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Evaluation of the measurement uncertainty of a high-precision telescopic instrument measuring the tool centre point of a robot

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Abstract
Industrial robots are widely used for applications that require flexibility, repeatability and, increasingly, positioning accuracy, especially when offline programming is performed. For any application, it is necessary to locate the end effector installed on the robot, relative to the robot frame, in order to establish the target positions reached by the robot. Tool Centre Point (TCP) measurement locates the end effector in the robot control. Defining the TCP of the robot in its control, and therefore, in the programs, is a key factor to avoid positioning errors. There are several standards for TCP measurement that are generally implemented in the robot control itself. These measurement procedures do not provide traceability of the results obtained. The work presented in this paper shows the modelling of the measurement system to evaluate its uncertainty measuring the TCP position of a robot. The measurement system calculates the position of the robot's end effector by performing multilateration from the simultaneous measurement of several distances with telescopic systems based on interferometry. The evaluation will include the effect of the performance of the robot in the measurement results.

Interferometry; Robot; Uncertainty; Tool-Centre-Point

1. Introduction

The use of flexible handling systems such as robotic arms has grown in recent years in the industrial sector [1]. Robots, given their programmability and high repeatability characteristics, improve the flexibility and capacity of production systems [2]. The automation of manufacturing systems within the framework of industry 4.0 is an important aspect in which improving the accuracy of production processes is becoming increasingly important [3, 4]. In this context, off-line programming of production systems aims to ensure accuracy in the measurement of their characteristics. In the case of robots, the measurement of the tool centre point (TCP) using a traceable procedure is essential in the off-line programming stage to prevent the necessary adjustments with the system stopped from consuming excessively long time [5, 6].

The calibration of the TCP, as the process of measure the TCP position in the reference system of the robot is known [6], has been studied in previous work using laser displacement sensors [4, 6], cameras and non-contact sensors [5, 7] and procedures based in spheres fitting algorithm [8]. Although, the TCP is traditionally measured using the robot and an external fixed point: positioning the TCP (a physical part of the tool) in the external fixed point with four different orientations. This procedure allows the estimation of the centre of a sphere locating the TCP but the measurement process lacks of traceability. In the context of improve the accuracy of the manufacturing systems and increase their productivity the quality in the production process depends on the traceability of the measurements [9].

The main goal of this work is to estimate the measurement uncertainty of a Telescopic Simultaneous Ballbar (TSB) measuring the TCP of a robot. To achieve this objective, a method for measuring the TCP is proposed simulating the behaviour of the robot and the TSB on the measurement process. The measurement uncertainty of the TSB measuring the TCP position of a robot tool has been estimated with the Monte Carlo method [10,11].

A calibration and uncertainty budget analysis for the TSB is presented in [12] but an analysis of the measurement uncertainty in a measurement application, in this case the TCP measurement, is needed to assure the traceability of the measurement results obtained with the TSB.

The paper is structured as follows: Section 2 describes the hardware proposed, the TSB and an anthropomorphic robot arm, the mathematical model developed to simulate the measurement and the methodology proposed to simulate the behaviour of the systems and to estimate the TCP measurement uncertainty. Section 3 details the characteristics of the simulation tests, their analysis and settings. Section 4 present the main results of the tests including the measurement uncertainty obtained for the TCP measurement. Finally, Section 5 shows the main conclusions of the study.

2. Hardware model and measurement methodology

This section describes the use of a TSB [12] to measure the TCP of a robot. The first subsection is focus in the TSB, and the mathematical model used to simulate its behaviour measuring XYZ coordinates of space points call Measurement Points (MP). The second subsection describes the robotic arm proposed to test the application, its mathematical model and the methodology for the TCP measurement.

2.1. Telescopic Simultaneous Ballbar and multilateration methodology

The TSB is a measurement device consisting of several telescopic ballbar, TB [12]. Each TB measures the distance between two steel balls using an interferometer. The special design of each TB allows three ends pointing to the same steel ball while the other end of each TB is located in a fixed kinematic support, KS [13]. This configuration allows the simultaneous tracking of a steel ball with the TSB: three TB simultaneously measuring the distance between three fixed points (the kinematic supports, KSi, with i from 1 to 3) and the steel ball. From three distances measured simultaneously, the coordinates of the centre of the steel ball, P ($P = \{X\ Y\ Z\}'$), can be multilatered as shown in equations 1 to 3.

