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Assessment of Geometric Errors in Coordinate Measuring Machines with Ring Gages

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Abstract

Coordinate Measuring Machines (CMMs) are affected to 21 geometric errors, including linear, angular, and squareness deviations, which may significantly affect measurement accuracy. To ensure robust metrological reliability, it is essential to perform interim checks capable of monitoring the machine's stability between calibrations. A wide range of methods and instruments is available to assess CMM performance. Some of these are highly sophisticated, enabling the simultaneous identification of multiple geometric errors with high precision; others are simpler and lower cost, but typically limited to detecting only one or a few error types. This paper presents and discusses an experimental approach employing ring gages as a practical means of evaluating linear positioning errors (XPX, YPY, ZPZ) and squareness errors (XWY, XWZ, YWZ). Ring gages are low-cost precision artefacts widely available in industrial and research metrology laboratories. Their ease of handling, high dimensional stability, and broad availability make them an attractive alternative for implementing efficient, low-cost interim checks of their measuring equipment. The proposed methodology and its mathematical principles are presented, along with the results from experiments compared with more sophisticated and commercially available testing techniques, highlighting its strengths and limitations.

Keywords: Coordinate Measuring Machines; Dimensional Metrology; Calibration; Interim checks.

1. Assessment of Geometric Errors in CMMs

Coordinate Measuring Machines (CMMs) are among the most important measurement systems used in dimensional control of products and processes in industry, and the accuracy of their results is essential to achieve high standards of quality and productivity.

Measurement accuracy depends on several factors, including the geometric conformity of the CMM. Three-axis Cartesian CMMs exhibit 21 geometric errors arising from imperfections in their mechanical structure and measurement scales [1]. Each Cartesian axis exhibits six geometric errors, and an additional three squareness errors exist between the axes. Figure 1 illustrates these errors.

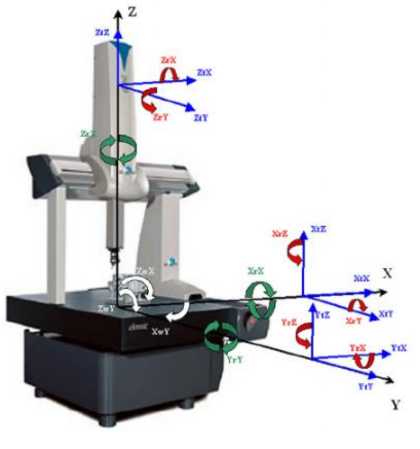


Figure 1. Geometric errors of a CMM [1]

According to ISO 230, the 21 geometric error components of a three-axis machine are designated as follows:

- 3 linear positioning errors: XPX, YPY, ZPZ
- 6 straightness errors: XTY, XTZ, YTX, YTZ, ZTX, ZTY
- 9 angular errors: XRX, XRY, XRZ, YRX, YRY, YRZ, ZRX, ZRY, ZRZ
- 3 squareness errors: XWY, XWZ, YWZ

These geometric errors, acting in combination, introduce deviations when measuring different features and tolerances on workpieces. Some errors mainly affect size measurements, while others have a stronger influence on location, form, and orientation characteristics.

To ensure adequate metrological accuracy, CMMs undergo periodic calibration and adjustment, typically at one-year intervals. In addition, periodic tests carried out at shorter intervals are essential to verify that the machine remains within acceptable error limits.

Although calibration and adjustment are necessary, periodic testing is indispensable to assess machine accuracy between calibrations. Such tests can identify potential problems in the CMM and provide valuable information to optimize calibration and adjustment intervals.

Several test systems and methods are available for evaluating CMMs, ranging from simple dimensional artefacts to sophisticated systems such as laser interferometers. While all are useful, some methods are limited to assessing only a subset of geometric errors, whereas others are capable of separately measuring all 21 parametric errors.

2. Methodology of CMM evaluation with ring gages

Different testing methods exhibit varying capabilities for measuring the 21 geometric errors. Some methods, such as ball plates [2] and laser interferometers [3], are capable of separately quantifying all parametric error components. Other methods focus on the most significant errors for CMM performance, namely linear positioning errors and squareness errors between the Cartesian axes. Ring gage testing falls into this latter category.

