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Transmissibility Based Sub-structure Coupling Method for Precision Metal Cutting

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Abstract

Simulation of metal cutting to predict resulting tolerances requires modeling the system's dynamics. The mathematical characterization of the tool center point (TCP) of the machine requires complex experimental procedures. The receptance coupling method enables analytical modeling of the free-free tool pairs to the spindle with the utilization of numerical methods and experimental methods respectively. The two substructures enable the mathematical assembly to avoid time-consuming experimental impact hammer-type tests for each specific cutting tool in the machine magazine. For experimental evaluations of the spindle nose, impact tests are conducted by utilizing position derivative tracking sensors. Such experimental procedures that employ multiple sensors are not only costly but are also cumbersome and time-consuming. Especially, if multiple points in the machining volume are to be measured as dictated by ISO 230. This results in a low uptake of industrial machines. In this paper, we propose a novel methodology for the characterization of the frequency response function (FRF) of the tool center point. The new method is based on the automated identification of the transmissibility of the spindle nose. This transmissibility is identified between the machine's NC axis and spindle nose and exploits the inertia of the axis as excitation. The cutting tool and tool holder are modeled numerically as traditional methods. The new methodology enables the use of NC motor encoders and cutter forces to identify the response of the tool's endpoint during machining operations while enabling easier and more repeatable experimentations for frequency response function identification. To showcase the validity of the new methodology a set of simulations are conducted.

Keywords: Receptance Coupling, Frequency Response Function, Substructure Coupling, Transmissibility, Cutting Process Simulation

1. Introduction

Production capacity, efficiency, and performance of machining operations heavily depend on the system dynamics of the machine-tool pair [1]. Vibrations generated by cutter-workpiece engagement (CWE) and drive excitations affect the quality of the production. The tool center point's (TCP) behavior depends on the cascaded dynamics of the drive, machine tool dynamics, and the cutting tool. While the drive system might have its first pole on the order of 10 Hz, the spindle and the tool have their first structural modes in between hundreds to thousands of Hertz [2,3,4].

Constructing a virtual CNC (VCNC) can help with the evaluation of the feasibility of tolerances on a time and cost basis, however, predicting the manufacturing process requires accurate modeling of the machine tool. The usage of FRFs in time domain analysis still remains a big challenge due to the complexity of experimental characterization. Impact testing is one of the most utilized methods for system identification of spindle units that come with a number of caveats such as poor signal to noise ratios and overload problems [1]. When nonlinear systems are analyzed the shaker inputs are preferred however require a complex setup and therefore are more time-consuming as compared to the hammer tests. Machine drive excitations on the other hand require overcoming dynamic friction conditions, requiring a constant velocity over time.

There exist a number of other FRF characterization methods proposed in the literature. The inverse Receptance Coupling Method is utilized by enforcing compatibility and equilibrium at the coupling interfaces, however, requires accurate modeling of receptance matrices and is prone to coupling errors [5]. Lately, the usage of transmissibility in the system identification of machine tools gained attention. Transmissibility function-based operational modal analysis (TOMA) employs accelerometers on the spindle side in order to estimate approximate FRFs. However, the method involves the inclusion of several accelerometers for all axes and is prone to fictitious modes [6].

In this study, we present Transmissibility Based Receptance Coupling Method(TBRCM). The mathematical coupling of the transmissibility response of the spindle and the FRF of the tool and tool holder has the potential to decrease the required experimental identification time. We derive the equations for the FRF of the coupled system using the transmissibility transfer function and use numerical simulations to investigate the validity of the Transmissibility Based Receptance Coupling Method.

2. Transmissibility Based Receptance Coupling Method

In this section, we present the Transmissibility Based Receptance Coupling Method. Even though the Receptance Coupling Method allows for shortened experimental procedures than traditional system identification methods and is more computationally efficient compared to the conventional finite element modeling approach used in machine tool design [7], there are several setbacks. Mainly the industry standards setting the number of required testing points to 10 or more points for detailed modal analysis in each axis direction [8] resulting in still time-consuming and cost-ineffective procedures, and the apparent model lacking drive inputs. TBRCM aims to fill in the gap by introducing machine excitations and capturing easily measurable position derivative signals on the spindle and drive system, utilizing the transmissibility response between the encoder of the drive system and accelerometers on the spindle. Consequently, time-consuming impact experiments can be omitted while modeling the input from the drive train.