$$X = \frac{{L_{1,p}}^2 - {L_{2,p}}^2 + {X_{KS2}}^2}{2 \cdot X_{KS2}} \quad (1)$$

$$Y = \frac{{L_{1,p}}^2 - {L_{3,p}}^2 + {X_{KS3}}^2 + {Y_{KS3}}^2}{2 \cdot Y_{KS2}} - \frac{X_{KS3}}{Y_{KS3}} \cdot X \quad (2)$$

$$Z = \sqrt{{L_{1,p}}^2 - X^2 - Y^2} \quad (3)$$

Where: $L_{i,p}$, with i from 1 to 3, are the distances simultaneously measured between each KSi and the centre of the steel ball atached to the system under verification, P; and $X_{KSi}\ Y_{KSi}\ Z_{KSi}$, with i from 1 to 3, are the coordinates of the centre of the ball located in each KS expressed in the TSB reference system (RS_{TSB}). The origin of the RS_{TSB} is the centre of the ball in $KS1$, the x-axis is defined with the segment between $KS1$and $KS2$, and the z-axis is defined with the normal vector of the plane containing the three KSs. The coordinates of $KS2$ and $KS3$ are calculated from the distances between each KS, $L_{i,j}$, with i and j from 1 to 3, as show in eq 4 to 6 and Table 1.

$$X_{KS2} = L_{1,2} \quad (4)$$

$$X_{KS3} = L_{1,3} \cdot \cos(\beta_1) \quad (5)$$

$$Y_{KS3} = L_{1,3} \cdot \sin(\beta_1) \quad (6)$$

Where β_1 is the angle between the segment joining $KS1$ and $KS3$ with the segment joining $KS1$ and $KS2$, Figure 1, and its cosine can be calculated from the cosine theorem with the three distances between the KS ($L_{1,2}$, $L_{2,3}$ and $L_{1,3}$).

$$\cos(\beta_1) = \frac{L_{2,3}^2 - L_{1,2}^2 - L_{1,3}^2}{-2 \cdot L_{1,2} \cdot L_{1,3}} \quad (7)$$

Each TB measures $L_{i,j}$, with i and j from 1 to 3, in an initial step of the measurement, and measures simultaneously $L_{i,p}$, with i from 1 to 3, in the tracking process, with a measurement uncertainty of 4 µm (with a confidence level of two, k=2) [12].

2.2. Robot arm and TCP measurement methodology

The robot arm modelled in this paper is a six degrees of freedom manipulator with a range of 650 mm and a positioning repeatability error (PRE) of ±0.02 mm according with ISO 9283 [14]. The mathematical model used in this work for the robot, the model proposed by Denavit-Hartenberg modified by Hayati-Mirmirani, can be found in [15]. The PRE of the robot will affect the behaviour of the method proposed for the TCP measurement.

The initial setting for the proposed method needs a steel ball machined so that it could be joined to the end effector of the robot, being the centre of the ball the TCP of the robot end effector. The diameter of the steel ball will be 1.5' to fit the kinematic coupling of the three TBs of the TSB. the three kinematic coupling of the TSB will be located simultaneously in the ball centred in the TCP. The TSB's KS will be fixed in a position in the Robot Reference System (RS_R). The position of the TSB in the RS_R is set as the position of RS_{TSB} in RS_R. This position affects the common space between the reach of the TSB and the positioning reach of the robot. The points measured to obtain the TCP must be within the common space between both. The TSB will measure the points reached (their coordinates expressed in the RS_{TSB}) by the robot's TCP when it moves between the programmed target positions.

The proposed method for estimating TCP consists of moving the robot end effector in such a way that it describes circles whose centre is the origin of the robot mounting flange reference system (RS_{A6}). The target positions in this method set the position of the RS_{A6} in the RS_R by holding the origin of the RS_{A6}fixed (with the same XYZ coordinates) and introducing relative motions in the orientation of the RS_{A6}axes. When the RS_{A6}reaches these positions, the TCP will rotate around the RS_{A6} axes.

The sequence of points measured at these positions traces the arc of a circle (more or less complete depending on the reach of the robot) for each sequence of turns about one of the axes. The positions reached by the TCP (Measurement Points, MP) will be measured with the TSB, obtaining the TCP coordinates at each position expressed in the RS_{TSB}. These positions will trace an arc of a circle (whose amplitude depends on the reach of the robot) for each rotation sequence around the axes of the RS_{A6}. The points measured when the robot rotates its TCP around one of the axes of the $RS_{A6}$are located on the same plane. The normal vector of the plane indicates the direction of the corresponding axis of the RS_{A6}. In addition, the points measured for all the target positions are located on the surface of a sphere whose centre is the origin of the RS_{A6}. Measuring the MP with the TSB allows the calculation of the coordinates of the origin and the direction cosines of the RS_{A6} axes expressed in the RS_{TSB} Due to robot positioning errors and TSB measurement errors, the plane to obtain the direction cosines and the sphere to obtain the centre are estimated by performing a least squares fit. With the TCP coordinates at each MP, expressed in the RS_{TSB}, the origin of the RS_{A6} and the direction of the RS_{A6} axes in the RS_{TSB} are obtained and equation 8 allows extracting the TCP coordinates expressed in the RS_{A6}.

