euspen's 25th International Conference & Exhibition, Zaragoza, ES, June 2025
www.euspen.eu

Towards enhanced quality control in additive manufacturing: a comparative study of spline reconstructions and discrete interpolation for surface texture assessment

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Abstract

Dealing with freeform complex surfaces in order to characterize their surface texture poses significant challenges in manufacturing, especially for quality control of complex parts from additive manufacturing. Traditional texture assessment methods often lack the precision needed to capture the details of these complex geometries, underlining the demand for advanced techniques that offer a more thorough representation of surface features.

A specialized approach for extracting surface texture parameters for freeform surfaces common in additive manufacturing is proposed in this work. Tailored for mechanical engineering applications, this methodology broadens traditional surface characterization by addressing the distinctive features of complex profiles. Through parametric modeling, B-spline and GB-spline reconstructions, and discrete smooth interpolation. Testing on unstructured datasets with comparative evaluations with Montains software confirms its robustness, accuracy, and reliability.

Surface texture, manufacturing, surface reconstruction, complex surfaces

1. Introduction

In recent years, advancements in manufacturing technologies have facilitated the design and production of complex and freeform surfaces with complex geometries. Despite these advancements, working with such geometries through additive manufacturing (AM) technologies poses significant challenges, particularly in the characterization of these complex surfaces. The irregular and complex nature of these surfaces, coupled with the lack of standardized of some approaches, makes it difficult to define and measure surface texture parameters. Furthermore, traditional texture analysis techniques, developed primarily for standard surfaces, often fall short when applied to these advanced geometries.

Numerous studies have been conducted on the analysis of surface texture for canonical surfaces, such as planar, cylindrical, and spherical surfaces [1,2,3,4]. However, relatively few works have focused on the case of complex surfaces, particularly in [5,6,7], where the authors extended the traditional definitions of AST parameters represented in the ISO 25178-2 [8] to accommodate parametric surfaces representing manufactured objects. In these works, the authors employed various surface reconstruction techniques, notably B-spline reconstructions and triangular meshes for a limited set of AST parameters. This area of research can be further expanded by investigating new reconstruction methods and incorporating a broader set of AST parameters.

This paper focuses on the study of surface texture for complex geometries. The study utilizes three approaches for surface reconstruction and characterization: B-spline surfaces, GB-spline surfaces, and the Discrete Smooth Interpolation (DSI) method, which, to the best of our knowledge, is being applied for the first time to address surface texture challenges in complex surfaces in comparison which the literature. Section 2 introduces the definition of the AST parameters of interest, grounded in a parametric surface representation. Section 3 describes the three reconstruction techniques utilized, and Section 4 presents experimental results based on surface measurements obtained from xCT using the different reconstruction methods presented in section 3 against Mountains software.

2. Selected AST parameters definition

assuming that a manufactured object can be described with a parametric surface $\sum \subset \mathbb{R}^3$ as:

$$r(u,v) = \begin{cases} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{cases} \qquad (1)$$

And it can be decomposed into to parts

$$r(u,v) = \boldsymbol{r}_{form}(u,v) + \boldsymbol{r}_{res}(u,v) \qquad (2)$$

$\sum_{form}$: $\boldsymbol{r}_{form}(u,v)$ represents the form surface.
$\sum_{res}$: $\boldsymbol{r}_{res}(u,v)$ represents the residual surface.
According to the definition of the parameters in the ISO 25178-2 [8], a generalisation of some surface texture parameters is given in [5,6,7] as:
The arithmetic mean of the absolute value of the height (Sa) and the root mean square height (Sq) are given as:

$$Sa = \frac{1}{|A|} \iint_{\Sigma_{form}} |r_{res}(u,v)| d\sigma_{form} \qquad (3)$$

$$Sq = \sqrt{\frac{1}{|A|} \iint_{\Sigma_{form}} |r_{res}(u,v)|^2 d\sigma_{form}} \qquad (4)$$

