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# Practicality and applications of body and face diagonal tests on machine tools

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**Abstract**

International standard ISO 230-6:2002 and American standard ASME B5.54-2005 (R2020) address body and face diagonal tests on machine tools and machining centres, respectively. ISO machine-specific standards do not include tests for checking machine performance when moving along diagonal trajectories across the full stroke of the axes. Consequently, there is no specified tolerance for these diagonal tests. This raises the need to investigate the applicability of these tests. ISO 230-6 highlights several potential uses for diagonal tests including "estimation of volumetric performance", "acceptance tests" and "health check or reassurance" of machine tools. It also mentions that the results of face diagonal tests can be used to derive the mutual squareness of linear axes of the machine under test. This paper explores the potential applications of diagonal tests alongside their practicality in an industrial environment. Approximately 600 tests were conducted on a vertical machining centre with the kinematic chain of [w X' Y' b Z (C) t] at various trajectories with different angles within the machine's working volume. Additionally, this paper explains how to select appropriate diagonal trajectory angles using Pythagorean triples and quadruples for ease of machine programming, convenient set up of laser software, and the elimination of rounding errors caused by NC algorithms or the machine's encoders. Experimental results are compared with predicted outputs obtained by simulations based on the Homogeneous Transformation Matrices (HTM) method which is a robust technique for modelling volumetric errors of machine tools under rigid body assumptions.

Machine tools volumetric performance, ISO standards, diagonal tests, body diagonal straightness, body diagonal positioning, face diagonal straightness, face diagonal positioning, Pythagorean triples, Pythagorean quadruples, Type A uncertainty, non-rigid behaviour of machine tools

## 1. Introduction

ISO 230-6:2002 [1] and American standard ASME B5.54-2005 [2] recommend diagonal positioning tests to evaluate planar and volumetric performance of machine tools. Wang and Liotto [3] and Chapman [4] had opposing views on the usage of diagonal tests. Wang aimed to use step body diagonal tests to identify machine error motions and for compensation purposes, whereas Chapman emphasized the limitation of the diagonal tests by presenting an example to argue that diagonal test results alone are insufficient for evaluating machine tool accuracy. By introducing intentional errors into the controller during diagonal tests, Svoboda [5] challenged and ultimately rejected Wang's method by comparing the obtained experimental results. In addition to diagonal positioning errors, ISO 230-1:2012 [6] addresses straightness of linear trajectories constructed by interpolation of multiple linear axes but does not specify the directions of the two straightness errors. Dashtizadeh et al. [7] defined direction and positive sign of both body diagonal straightness errors. They also investigated face diagonal straightness errors through some experiments and comparing the results with HTM simulations. Furthermore, Dashtizadeh et al. [8] demonstrated the advantage of measuring diagonal straightness in characterising the volumetric performance of machine tools by conducting statistical simulations and providing a conceptual analysis of the necessity of straightness measurements alongside diagonal positioning.

This research explores the practicality of diagonal tests, with a particular focus on straightness measurements. It also examines the limitations of mathematical error models, specifically the Homogeneous Transformation Matrices (HTM) method, in predicting the volumetric performance of machine tools. It argues that diagonal tests provide a direct evaluation of volumetric performance despite their limitations in selecting multiple trajectories. Moreover, the potential applications of the diagonal tests are discussed.

## 2. Utilising Pythagorean triples in face diagonal test

Neither ISO 230-6 nor ASME B5.54 provide specific guidance on choosing start and end points or target positions for face and body diagonal tests. A practical approach for face diagonal tests is using Pythagorean triples, ensuring all integer values along two linear axes and the diagonal itself. This simplifies machine programming and laser software setup, while avoiding interpolation errors due to decimal values. However, this method often does not align exactly with the face diagonal. Instead, the test can be performed along a close trajectory, yielding similar results due to nearly identical axis travel.

**Figure *1*** (left) illustrates a face diagonal (FD) in the XY plane with maximum X and Y-axis travel of $L_1$ and $L_2$, respectively. The angle $\alpha$ is the angle between the positive direction of the primary axis (X) and positive direction of FD. Smaller $\alpha$ requires larger X travel with smaller Y travel. To apply Pythagorean triples, all $L_1$, $L_2$ and FD must be integers. As this is often impractical over the full axis stroke, partial strokes are used, offering many possible combinations.

