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Application of generalized Union-Jack technique to measurements with laser line triangulation sensors

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Abstract

Geometrical control of parts plays a critical role in actual manufacturing and assembly processes. Laser line triangulation devices are widely adopted for dense point cloud generation with both geometrical control as well as digitizing purposes. They provide an intrinsic reference for the points measured in one acquisition, however, the generation of an accurate 3D point cloud requires providing the pose, position and orientation, of the laser line triangulation device to transform all individual acquisitions to a common reference frame. In automated scenarios, where the laser line triangulation device is moved by a manipulator, such as an industrial robot, the pose of the manipulator is used to provide this common reference frame to individual acquisitions. In this approach the uncertainty of the position awareness of the manipulator contributes directly to the uncertainty budget of the generated point cloud.

The paper presents the analysis of the application of a generalized Union-Jack measurement technique, ussually carried out with levels to measure flatness standards, to laser triangulation sensors to improve the out-of-plane measurement performance of quasiflat parts. The possibilities of the approach have been explored in simulation and presented regarding the uncertainty propagation properties.

Metrology, laser, robot, dimensional

1. Introduction

Dense point cloud measurements performed by scanning techniques are common in industry nowadays with different performance levels regarding measurement uncertainty [1]. Among different sensing devices laser line triangulation sensors are very widely used because they provide quite high accuracy, relative accuracy to range in the order or 1:30000 [2], and a very high throughput of several million points/s. However, in order to digitalize 3D scenes, a laser line sensor must rely on a manipulator to move it around the part under test. For some applications, the relatively low positioning accuracy of the manipulator degrades the performance of the laser line sensor. For example, an industrial robot can achieve anything between 1:2000 and 1:10000 relative accuracy (0.5 mm for a working volume of 2500 mm in radius [3,4]).

The work presented in this paper adopts the basic idea of the Union-Jack measurement technique of flatness standards based on levels [5], which was already generalized to more dense grids of points [6] and applies it to measurements realized with laser triangulation sensors. The technique allows to extend the lateral range of the sensor from a short segment to a large area keeping the resulting uncertainties low even if the manipulator is considerably less accurate than the laser line sensor.

The approach is presented, the uncertainty propagation characteristics are analysed and some numerical experiments and results are finally included.

2. Measurement strategy

The goal of the measurement strategy is to measure the outof-plane profile of a given surface, direction z as in Figure 1. It takes advantage of the redundancy between the capabilities of the laser line sensor to provide measurements along a segment and the capability of a manipulation device, such as a robot, to move the sensor along the line containing the segment.

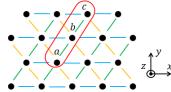


Figure 1. Triangular grid of control points and exemplary segment measurent with a laser line sensor, in red.

By creating a grid of control points or nodes, see figure 1, so that nodes belong to several lines, it is possible to build a system of linear equations that implicitly performs the stitching of all measurements along the plane without out-of-plane information about the position of the manipulator.

In Figure 1 an exemplary measurement with the laser line sensor is denoted in red. It comprises the segment \overline{abc} . For the heigths of the nodes of this segment, it can be stated that,

$$h_b=\frac{h_a+h_c}{2}+m_{ac}(s=0.5) \eqno(1)$$
 Where h_i is the height of the i-th node and $m_{i,j}(s)$ is the

Where h_i is the height of the i-th node and $m_{i,j}(s)$ is the reading of the laser line sensor along the segment connecting nodes i and j. The parameter 0 < s < 1 defines the sampling point along the segment. The reading must be adjusted so that $h_{i,j}(0) = h_{i,j}(1) = 0$.

The complete set of measurements can be feed to a bundle adjustment to obtain the node heights by least squares, equation (3). It is convenient to rewrite equation (1) to build the set of linear equations as:

$$2m_{a,c}(s=0.5) = -h_a + 2h_b - h_c \tag{2}$$

The set of linear equations and its sparse system matrix, A, resulting from the approach applied to the grid in Figure 1 are shown in equation (3),

$${2m_{1,3}(0.5) \atop \vdots \atop 2m_{l,J}(0.5)} = [A] {h_1 \atop \vdots \atop h_N} = \begin{bmatrix} h_1 \\ \vdots \\ h_N \end{bmatrix} = \begin{bmatrix} h_1 \\ \vdots \\ h_N \end{bmatrix}$$
(3)

In case of a rectangular grid, readings along diagonals in one direction are necessary to obtain a properly conditioned matrix. The geometrical interpretation of the diagonal measurements is to constrain the torsion of the surface. As an alternative it is possible to use a triangular grid, which in essence, topologycally, is similar to an skewed rectangular grid with diagonals in one direction. The connectivity of the three sets of lines can be clearly recognized in equation (3).