The skewness (Ssk) and the kurtosis (Sku) are given as:

$$Ssk = \frac{1}{|A|Sq^3} \iint_{\Sigma_{form}} |r_{res}(u,v)|^3 d\sigma_{form} \quad (5)$$

$$Sku = \frac{1}{|A|Sq^4} \iint_{\Sigma_{form}} |r_{res}(u,v)|^4 d\sigma_{form} \quad (6)$$

And

$$Rz = \max_{u,v} r_{res}(u,v) - \min_{u,v} r_{res}(u,v) \quad (7)$$

Where

$$d\sigma_{form} = \left\| \boldsymbol{r}_{form,u}(u,v) \times \boldsymbol{r}_{form,v}(u,v) \right\| du dv \quad (8)$$

And $|A|$ is the area of the form profile given by:

$$|A| = \iint_{\Sigma_{form}} 1 \, d\sigma_{form} \quad (9)$$

3. Surface reconstruction methods

In order to fit a data point $(P_i)_{i=0}^{N}$ to a parametric function it has to be parameterized, which means describing it with a function $r = r(u,v)$, where u, v are abstract parameters. In this paper three surface reconstruction methods are used.

3.1. B-spline reconstruction

The equation of a B-spline surface of degree $k_1 \times k_2$ as defined in [9] is given as follows:

$$S(u,v) = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} N_{i,k_1}(u) N_{j,k_2}(v) Q_{ij} \quad (10)$$

Where $(Q_{ij})_{i=0,j=0}^{n_1,n_2}$ are control points and $N_{i,k_1}(u), N_{j,k_2}(v)$ are the basis functions in the u-direction and the v-direction, the B-spline basis functions are defined as follows:

$$N_{i,0}(t) = \begin{cases} 1; & if\ t_i \le t < t_{i+1} \\ 0; & else \end{cases} \quad (11)$$

And for $j > 0$:

$$N_{i,j}(t) = \frac{t - t_i}{t_{i+j} - t_i} N_{i,j-1}(t) + \frac{t_{i+j+1} - t}{t_{i+j+1} - t_{i+1}} N_{i+1,j-1}(t) \quad (12)$$

Where the values t_i are taken from two sequences called the knot vectors: the knot vector in the u-direction

$$T_1 = (t_0, t_1, \dots, t_{m_1})$$

the knot vector in the v-direction

$$T_2 = (t_0, t_1, \dots, t_{m_2})$$

An iterative least square method is used to fit a B-spline surface to the data.

3.2. GB-spline reconstruction

The GB-spline surface is a generalization of the B-spline surface. The equation of a GB-spline surface of degree $k_1 \times k_2$ as defined in [10] is given as follows:

$$S(u,v) = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} G_{i,k_1}(u) G_{j,k_2}(v) Q_{ij} \quad (13)$$

Where $(Q_{ij})_{i=0,j=0}^{n_1,n_2}$ are control points and $G_{i,k_1}(u), G_{j,k_2}(v)$ are the GB-spline basis functions in the u-direction and the v-direction, the basis functions are defined as follows:

$$G_{i,0}(t) = \begin{cases} 1; & if\ t_i \le t < t_{i+1} \\ 0; & else \end{cases} \quad (14)$$

And for $j > 0$:

$$G_{i,j}(t) = \delta_i^{j-1}(t) G_{i,j-1}(t) + \left(1 - \delta_{i+1}^{j-1}(t)\right) G_{i+1,j-1}(t) \quad (15)$$

With:

$$\delta_i^{\alpha}(t) = \begin{cases} \delta\left(\frac{t - t_i}{t_{i+\alpha+1} - t_i}\right); & if\ \ t_i \le t < t_{i+\alpha+1} \\ 0; & else \end{cases} \quad (16)$$

With $\delta: [0,1] \to [0,1]$ is an increasing function, smooth and satisfies $\delta(0) = 0$ and $\delta(1) = 0$, δ is called a core function. And the values t_i are taken from two sequences called the knot vectors: the knot vector in the u-direction

$$T_1 = (t_0, t_1, \dots, t_{m_1})$$

the knot vector in the v-direction

$$T_2 = (t_0, t_1, \dots, t_{m_2})$$

The core function

$$\delta(t) = 0.5(2t^3 - 3t^2) + 1.5t$$

is used in this work due to its robustness showed in [11].