$$X_{A6} = M_{ABC} \cdot {}^{A6}M_{TSB} \cdot X_{TSB} \quad (8)$$

Where: X_{TSB} are the homogeneous coordinates of the TCP expressed in RS_{TSB} (x y z 1)'; X_{A6} are the homogeneous coordinates of the TCP expressed in RS_{A6} (the TCP coordinates needed for robot programing); ${}^{A6}M_{TSB}$ is the homogeneous transformation matrix that allows to express the coordinates of a point, known in the RS_{TSB}, in the RS_{A6}; and M_{ABC} is the homogeneous transformation matrix that introduces the rotation to express the coordinates of the point in the RS_{A6} when A, B and C rotations are zero (A is the rotation on the z axis of RS_{A6}, B is the rotation on the y-axis and C is the rotation on the x-axis).

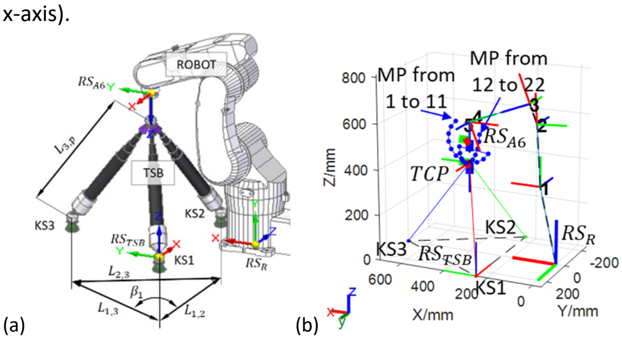

Figure 1. TSB and robot models main characteristics. Reference systems, linked to the robot: RS_R, joints reference systems from 1 to 5, RS_{A6} and the TCP. Reference systems and characteristics of the TSB: RS_{TSB}, KC, $L_{i,p}$ and $L_{i,j}$. (a) CAD model. (b) Mathematical model scheme. In the simulation tests, the model of the robot will reach each MP (blue dots) and the TSB will measure the coordinates of each MP.

3. Simulation test

The simulation environment begins with the robot and the TSB modelled as indicated in section 2.

The KSi and the TSB are placed in an equilateral triangle of 520 mm of side. The origin of the RS_{TSB} is materialised in the X_{KS1} as indicated in section 2.1. The coordinates of the origin of the RS_{TSB} (point $KS1$ and) expressed in the RS_R are indicated in Table 1. The RS_{TSB} is rotated -90° with respect to the z-axis of the RS_R. The coordinates of the centre of the balls placed on the KS that form the base of the TSB are indicated in Table 1.

Table 1 Coordinates of the ball centre in each kinematic support, KSi, with i from 1 to 3. The coordinates of the points are expressed in RS_{TSB} and in RS_R.

i	RS_{TSB}			RS_R		
	X_{KSi}	Y_{KSi}	Z_{KSi}	X_{KSi}	Y_{KSi}	Z_{KSi}
1	0	0	0	250	260	0
2	520	0	0	250	-260	0
3	260	450	0	700	0	0

The robot target positions reached during the measurement must be selected to avoid collision between the robot and the TSB but also to ensure a sufficient amplitude for TCP estimation. In this work, the robot flange (axis A6, RS_{A6}) is simulated at position (400, 0, 500 mm), in RS_R. The target positions are programmed so that the TCP moves along two trajectories of 11 points (MP) homogeneously distributed on each trajectory: an arc of 200° rotating around the x-axis of the $RS_{A6}$and another arc of 198° rotating around the y-axis of the RS_{A6}, Figure 3. In this way, the nominal data for the simulation are obtained.

The procedure to estimate the TCP described in subsection 2.2 will be affected by the robot PRE and the measurement uncertainty of the TSB. The effects of the robot PRE and the measurement uncertainty of each TB measuring lengths ($L_{i,p}$ and $L_{i,j}$) in the estimation of the TCP are simulated in order to estimate the uncertainty of the measurement of the TCP in RS_{A6} with the Monte Carlo Method [10,11].