The proposed methodology using a ring gage considers a circle with a calibrated diameter and roundness error small compared to the measurement accuracy of the CMM. The circle is automatically probed at eight points equally spaced by 45°, and four diametral distances between opposite points are calculated, designated as D1, D2, D3, and D4, in mm.

Figure 2 illustrates this concept in the XY plane of the CMM; the same approach applies to the XZ and YZ planes.

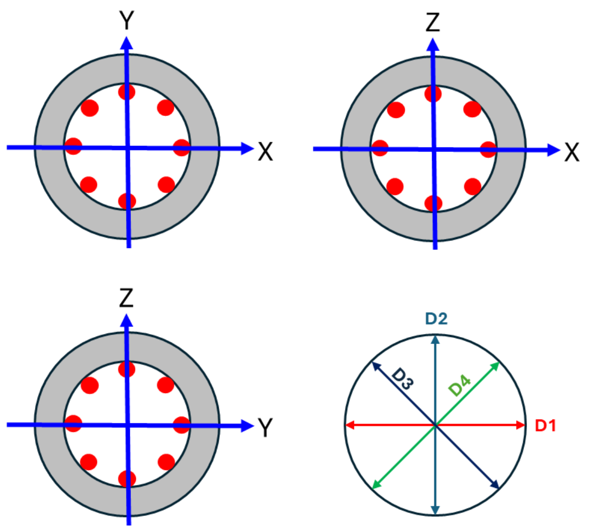


Figure 2. Probing strategy and distances extracted

In practice, any measurement result obtained with a CMM is influenced by the combined effect of the 21 geometric error components of the machine. However, for the purpose of estimating specific parametric errors using a simple artefact-based test, a simplified model is adopted.

The equations presented in this work assume that the measured distances are predominantly influenced by the geometric error under investigation (linear positioning or squareness), while the remaining error components are considered to have a secondary influence within the test configuration. This simplification is commonly adopted in practical interim tests, where the objective is not the complete identification of all parametric errors but rather the detection of variations in the dominant error components affecting the measurement result.

Under this assumption, the distances obtained from the probing strategy allow the estimation of the following geometric errors:

- **3 linear positioning errors:** XPX, YPY, ZPZ
- **3 squareness errors:** XWY, XWZ, YWZ

The linear positioning errors (scale errors, in µm/m) are determined by comparing the distances along the Cartesian axes (DX and DY) with the calibrated diameter of the ring gage, as shown in Equations (1) and (2) for the XY plane:

$$XPX = (D1 \,/\, \text{Calibrated Diameter} - 1) \times 1000 \quad (1)$$

$$YPY = (D2 \,/\, \text{Calibrated Diameter} - 1) \times 1000 \quad (2)$$

The squareness error, expressed in µm/m, is determined from the difference between the two diagonal distances D3 and D4, according to Equation (3):

$$XWY = (2.10^6 \times |D3 - D4|) \,/\, (D3 + D4) \quad (3)$$

For testing in the XZ plane, the errors are calculated as:

$$ZPZ = (D2 \,/\, \text{Calibrated Diameter} - 1) \times 1000 \quad (4)$$

$$XWZ = (2.10^6 * |D3 - D4|) \,/\, (D3 + D4) \quad (5)$$

In the YZ plane, the squareness error, in µm/m, is given by:

$$YWZ = (2.10^6 \times |D3 - D4|) \,/\, (D3 + D4) \quad (6)$$

As an accuracy criterion for practical implementation, the uncertainty of the calibrated diameter and the roundness error of the ring gage must be less than 20% of the CMM measurement uncertainty for size measurements. This condition is necessary to ensure the validity of the experimental results.

3. Experimental evaluation of a CMM using a ring gage

The tests were carried out on a CNC CMM with a maximum permissible error MPE = (2 + L / 500) µm, in accordance with ISO 10360-2 [4]. A 200 mm diameter ring gage was used, as this was the largest available artefact in the metrology laboratory at the time of testing.