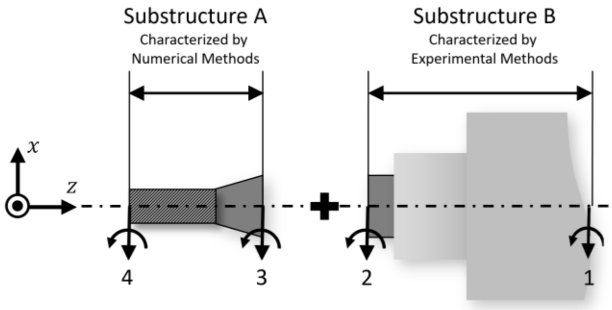

Figure 1. Modeling of substructure coupling between the tool holder and the spindle.

Figure 1 depicts the substructure coupling between the spindle and the tool holder. The assembly of the machine and the tool is divided into two substructures denoting the free-free end mill's cutting edge by point 4, the tool holder with point 3, the spindle end unit denoted by point 2, and the encoder denoted by point 1. Consider the FRF of the free-free end mill and tool holder structure as denoted in:

$$\begin{bmatrix} X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} H_{33} & H_{34} \\ H_{43} & H_{44} \end{bmatrix} \begin{bmatrix} F_3 \\ F_4 \end{bmatrix}, \quad (1)$$

where, X_3 and X_4 represent the displacement vectors while F_3 and F_4 represent the applied forces at points 3 and 4, respectively. H_{ij} terms represent the FRF between the two points. The relationship between points 1 and 2 can be represented with the transmissibility transfer function as given in Eqn. (2).

$$T_{21}(s) = \frac{X_2(s)}{X_1(s)} \quad (2)$$

The FRF of points 3 and 4, denoted in Eqn. (1), is either found by finite element methods or beam theory while the transmissibility response between points 1 and 2 is characterized by experimental methods. The receptance coupling method allows for analytical coupling of the free-end tool holder represented by point 3 to the free-end spindle represented by point 2. The rigid coupling conditions of the two structures are as follows:

$$F_t = F_2 + F_3, \quad (3)$$

$$X_2 = X_3. \quad (4)$$

Substituting Eqn. (1), Eqn. (2) and Eqn. (3) into Eqn. (4) and solving for F_2 results in:

$$F_2 = F_t + H_{33}^{-1} H_{34} F_4 - H_{33}^{-1} T_{21} X_1. \quad (5)$$

Rewriting X_4 from Eqn (1) and combining with Eqn. (5) with basic mathematical manipulation results in:

$$X_4 = (H_{43} H_{33}^{-1} H_{34} + H_{44}) F_4 - H_{43} H_{33}^{-1} T_{21} X_1. \quad (6)$$

While, F_4 and X_1 are the parameters that can be measured during operation; F_4 is the cutting force at the TCP and X_1 is the encoder displacement, hence if the system defined by H_{ij} and T_{21} is identified, the TCP deflection can be calculated. Two main applications exist for the utilization of TCP estimation, the first being the utilization of a force sensor and direct readings of the encoder position in order to model in-process material removal simulation, the second being the employment of mechanistic cutting force model and simulation of drive dynamics in order to estimate the tolerance levels before the deployment of the operation.

4. Simulations of Transmissibility Based Receptance Coupling Method

In this section, we present simulation results of the TBRCM. First, a modal analysis was conducted using Ansys® Workbench in order to evaluate the structural dynamics of the model of a tool and tool holder referred to as Substructure A in **Figure 1**.

The expanded version of the FRF of a structural system can be obtained from its structural modes and mass-normalized mode shapes[9]. FRF matrix is expressed in Eqn. (7) in terms of modal poles and mode shapes

$$H(\omega) = \sum_{k=1}^{Modes} \frac{[\Psi(k)]_N \, [\Psi(k)]'_N}{(m(k) - \omega^2)}, \quad (7)$$

where *m(k)* is the modes, $[\Psi(k)]_N$ is the mass-normalized mode shape, and $H(\omega)$ is the FRF of the structural system. FRF of the tool and tool holder can be therefore found with the extraction of modes and mode shapes from Ansys® Modal Analysis. The structure is modeled as a cylinder by taking 80% of the 10 mm diameter of an end mill. Two virtual topologies are created on the circular faces of the cylinder. For each mode, the average displacement is calculated from the nodes which lie inside the created virtual topologies. The modes and mode shapes are automatically extracted with Ansys® APDL commands. The first 12 modes are selected to formulate the FRF of the structure.

Table 1 Modes of the tool and tool holder.