A uniform distribution simulates the PRE of the robot. The robot reaches the points, affected by its PRE , forming a sphere around the nominal point, the radius of the sphere being the PRE of the robot according to ISO 9283 [14]. The uncertainty of each TB measuring lengths has been taken into account by introducing a noise in the nominal values of the lengths. The noise has a normal distribution with a standard deviation equal to the typical uncertainty of the TB, Figure 2. The noise is introduced randomly each iteration of the simulation in the nominal measurements of $L_{i,p}$ and $L_{i,j}$. The $L_{i,p}$ measurements are updated in each iteration based on the modification of the nominal positions of the robot introduced by the PRE of the robot.

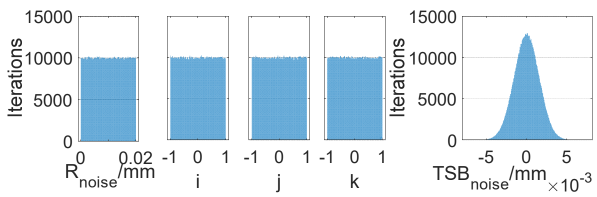

Figure 2. Input noise probability distributions, noise amplitude, R_{noise}, and orientation, i, j, and k, to simulate the PRE of the robot, and noise due to the measurement uncertainty of each TB measuring length, TB_{noise}. Values i, j, and k: director cosines for the orientation of R_{noise} in a Cartesian reference system. The probability distribution of R_{noise}, i, j, and k are those obtained for MP 1, the 22 MPs have similar probability distributions. TB_{noise} is the error probability distribution due to TB uncertainty for the measurement of $L_{1,2}$. The length measurements ($L_{i,p}$ and $L_{i,j}$) have similar probability distributions.

The number of iterations of the Monte Carlo simulation is an important parameter to obtain results that converge to the same solution. Previous works have shown the convergence of the Monte Carlo simulation using a value of 10^6 iterations [12].

4. Simulation results

The simulation tests follow the methodology described in the previous sections. Generating the 22 target points of the robot using an inverse kinematic algorithm, took 40s with a 2.3 GHz CPU clock speed. The 10^6 iterations to apply the Monte Carlo Method took 11 minutes with the same CPU.

The measurement uncertainty for the TSB measuring the coordinates of the robot target points has been estimated. The uncertainty variation on the measurement of the points within the two trajectories is shown in Figure 3. The maximum value for the measurement uncertainty of the target points in x and y axes is 5.7μm and 2.4 μm for z-axis. All the uncertainty values presented in this section have been calculated using a confidence level of two, k=2, according with [10].

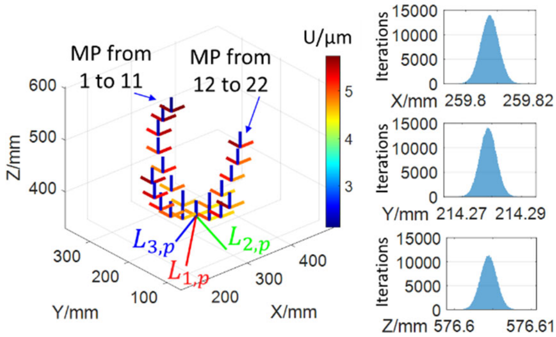

Figure 3. TSB measurement uncertainty in the MP locations (22 points, U_x, U_y and U_z /mm) and probability distribution of the MP coordinates measured by the TSB for the 10^6 iterations of MP 1. The 22 MPs have similar probability distributions.

The measurement of the z-coordinate presents lower uncertainty values when MP increases its z-coordinate, while the measurement uncertainty for x and y-coordinates follows the opposite trend: it decreases when z decreases (Figure 3).

From these MPs in each iteration, the TCP position in the RS_{A6} is obtained. In this process, in addition to the measurement uncertainty of the TSB, the PRE of the robot is taken into account, which will reach a position slightly different from the nominal target position in each iteration, always depending on the PRE of the robot.

The PRE of the robot and the measurement uncertainty of the TSB cause the coordinates measured by the TSB to present a probability distribution around the nominal values. Figure 3 shows the probability distribution of the coordinates measured for the first MP. The 22 MPs have similar probability distributions.

The first 11 MPs allow the estimation of the direction of the RS_{A6} x-axis expressed in the RS_{TSB}. With the remaining 22 points (second group of 11 points) the orientation of the RS_{A6} y-axis in the RS_{TSB} is estimated. The uncertainty of the measurement process of the RS_{A6} axes orientation expressed in the RS_{TSB} is expressed as the variation of the angles projected in the XY and XZ planes for the x-axis (α_{XY} y α_{XZ}) and in the YX and YZ planes for the y-axis (α_{YX} y α_{yZ}), Figure 4.