The roundness of the ring gage was measured to verify that the form error was compatible with the accuracy requirements of the CMM. Measurements were performed using a Mitutoyo RA1600 form measuring machine, resulting in a roundness deviation of 0,27 ± 0,05 µm (Figure 3), after applying a 50 UPR low-pass filter and calculating the reference circle using the minimum zone (Chebyshev) method, in accordance with ISO 12181 [5].

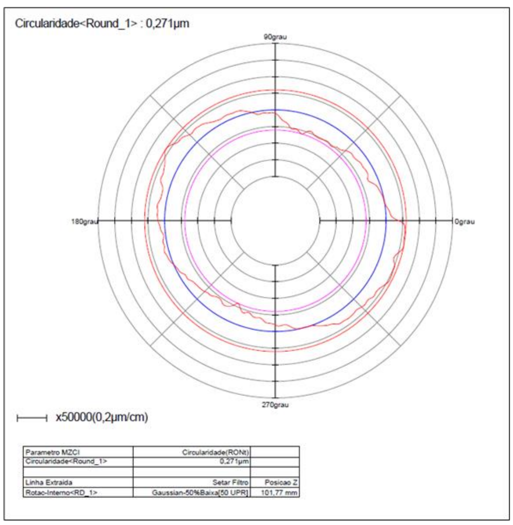


Figure 3. Ring gage roundness error

The diameter of the ring gage was calibrated using the comparison method with thermal expansion correction. The calibrated diameter obtained under these conditions was 199,997 ± 0,001 mm.

After calibration in terms of size and form, the ring gage was used to evaluate the geometric errors of the CMM. The ring gage was positioned in each Cartesian plane, aligned as closely as possible with the machine coordinate system. A mathematical leveling and centering procedure was then performed, using the top plane of the ring for leveling and the fitted circle for coordinate system localization. Both plane and circle were fitted using the least-squares method. Figure 4 illustrates the alignment and measurement procedure in the three Cartesian planes.

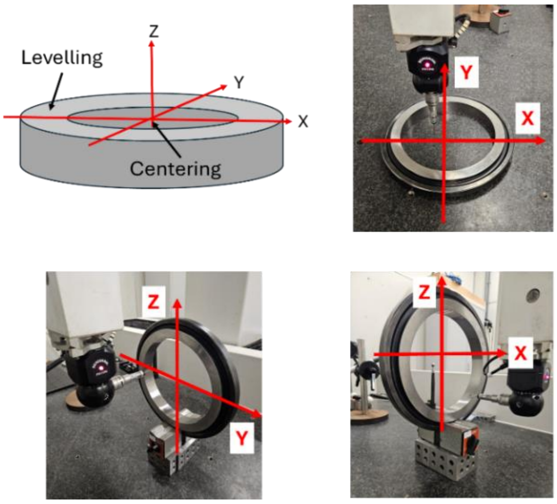
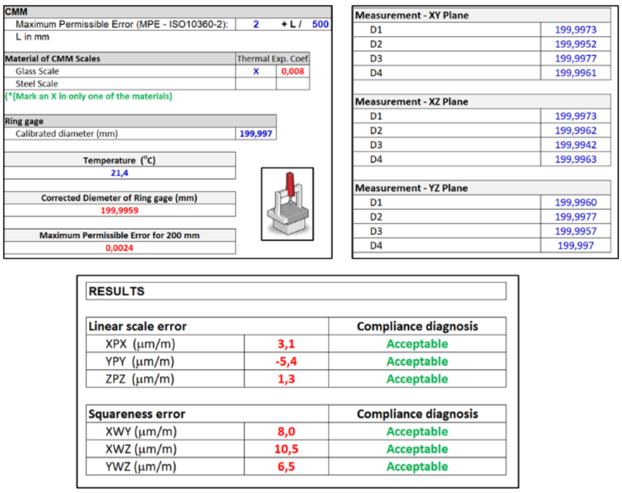


Figure 4. Alignment and measurement procedure

Eight probing points were measured as described in Section 2, and the data were processed using a dedicated spreadsheet developed to estimate linear positioning and squareness errors (Figure 5).