An iterative least square method is used to fit a GB-spline surface to the data.

3.3. Discrete Smooth Interpolation (DSI) reconstruction

Discrete smooth interpolation (DSI) as introduced by Jean-Laurent Mallet [12,13] is a method used to interpolate scattered data points while ensuring a smooth transition between them. The technique fit the data using a triangular mesh obtained by minimizing a functional $\mathcal{R}^*(g)$. This minimization problem is solved in [14] using the following iterative method:
Let (ξ_0) be the initial approximated mesh.
Define g as (ξ_0).
Let I denote the set of nodes x such that at least one data point is projected onto a triangle containing x as one of its vertices. The nodes of the mesh $g(x)$ are updated when $x \in I$ as follows:

While{more iterations are needed}
for all ($v \in I$)
  for $s \in A$

$$\gamma^s(v) = \sum_{c \in \mathcal{C}} \omega_c \left(A_c^s(v)\right)^2$$

$$\Gamma^s(v) = \sum_{c \in \mathcal{C}} \omega_c A_c^s(v) \left(\sum_{\alpha \in G - \{v\}} A_c^s(\alpha).\, g^s(\alpha) - d_c + \sum_{\varepsilon \neq s} \sum_{\alpha \in G} A_c^{\varepsilon}(\alpha).\, g^{\varepsilon}(\alpha) \right)$$

$$h^s(v) = \sum_{k \in N(v)} \mu(k) (\rho^v(k))^2$$

$$H^s(v) = \sum_{k \in N(v)} \left\{ \mu(k) \rho^v(k) \sum_{\beta \in N(k) - \{v\}} \rho^{\beta}(k) g(\beta) \right\}$$

$$g^s(v) = -\frac{H^s(v) + \phi \Gamma^s(v)}{h^s(v) + \phi \gamma^s(v)}$$

  end
end
end

where $A = \{x, y, z\}$, and $\mu(k), \rho^{\beta}(k), \phi$ are some weights. $N(v)$ is the neighbourhood of the node v. $A_c^s(v)$ is a constraint formed by tow normal and positional constraints of the node v. And d_c are constants related to each constraint. In this work In this work the weights ϕ, $\mu(a)$ are choosed to be 1 for all nodes, and the weights $\{\rho^i(a)\}$ are set according to the following
harmonic weighting scheme:

$$\rho^i(a) = \begin{cases} -|N(a) - \{a\}|, & i = a \\ 1, & i \in N(a) - \{a\} \end{cases} \quad (17)$$

4. Experimental results

The proposed method was evaluated using two examples of non-structered datasets consisting of points extracted from complex objects manufactured through additive manufacturing techniques. These data points were acquired using an xCT system, the first data set contains 996 point and the second data set contains 1455 point. The AST parameters were computed using the different proposed reconstruction methods, as well as with Mountains software. A comparison of the results obtained from the proposed methods against Mountains software is provided. For the three methods, the integrals were approximated using discrete Riemann sums to calculate the AST parameters.

Figure1 presents the first measured surface and the reconstructed surface for both the B-spline and DSI reconstruction methods.

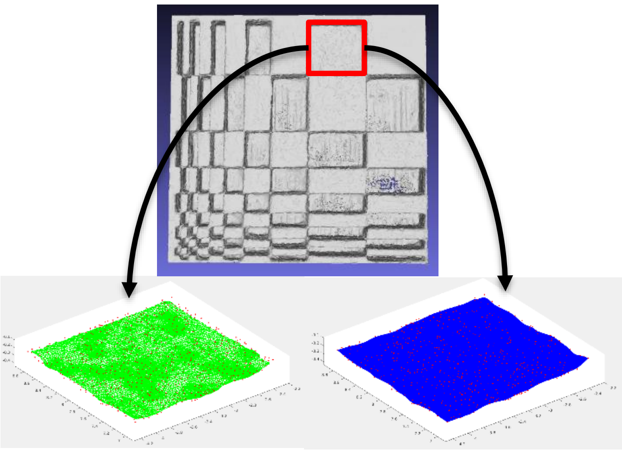

Figure 1: An extracted region from an additive manufactured object with its DSI surface (green) and B-spline surface (blue) reconstructions.