The $m_{i,j}(s)$ readings are of outmost importance for this measurement strategy. A naive implementation can sample just three points, s=0, s=0.5 and s=1. However more sophisticated approaches could benefit from the large amount of data generated by the sensor, such as fitting a common curve to last-half and first-half of two consecutive segments in a line.

The minimal fitting to make the approach work considers adjusting just 2 Degrees of Freefom, DOFs, height and pitch, to stitch two segments. However, in case the surface under measurement provides enough details along the length of the line, a 3 DOF fitting could improve in-plane performance of the results.

In the case the grid is coarse and several measurements are carried out along a line before reaching a node belonging to other lines, the sparsity of the A matrix would rise in detriment of overall accuracy but reduction in measurement time.

2.1. Propagation of uncertainty

Considering the covariance of the measurements, cov_m , the covariance of the best fitted node height values can be obtained as [3]:

$$cov_h = (A^T \cdot cov_m^{-1} \cdot A)^{-1} \tag{4}$$

The square root of the diagonal of this covariance matrix can be interpreted as the uncertainty of each node value. Considering uncorrelated measurements and unitary uncertainty for each of them, it is possible to compute the Uncertainty Propagation Factors, UPFs, as:

$$UPFs = \sqrt{diag((A^T \cdot A)^{-1})}$$
 (5)

The positioning errors of the manipulator carrying the laser line sensor as well as the errors in the hand-eye calibration of the sensor on top of it impact the measurement as soon as the reading from segment \overline{bc} by \overline{abc} are not the same as by measuring \overline{bcd} . The sensitivity increases by strongly curved surfaces. More work is needed to understood this sensitivity. On the other hand, the systematic errors of the laser line sensor lead to correlations in the measurements and finally to curvature on the results. It can be corrected by calibrating the system with a planar material standard of known curvature or flatness.

3. Numerical experiments

Figure 2 presents an exemplary result of the Uncertainty Propagation Factors for a grid of 10x10 nodes. The geometry of the part is considered flat and smooth, this means that positioning errors of the manipulator play a minimal role. The numbers are UPF and colors are scaled according to this values. The height of three nodes have been defined to constrain the solution in space, lower-left, lower-right and upper-mid nodes.

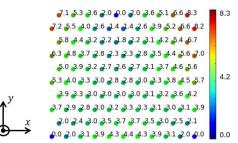


Figure 2. Uncertainty Propagation Factors for each node of a 10x10 grid considering equal uncertainty for all measurements. The nodes are represented in the corresponding xy position.

Table 1 summarizes the UPFs for grids of different sizes. Many applications only require the evaluation of flatness errors which are obtained after removal of the best fit plane to the data. The UPFs for this use case are also summarized in Table 1.

Table 1 Uncertainty Propagation Factors considering grid size and whether a plane is fitted to the results or not

Size of grid	No plane fit		plane fit	
	mean	max	mean	max
4x4	1.84	3.58	1.00	1.62
10x10	3.78	8.36	2.04	<mark>4.30</mark>
20x20	7.22	16.58	3.86	9.04
50x50	17.58	41.82	9.34	23.66
100x100	35.20	82.64	18.40	47.50

Application example: An μ Epsilon LLT30x0-430 laser line sensor with a mid measuring range of 515 mm and a linearity of 0.012 mm (z-axis) in a grid of 10x10 nodes, yellow marked result in Table 1, spanning $10\cdot515/2 =$ **2575 mm** and $2575\cdot\sin(60)=$ **2230 mm** in each coordinate direction, could achieve an out-of-plane measurement uncertainty of $4.30\cdot12$ um = **0.052 mm** (flatness of a flat part).

4. Summary, conclusions and future work

The paper presents a novel measuring strategy for laser line sensors mounted on top of low accuracy manipulation devices which enable the out-of-plane measurement of large quasi-flat parts without degrading substantially the measurement accuracy of the sensor. In the analysed case from 0.012 mm linearity in a 515 mm line to 0.052 mm flatness in a 2500 x 2200 mm area which shows the very favourable uncertainty propagation properties of the approach.

Future work will experimentally prove these findings and explore the application to curved parts.

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