CMM		
Maximum Permissible Error (MPE - ISO10360-2):	2	+ L / 500
L in mm		

Material of CMM Scales	Thermal Exp. Coef.	
Glass Scale	X	0,008
Steel Scale		
(*(Mark an X in only one of the materials)		

Ring gage	
Calibrated diameter (mm)	199,997

Temperature (°C)
21,4

Corrected Diemeter of Ring gage (mm)
199,9959

Maximum Permissible Error for 200 mm
0,0024

Measurement - XY Plane	
D1	199,9973
D2	199,9952
D3	199,9977
D4	199,9961

Measurement - XZ Plane	
D1	199,9973
D2	199,9962
D3	199,9942
D4	199,9963

Measurement - YZ Plane	
D1	199,9960
D2	199,9977
D3	199,9957
D4	199,997

RESULTS

Linear scale error		Compliance diagnosis
XPX (μm/m)	3,1	Acceptable
YPY (μm/m)	-5,4	Acceptable
ZPZ (μm/m)	1,3	Acceptable

Squareness error		Compliance diagnosis
XWY (μm/m)	8,0	Acceptable
XWZ (μm/m)	10,5	Acceptable
YWZ (μm/m)	6,5	Acceptable

Figure 5. Test management spreadsheet and error calculation

In addition to calculating geometric errors, the spreadsheet evaluates whether these errors are significant based on the CMM accuracy specification, using the MPE equation for the ring gage size (approximately 200 mm):

$$MPE = \pm (2{,}0 + L/500)\ [mm] \qquad (12)$$

$$MPE = \pm (2{,}0 + 200 / 500) = \pm 0{,}0024\ mm$$

Thus, for a linear positioning error to be acceptable, it must be below the MPE value of 2,4 μm for the ring gage size. For squareness errors, the difference between the diagonal measurements must also be less than 2,4 μm.

Five repetitions were performed in each Cartesian plane, producing the average results shown in Table 1. The measurement repeatability observed in the tests was better than 1,2 μm/m.

Table 1 – Geometric errors measured with the ring gage

Geometric Error	Result (μm/m)
XPX	3,1
YPY	-5,4
ZPZ	1,3
XWY	8,0
XWZ	10,5
YWZ	6,5

After the tests performed with the ring gage, the same CMM was evaluated using two different methods. One of them was the orbital testing system Machine Checking Gage (MCG) from Renishaw (Figure 6), which is widely used in industry, with the objective of comparing the errors obtained by different methods and validating the previous results.

The MCG consists of a fixed-length carbon fiber bar, supported by a spherical joint at one end and by the CMM probe at the other, allowing the machine to perform orbital movements in different spatial orientations. Using the MCG, it is possible to identify scale errors, volumetric errors, and squareness deviations between the machine axes.

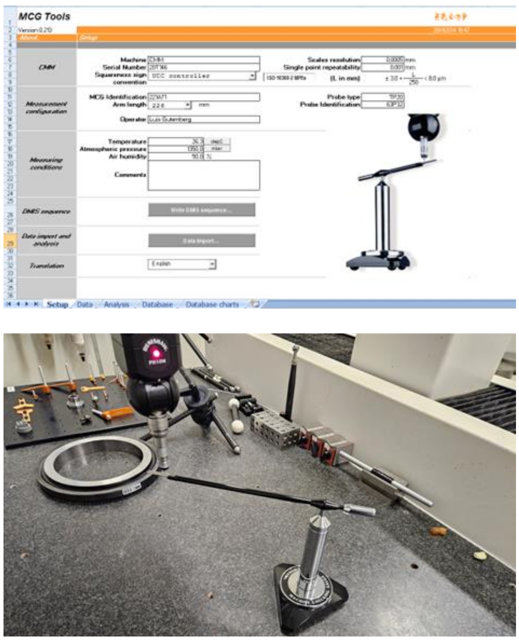


Figure 6. CMM tested with machine checking gage

In addition to the MCG, the CMM was evaluated using a dimensional artefact in the form of a tetrahedron (Figure 7). The system, developed by the company Trapet Precision Engineering, consists of a carbon fiber tetrahedron with four spheres at its vertices, with the distances between the vertices being calibrated.

The CMM under test measures the distances between each pair of spheres in six different directions, compares these measurements with the calibrated values, and, based on the resulting differences, calculates scale errors along the three Cartesian axes as well as squareness errors between the axes.