Mode no.	Frequency [Hz]	Mode no.	Frequency [Hz]
1	0	7	6.8e+3
2	1.2e-4	8	6.8e+3
3	7.5e-4	9	7.8e+3
4	1.0e-3	10	1.25e+4
5	1.2e-3	11	1.38e+4
6	1.2e-3	12	1.39e+4

These modes and mode shapes are then used for the formulation of FRF of the structural system following Eqn. (7). **Figure 2.** depicts the simulated FRF of tool tip (point 4) with a collocated force input, H_{44}, from the extracted modes and mode shapes compared to frequency response acquired from Ansys® Harmonic Response software. The resultant FRF formulated with the analytical method has been accepted to be precise enough for the estimation of TCP with the utilization of the TBRCM.

3.1.Simulations of Transmissibility Based Receptance Coupling Method with Unity Substructure B

With the acquired FRF of the tool and tool holder, we first consider the transmissibility between points 1 and 2, the encoder and spindle nose respectively, to be infinitely stiff. This assumption implies that for such a system the gain relating two points is unity. Assuming there is no apparent force on the tooltip, meaning $F_4 = 0$ and setting T_{21} = 1 results in the following equality derived from Eqn. (11):

$$X_4 = (H_{43} H_{33}^{-1}) \, X_1. \quad (8)$$

The equality shown in Eqn. (8) is simulated in Ansys® Harmonic Response software and Matlab R2024b with a unit input. The displacement frequency is swept from 0 Hz to 15000 Hz. **Figure 3** shows the resultant FRFs that are acquired from the FEM software and TBRCM. Two results show similar characteristics with a non-perfect pole-zero cancelation of TBRCMat around 7800 Hz.

3.2.Simulations of Transmissibility Based Receptance Coupling Method when Substructure B is compliant

In this section, we consider the model of the structure that lies in between the spindle is not infinitely stiff. **Figure 4.** depicts the system used for the simulations. The encoder-spindle unit (Substructure B) is modeled as a free-free structure and is simulated when a displacement sweep signal in direction up to

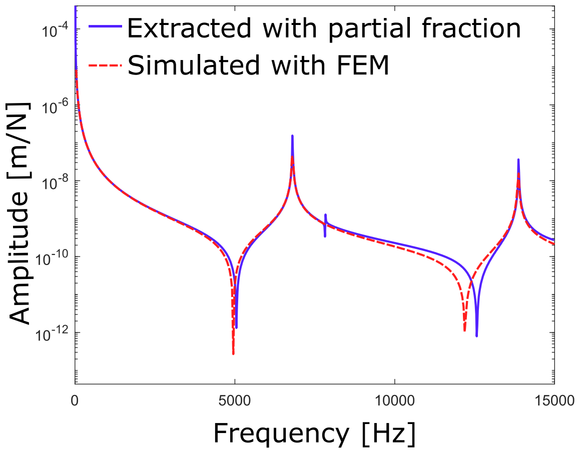

Figure 2. The extracted FRF of the tool and tool holder model in the x-axis.

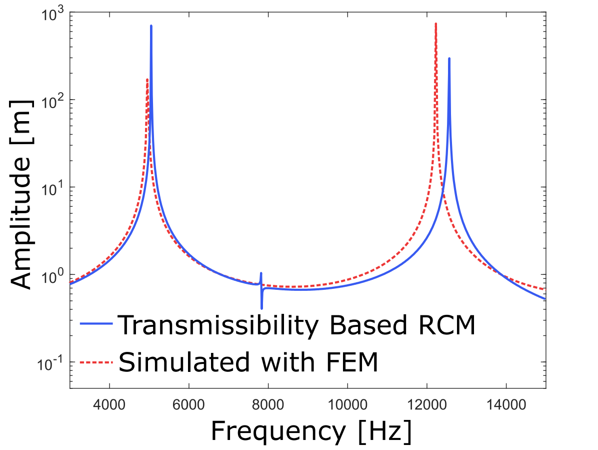

Figure 3. The resultant frequency response of the tooltip with unity transmissibility and zero apparent force.

15000 Hz is present on point 1 in Ansys® Harmonic Response software. The frequency response of point 2 is saved and used for the estimation of the transmissibility transfer function T_{21}. For the estimation of the transmissibility transfer function Matlab 2024b System identification toolbox is used. The order of the used transfer function varies between 4 and 12 with a relative degree of zero and demonstrated an average fit to estimation data of 99.3%.