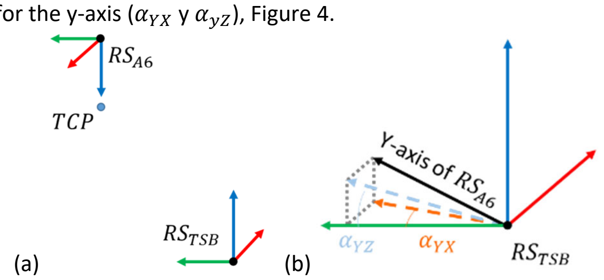

Figure 4. (a) Relative orientation between RS_{TSB} and RS_{A6} including the TCP. (b) Y-axis of RS_{A6} measured in RS_{TSB} and α_{YX} y α_{yz} angles.

The maximum uncertainty obtained in the simulation of the measurement of the RS_{A6} orientation is 123 μrad (25 s), Table 2.

Table 2. Measurement uncertainty for α_{XY}, α_{XZ}, α_{YX} y α_{YZ} angles in µrad.

RS_{A6} x-axis measurement		RS_{A6} y-axis measurement	
$U\alpha_{XY}/\mu rad$	$U\alpha_{XZ}/\mu rad$	$U\alpha_{YX}/\mu rad$	$U\alpha_{YZ}/\mu rad$
79	87	59	123

With all the MP (22 points) an estimation of the TCP (centre of a sphere) is obtained in each iteration, the results obtained present the probability distributions shown in Figure 5 for each coordinate. The measurement uncertainty of the TCP coordinates of the robot with the TSB is U_x = 8.2 µm, U_y = 11.1 µm y U_z = 5.1 µm.

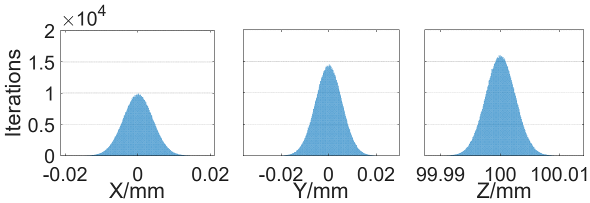

Figure 5. Probability distribution of the TCP XYZ coordinates measured by the TSB for the 10^6 iterations (coordinates in RS_{A6}).

The programmed simulation tool allows estimating the uncertainty for different cases, so the result is estimated for the case of using an ideal robot with PRE equal to zero). Even though it is a case with an ideal manufacturing system, in this case with a robot, this simulation allows establishing the value of the measurement uncertainty of the TSB by measuring the TCP, decoupling the influence of the robot's PRE. In this case, the measurement uncertainty of the TCP coordinates of the robot with the TSB results in: U_x = 2.8 µm, U_y = 3.7 µm y U_z = 1.6 µm. This result allows establishing the minimum uncertainty achievable on the measurement of the TCP of a robot in the established position, by a simultaneous multiateration measurement system based on the measurement uncertainty of each TB.

5. Conclusions

The behaviour of the robot and the TSB have been simulated and the TSB measurement uncertainty measuring the TCP of the robot has been estimated. The mathematical model of the systems and a methodology for the simulation of the measurement process have been presented. A method has been presented for the estimation of the TCP from the measurement of its position when the robot reaches several target points. The simulation of the presented procedure consider the PRE of the robot as an influencing factor on the measurement, estimating uncertainty values less than 12 µm for the coordinates. The simulation tool also estimates the measurement uncertainty on the measurement of the orientation of the reference system were the TCP is mounted (RS_{A6}), being 123 µrad in the worst case.

The simulation tool can evaluate cases that can show the limitations of use of the system depending on the application. The case where the PRE of the robot is zero (ideal case) has been simulated to decouple the influence of the robot's PRE on the measurement uncertainty of the TSB obtaining uncertainty values of 3.7 µm in the worst case.

As future work, The simulation tool needs to be experimentally validated by performing the TCP measurement of a robot of known PRE with a TSB of also known measurement uncertainty and evaluating the measurement uncertainty obtained for the TCP measurement. Once validated, the tool could be used to evaluate the effect on the measurement results when the angle range and the number of points used for the trajectories changes. It is also worth evaluating the effect of the size and position of the TCP (sphere drawn with the MPs) in the uncertainty results. Finally, implement the methodology to the MPs in the TCP measurement process to optimize the selection of the points depending on the measurement uncertainty.

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