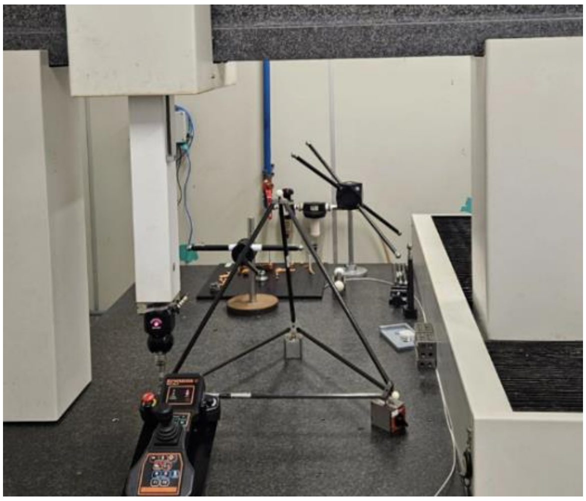


Figure 7. CMM tested using tetrahedron

The errors provided by both the MCG and the tetrahedron systems are the linear positioning errors (scale errors XPX, YPY, and ZPZ) and the squareness errors between the Cartesian axes (XWY, XWZ, and YWZ). Table 2 presents the results obtained using these systems. The repeatability observed in the tests performed with the MCG system was better than 3,0 µm/m, while the tests performed with the tetrahedron artefact showed a repeatability better than 1,8 µm/m.

Table 2 – Errors measured with the MCG System

Geometric Error	Result (µm/m)	
	MCG	Tetrahedron
XPX	6,0	4,7
YPY	-5,3	-5,9
ZPZ	-1,2	2,1
XWY	6,9	5,3
XWZ	11,3	8,7
YWZ	4,8	5,7

Based on these values, a comparison between the methods was established, as shown in Figure 8. Although the procedures are different and involve different measurement volumes, the results exhibited similar values, with differences compatible with the uncertainty of the CMM, indicating that the ring gage testing method proved capable of revealing the geometric condition of the measuring machine. Maximum deviations between methods were below 3 µm/m.

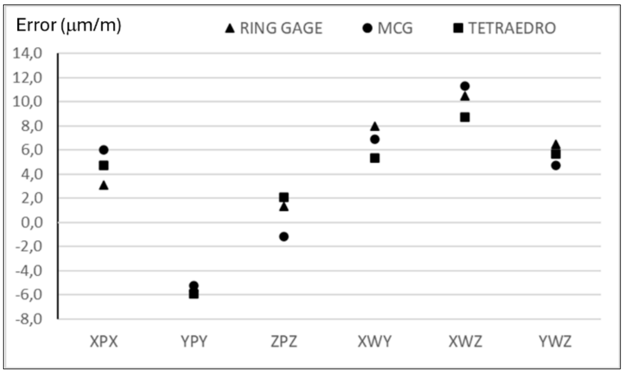


Figure 8. Comparison of results between the evaluated systems

4. Conclusions

This study evaluated the use of a ring gage to assess linear scale errors and squareness errors in a Coordinate Measuring Machine, with the primary objective of investigating the potential of a practical and low-cost method for periodic CMM testing.

The results indicate that the method is promising when compared with well-established industrial testing techniques. The values obtained were similar, even for a CMM in good metrological condition. For machines with larger errors, an even better agreement between methods is expected.

The test proved to be practical and fast, although an important limitation is the size of the ring gage relative to the measurement volume of the CMM. In the present study, a 200 mm ring gage was used, but a diameter of at least 300 mm would be recommended.

The method provides an approximate estimation of dominant geometric errors, but its main purpose is the detection of variations in machine behaviour during periodic stability checks. For this purpose, the method demonstrated a good capability to reveal errors in the measuring machine, with a very short testing time, using a dimensional artefact that is widely available in industry.

Future work will focus on evaluating the reliability of angular error assessment using the ring gage method, based on a procedure similar to that described in this work, as well as on a detailed estimation of the measurement uncertainty associated with the test results.

In this study, the test was performed near the centre of the CMM measuring volume. Evaluating the method at different locations within the measuring volume is also a relevant topic for future work.

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