\mathrm{H}$. The measured displacement $d_i$ for the $i$-th sensor is modelled as:

$$d_i = \mathrm{h}_i \cdot \mathrm{u} \quad (1)$$

where $\mathrm{h}_i$ denotes the $i$-th row of the configuration matrix $\mathrm{H} \in \mathbb{R}^{8\times 6}$, and $\mathrm{u} = [T_x, T_y, T_z, R_x, R_y, R_z]^T$ represents the parasitic translations and rotations to be estimated at the measured position angle $\theta$.

Extending Equation (1) to all eight sensors yields the overdetermined linear system:

$$\mathrm{d} = \mathrm{Hu} \quad (2)$$

with solution $\hat{\mathrm{u}} = (\mathrm{H}^T\mathrm{H})^{-1}\mathrm{H}^T\mathrm{d}$.

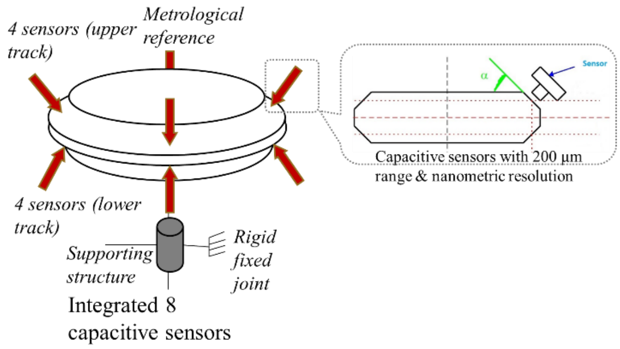


*Figure 2: Capacitive sensing principle of the metrological reference*

### *2.2. Geometrical error effect on angle measurement*

The geometric error model is expressed as:

$$e_g(\theta) = \sum_{k=1}^{N} [a_k \cos(k\theta) + b_k \sin(k\theta)] \quad (3)$$

This model characterises the angular signature of geometric errors as a function of the nominal angular position $\theta$. It decomposes the error into harmonic contributions primarily associated with radial runout, tilt, and axial motion of the rotary table [2].

### 3. Thermal drift model

the thermally induced angular error is expressed as a function of temperature proxies and sensitivity coefficients [3].

$e_{\text{th}} = k_0 + k_U \cdot U + k_{G1} \cdot G_1 + k_{G2} \cdot G_2$ (4)

where:

$k_0$ = constant angular offset (µrad)

$U = T - T_{\text{ref}}$ = uniform thermal expansion proxy (difference from reference temperature, °C)

$G_1 = \frac{dT}{dt}$ = rate of temperature change (°C/s ) captures transient gradient effects

$G_2 = H(T)$ = thermal imbalance function

$k_U, k_{G1}, k_{G2}$ = identified sensitivities (µrad/°C, µrad/(°C/s), µrad/°C respectively)

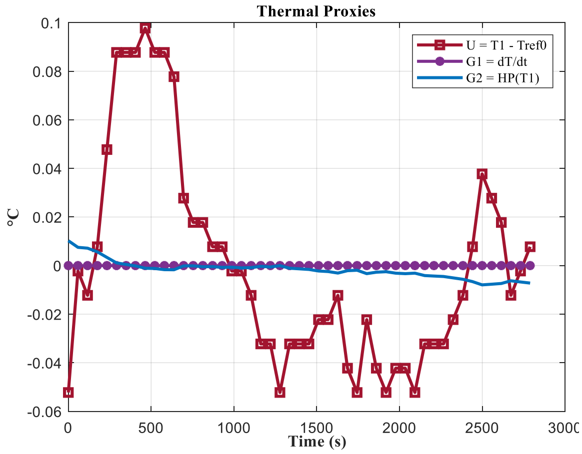


*Figure 3: Thermal proxies*

Figure 3 illustrates the thermal proxies, showing quasi-static behavior with limited dynamic variations.

### 4. Virtual dual encoder

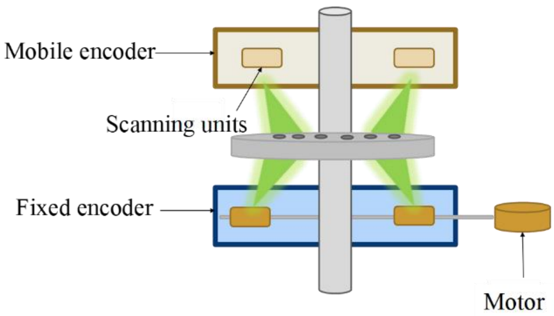


*Figure 4: Dual encoder*

The virtual dual encoder (Figure 4) reproduces the measurement sequence of the reference rotary table [1], including continuous shaft rotation, independent encoder readings, error superposition, and differential signal integration over one revolution.

Each encoder output includes:

- Graduation errors: $e_g(\theta) = A\sin(2\pi f\theta + \phi)$ (where A is the amplitude, f the frequency, and ϕ the phase shift)
- Eccentricity: first-harmonic $e_e \approx e/r$ (for small eccentricity $e$, radius $r$).
- Interpolation: high-frequency

The Fixed encoder measured angle is $\theta_f$ the mobile one is $\theta_m$ (offset $\alpha_0$). Differential signal: $\delta\theta = \theta_m - \theta_f = \alpha_0 + e_d$, where $e_d = e_m - e_f$ is periodic.

➔ $\hat{\alpha}_0 = \frac{1}{2\pi}\int_0^{2\pi} \delta\,\theta(\phi)d\phi$.

Table 1 below shows the results of the virtual dual encoder.

**Table 1** Virtual dual encoder simulation results

| Nominal angle (°) | Virtual measured angle (°) |
|---|---|
| 0 | 0.0098 |
| 15 | 14.9973 |
| 30 | 29.9888 |
| 45 | 44.9302 |

### 5. Results and discussion

A simulation and an experiment were conducted on a 24-face polygon (Figure 5). Table 2 compares estimated and measured angles.

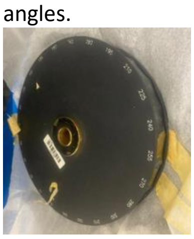

*Figure 5: 24 faces polygon*

The particle filter operates on a state vector that combines the true angle with key model parameters [4]. At discrete time step $k$, this is defined as:

$$x_k = [\theta_k, u_k^{\top}, \beta^{\top}, p^{\top}]^{\top}$$

where $\theta_k$ is the rotary table angle, $u_k$ represents the small-displacement torsor (SDT), β contains the thermal sensitivities of the proxy-based drift model, and p groups the dual encoder parameters.

**Table 2** Particle filter estimation of the corrected angle

| Estimated angle (°) | Measured angle (°) | Filter error(″) |
|---|---|---|
| 359.998831576929 | 359.999700000000 | 3.126 |
| 345.000800000000 | 344.998589418415 | 7.958 |
| 330.000100000000 | 329.998687987665 | 5.083 |
| 315.000100000000 | 314.998755776230 | 4.839 |

### 6. Conclusions and future work

This paper presents a digital twin framework for real-time angular error compensation of a reference rotary table. By combining geometrical, thermal, and sensor error models with particle-filter-based data fusion, the proposed approach enables improved estimation of the true angular position.

Preliminary results demonstrate the relevance of the framework for reducing angular measurement errors. However, remaining discrepancies highlight the influence of simplified thermal modeling and unmodeled dynamic effects.

Future work will focus on enhancing the thermal model using physics-based approaches. These developments aim to improve uncertainty evaluation and ensure traceability in high-precision angular metrology.

### References

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