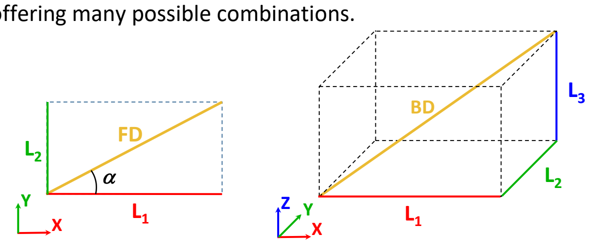


**Figure 1. Face (left) and body (right) diagonal designations using Pythagorean triples and quadruples**

For a machine with 1000 mm travel on both X and Y axes, Euclid's formula generates over 220 primitive Pythagorean triples. **Figure 2** plots the endpoints of these trajectories, assuming an initial position at (0,0). By shifting the start point to ($X_0$, $Y_0$), the endpoints translate accordingly. Connecting each endpoint to (0,0) defines unique test trajectory. When X and Y travel are equal, ISO 230-6:2002 suggests testing along the right-angle bisector. Selecting the closest Pythagorean triple to this bisector ensures compliance.

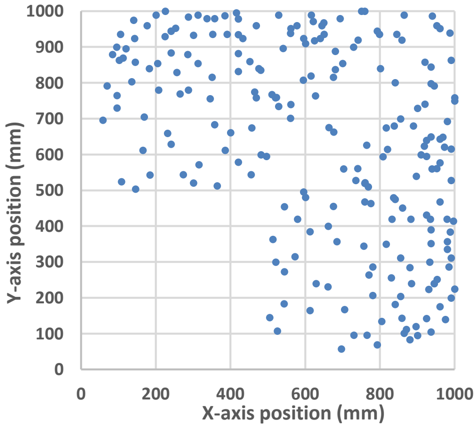


**Figure 2. Endpoints of face diagonal trajectories for linear axes upto 1000 mm**

Dashtizadeh et al [7] conducted 18 sets of face diagonal tests at 9 different angles along PP and PN trajectories. Except angles 3 and 87, the rest of 14 trajectories were carried out using Pythagorean triples in XY planes. **Table 1** summarises these triples. Their results show that the largest positioning error, $E_{DD}$, does not necessarily occur along the main face diagonal. Therefore, for machine planar performance evaluation, other triples may may lead to larger errors. However, for periodic health checks, repeating tests along the same trajectory suffices. Choosing a Pythagorean triple with higher error can enhance detection of machine changes due their higher sensitivity.

**Table 1. Pythagorean triples and axes travels used for face diagonal experiments on VMC Cincinnati Arrow500**

| Nr. | Pythagorean triple | $L_1$ (mm) | $L_2$ (mm) | FD (mm) | α (deg) |
|---|---|---|---|---|---|
| 1 | 60-11-61 | 480 | 88 | 488 | 10.39 |
| 2 | 77-36-85 | 462 | 216 | 510 | 25.06 |
| 3 | 55-48-73 | 495 | 432 | 657 | 41.11 |
| 4 | 20-21-29 | 480 | 504 | 696 | 46.40 |
| 5 | 3-4-5 | 360 | 480 | 600 | 53.13 |
| 6 | 36-77-85 | 216 | 462 | 510 | 64.94 |
| 7 | 11-60-61 | 88 | 480 | 488 | 79.61 |

## 3. Utilising Pythagorean quadruples in body diagonal test

Body diagonal tests involve more complex setup and trajectory selection. A study of primitive Pythagorean quadruples under 1000 reveals tens of thousands of integer combinations, making 3D visualisation and compilation of a complete list challenging. However, the abundance of quadruples in any 3D space allows for easy selection of suitable test trajectories. **Figure 1** (right) defines terminology for Pythagorean quadruples within a machine tool's working volume, where the X, Y, and Z axes have travels of $L_1$, $L_2$, and $L_3$, respectively. For a machining centre with 510 mm X/Y travel and 465 mm Z travel used in this research, six quadruples were selected for experimental tests along PPP (NNN) and NPP (PNN) directions to cover the maximum reachable volume of this VMC. **Table 2** summarises these quadruples and the used travels.