Table1 summarizes the different AST parameters values using the presented methods computed for the first dataset and Table2 summarizes the absolute errors between each method and Mountains software results.

	$Sa(\mu m)$	$Sq(\mu m)$	Ssk	Sku	$Sz(\mu m)$
B-spline Reconstruction	8	10.5	0.36	4.92	98.7
GB-spline Reconstruction	8.1	10.6	0.39	4.97	101.3
DSI Reconstruction	10.9	13.3	− 0.11	4.09	104.1
Mountains software	8.32	11.21	0.72	5.56	101.5

Table 1: AST parameters and different errors

	$Sa(\mu m)$	$Sq(\mu m)$	Ssk	Sku	$Sz(\mu m)$
B-spline Reconstruction	0.32	0.71	0.36	0.64	2.8
GB-spline Reconstruction	0.22	0.61	0.33	0.59	0.2
DSI Reconstruction	2.58	2.09	0.83	1.47	2.6

Table 2: Absolute errors between the three methods results and Mountains software results

Figure2 presents the second measured surface and the reconstructed surface for both the B-spline and DSI reconstruction methods.

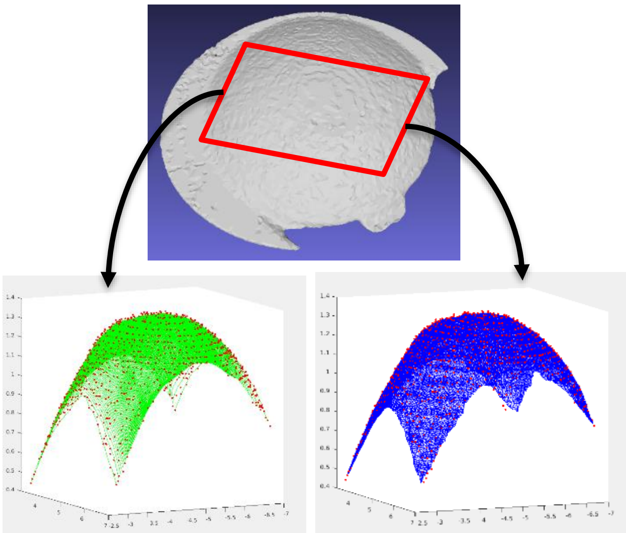

Figure 2: An extracted region from an additive manufactured object with its DSI surface (green) and B-spline surface (blue) reconstructions.

Table3 summarizes the different AST parameters values using the presented methods computed for the second dataset and Table4 summarizes the absolute errors between each method and Mountains software results.

	$Sa(\mu m)$	$Sq(\mu m)$	Ssk	Sku	$Sz(\mu m)$
B-spline Reconstruction	12.13	15.69	− 0.55	7.04	111.2
GB-spline Reconstruction	12.5	16.09	− 0.76	6.31	108.2
DSI Reconstruction	12.33	14.4	− 0.09	3.51	119.6
Mountains software	13.07	16.46	− 0.22	5.76	101.5

Table 3: AST parameters and different errors

	$Sa(\mu m)$	$Sq(\mu m)$	Ssk	Sku	$Sz(\mu m)$
B-spline Reconstruction	0.94	0.77	0.33	1.28	9.7
GB-spline Reconstruction	0.57	0.37	0.54	0.55	6.7
DSI Reconstruction	0.74	2.06	0.13	2.25	18.1

Table 4: Absolute errors between the three methods results and Mountains software results

The experimental results showed that the errors between each method and Mountains software are negligible, specialy for for B-spline and GB-spline surfaces. DSI reconstruction showed larger errors.

Table 5 provide some remarks and deductions on the the different methods based on the obtained results in Tables 1,2,3,4.