In accordance with Eqn. (6), the mathematical coupling is simulated in Matlab 2024b with unity displacement in the x direction. The coupled system is also simulated in Ansys® Harmonic Response software, with unity displacement applied from point 1 with no force input. The frequency response of point 4 is saved.

Figure 5. shows the frequency responses of the FEM software and mathematical coupling. It is observed that the error between the FEM software and the mathematical coupling method stays under 1.5% up to 750 Hz. Most Industrial machines have interpolation cycles in the range of 500 Hz to 1000 Hz, therefore a maximum quasistatic range of 750 Hz is considered acceptable.

4. Simulations of Transmissibility Transfer Function Estimation

In this section, we present simulations of system identification of the The expanded version of the FRF of a structural system can be obtained from its structural modes and mass-normalized mode. **Figure 6.** presents the common feedback control structure used in industrial machine tools, a PD-PI position velocity cascade control when the spindle error acts as a disturbance to the control architecture. K_p is the proportional position control gain, D_p is the derivative position control gain, K_v is the proportional velocity gain and I_v is the integral velocity gain.

J is the system inertia seen from the motor side while B is the damping seen from the motor side. X_{ref} symbolizes the position reference while X_{enc} represents the value of the encoder. T and T^{-1} are the transmissibility transfer functions relating point 2 (spindle) to point 1 (encoder) and point 1 to point 2 respectively. X_{enc} and the output of T^{-1} are used for the calculation of the error between the spindle position and the encoder: e_{x2}. The system is simulated within Simulink (Mathworks) and tuned with the values provided in **Table 2**. Given that the error between the points 2 and 3 is zero or no disturbance occurs, the closed loop transfer function between X_{ref} and X_{enc} can be represented as in Eqn. (9).

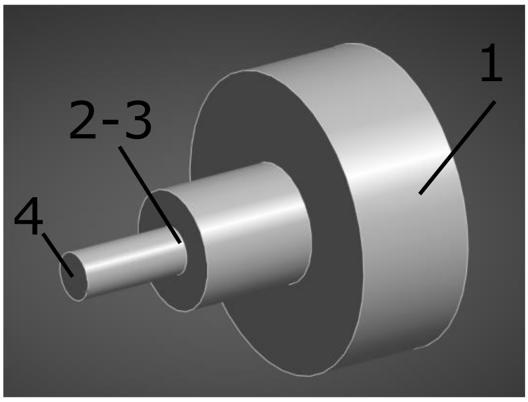

Figure 4. The points on the model of the coupled tool-tool holder and spindle-encoder unit.

$$\frac{(D_p + K_v)s^2 + (D_P I_v + K_p K_v)s + I_v K_p}{J\,s^3 + \left(B + K_v\left(1 + D_p\right)\right)s^2 + \left(I_v\left(1 + D_p\right) + K_v K_p\right)s + I_v K_p} \quad (9)$$

The transmissibility transfer function, T, is selected as a second-order low-pass filter with a relative degree of zero. The system is simulated with a chirp signal with a sweep in between 0 Hz to 40 Hz, when the disturbance is present, with a fixed step time of 10^{-4} seconds for 10 seconds. The error between the encoder and the reference, the error, e_{x2}, and the encoder, X_{enc}, values are saved.

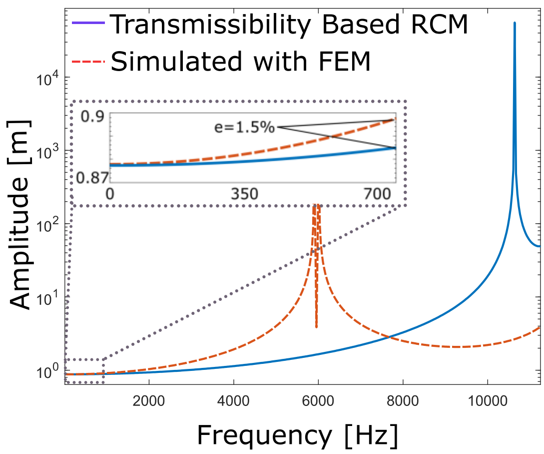

Figure 5. The frequency response of the coupled system with unity translation.