Method	**Method accuracy**	**General Remarks**
B-spline Reconstruction	Moderate errors, fairly close to Mountains software	Moderately accurate, for surfaces exhibiting slight roughness and discernible variations.
GB-spline Reconstruction	Smallest absolute errors, most accurate	Most accurate, with surfaces closely matching Mountains software
DSI Reconstruction	Larger errors, especially in roughness parameters	Produces smoother surfaces with lower overall roughness but larger roughness

Table 1: Accuracy and general remarks

The presented results can be regarded as a preliminary experimental outcome. The method requires further enhancement and testing on a variety of data sets with complex features in order to validate its effectiveness and to assess its robustness.

5. Conclusion

This study proposes a specialized approach for extracting surface texture parameters of freeform surfaces commonly encountered in additive manufacturing. The methodology focuses on characterizing surface texture parameters through parametric modeling, B-spline and GB-spline reconstructions, and discrete smooth interpolation. The results were compared to Mountains software results. This first comparison demonstrated that the errors among the three reconstruction methods against Mountains software were negligible, but the method still need to be improved.

Future work aims to enhance the presented method by addressing more complex geometries, exploring a new set of AST parameters, and incorporating filtration techniques to refine the analysis since in this paper the method was applied directly on the primary surface.

6. Acknowledgments

This work has been carried out within the project 23IND08 DI-VISION, which has received funding from the European Partnership on Metrology, co-financed from the European Union's Horizon Europe Research and Innovation Programme and by the Participating States.

References

[1] Harris P M, Smith I M, Leach R K, Giusca C, Jiang X and Scott P 2012 Software measurement standards for areal surface texture parameters: part 1—algorithms *Meas. Sci. Technol.* **23** 105008.

[2] Jung L, Spranger B, Krüger-Sehm R and Krystek M 2004 Reference software for roughness analysis—features and results *Proc. XI Int. Colloquium Surfaces, Chemnitz, Germany* pp 164–70.

[3] Bui S H and Vorburger T V 2007 Surface metrology algorithm testing system *Precis. Eng.* **31** 218–25.

[4] Harris P M, Smith I M, Chaoyi W, Leach R K and Giusca C 2012 Software measurement standards for areal surface texture parameters: part 2—comparison of software *Meas. Sci. Technol.* **23** 105009.

[5] Pagani, Luca and Scott, Paul J. (2016) Areal surface texture parameters on surface. In: Dimensional Accuracy and Surface Finish in Additive Manufacturing. American Society for Precision Engineering, North Carolina, USA.

[6] L. Pagani, Q. Qi, X. Jiang, P. J. Scott, Towards a new definition of areal surface texture parameters on freeform surface, Measurement 109 (2017) 281–291.

[7] Luca Pagani, Andrew Townsend, Wenhan Zeng, Shan Lou, Liam Blunt, Xiang Qian Jiang, Paul J. Scott, Towards a new definition of areal surface texture parameters on freeform surface: Re-entrant features and functional parameters, Measurement 141 (2019) 442–459.

[8] ISO 25178-2:2012 - Geometrical Product Specifications (GPS) - Surface Texture: Areal - Part 2: Terms, Definitions and Surface Texture Parameters.

[9] Piegl, L., & Tiller, W. (1997). The NURBS book (2nd ed.). Springer.

[10] Juhász, I., Róth, Á.: A class of generalized B-spline curves [J]. Comput. Aided Geom. Des. 30(1), 85–115 (2013)

[11] Li Zhang a, Jieqing Tan, Xianyu Ge, Guo Zheng, Generalized B-splines' geometric iterative fitting method with mutually different weights, Journal of Computational and Applied Mathematics 329 (2018) 331–343.

[12] Jean-Laurent Mallet, Discrete smooth interpolation, ACM Transactions on Graphics, 1989, 8 (2), pp.121-144.

[13] Jean-Laurent Mallet, Discrete smooth interpolation in geometric modelling, Computer-Aided Design Volume 24, Issue 4, April 1992, Pages 178-191.

[14] Bruno L´evy, Jean-Laurent Mallet, Discrete Smooth Interpolation: Constrained Discrete Fairing for Arbitrary Meshes, ASGA , 332-353 (1999).