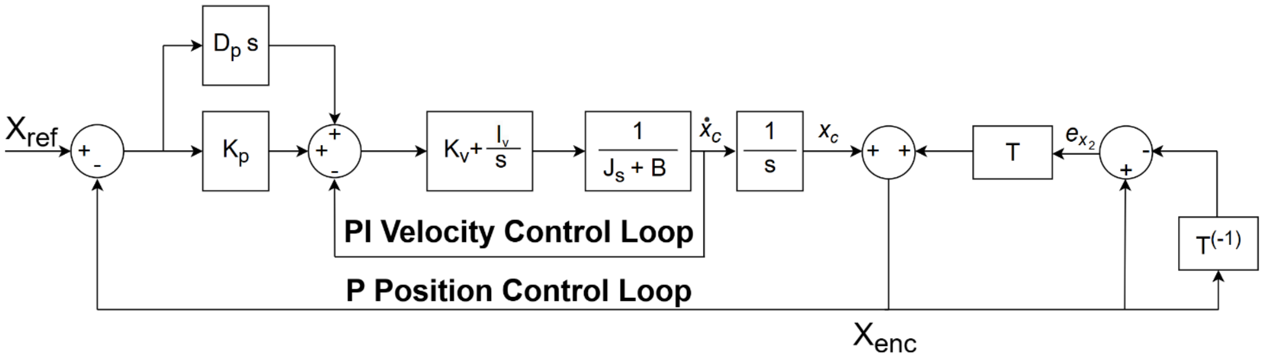

Figure 6. Cascaded closed-loop system when the spindle error acts as a disturbance.

Table 2 Control and Plant Parameters.

Parameter	K_p	D_p	K_v
Value	75 [s]	60 [-]	35 [N/ms]
Parameter	I_v	J	B
Value	10 [N/m]	0.01 [kg m^2]	10 [N/ms]

The saved values are used for system identification of the transfer function given in Eqn. (9) and transmissibility transfer function with the least squares method in Matlab 2024b. The numerically estimated systems show high accuracy with a fit to estimation of 98.2%. **Figure 7.** shows the estimated output and simulated output with the knowledge of the full-order system. **Figure 8.** shows the bode magnitude of the original and the estimated controller, the transfer function in Eqn. (9).

The average error in between the sweeping frequency is calculated as 3.2% and 14.6% for the controller and the transmissibility transfer function respectively.

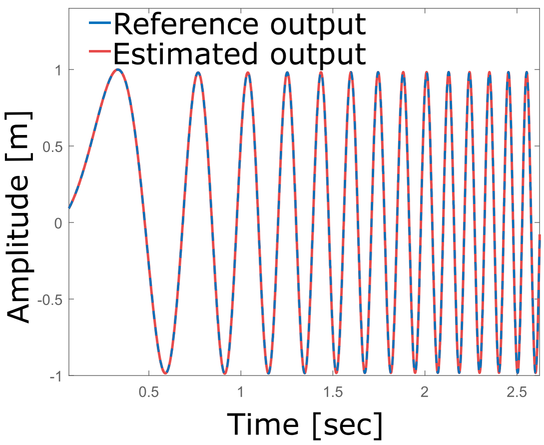

Figure 7. The comparison between the estimated output and the reference used for the estimation.

5. Conclusion and Future Work

In this paper, we presented Transmissibility Based Receptance Coupling Method, which shows promise for rapid experimental procedures for the identification of TCP dynamics. Transmissibility Based Receptance Coupling Method expands the traditional sub-structure methods by the inclusion of drive dynamics and motor inputs for the estimation of material removal rate, aiming to increase the efficiency of industrial machines. A set of simulations were conducted for both TCP deflection estimation and system identification of the control system and transmissibility transfer function. The consistency between the simulated and mathematically estimated response of the system is considered acceptable in the applicable frequency range of industrial machines.

The future work includes the utilization of more sophisticated system identification methods such as the Polymax method as well as system identification experiments.

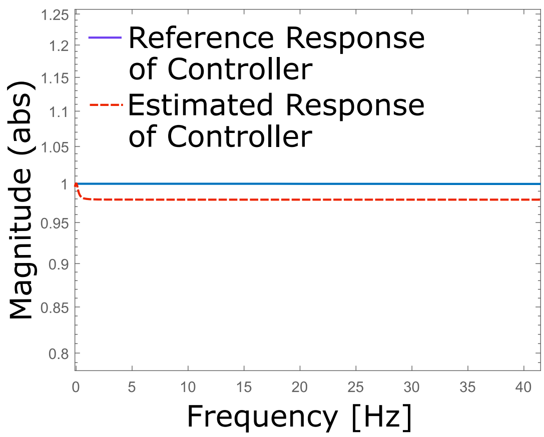

Figure 8. The comparison between the estimated response of the controller and the response used for the estimations.

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