**Table 2. Pythagorean quadruples and axes travels used for body diagonal experiments on VMC Cincinnati Arrow500**

| Nr. | Pyth. quadruple | $L_1$ | $L_2$ | $L_3$ | BD |
|---|---|---|---|---|---|
| 1 | 15-16-12-25 | 360 | 384 | 288 | 600 |
| 2 | 16-15-12-25 | 384 | 360 | 288 | 600 |
| 3 | 14-22-7-27 | 294 | 462 | 147 | 567 |
| 4 | 22-14-7-27 | 462 | 294 | 147 | 567 |
| 5 | 6-9-2-11 | 330 | 495 | 110 | 605 |
| 6 | 9-6-2-11 | 495 | 330 | 110 | 605 |

## 4. Terminologies and designations

The terminologies and designations in this research are identical with those defined by Dashtizadeh et al. [7]. One diagonal positioning error and two diagonal straightness errors are measured along the specified body diagonal trajectories. **Figure 3** illustrates the body diagonal positioning deviation, $e_{DD}$, and the two body diagonal straightness deviations, $e_{S1D}$ and $e_{S2D}$, at an arbitrary target position on the main body diagonal of a machine tool. This trajectory follows all positive axis directions (PPP) when the machine moves from the lower-left to the upper-right corner of its working volume. Conversely, when moving from the upper-right to the lower-left corner, all axes follow negative directions (NNN).

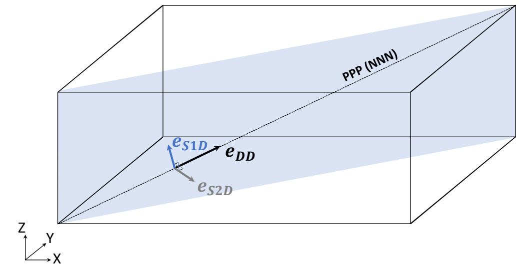


**Figure 3. Direction and sign of diagonal deviations [7]**

## 5. Volumetric error modelling

This research employs the HTM method to model machine tool volumetric errors, assuming rigid body behaviour. Donmez et al. [9] and Okafor and Ertekin [10] applied this method to 2-axis and 3-axis machines, respectively, while Dashtizadeh et al. [11] used it to statistically analyse probable volumetric errors in machining centres conforming to ISO 10791 tolerances.

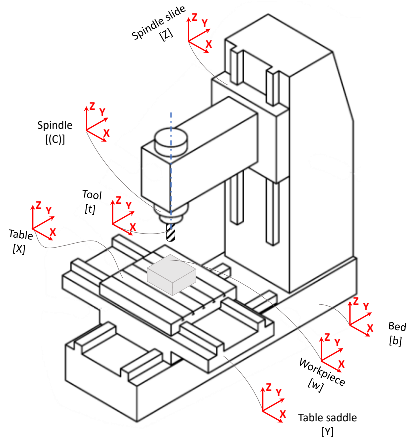


**Figure 4. VMC with kinematic chain of [w X' Y' b Z (C) t] and its coordinate frames (modified from ISO 10791-2 [12])**

For this study, a 3-axis vertical machining centre with the kinematic chain of [w X' Y' b Z (C) t] as depicted in **Figure 4** was modelled. A MATLAB code was developed to compute volumetric deviations across the working volume of this machining centre. By interpolating the HTM-derived volumetric deviations along the body diagonal trajectory, $e_{DD}$, $e_{S1D}$, and $e_{S2D}$ can be determined at all target positions. These values enable the calculation of body diagonal errors , $E_{DD}$, $E_{S1D}$, and $E_{S2D}$, over the full body diagonal travel, BD.

## 6. Experiments

12 sets of body diagonal tests were carried out on Cincinnati Arrow 500 3-axis VMC with the identical kinematic chain shown in **Figure 4**, using the Pythagorean quadruples listed in **Table *2***. A Renishaw XM-60 laser system was used for all measurements. **Figure *5*** shows the setup for a test along the 16-15-12-25 body diagonal trajectory. This setup allows direct measurement of the three diagonal deviations, $e_{DD}$, $e_{S1D}$, and $e_{S2D}$ along any body diagonal trajectory.

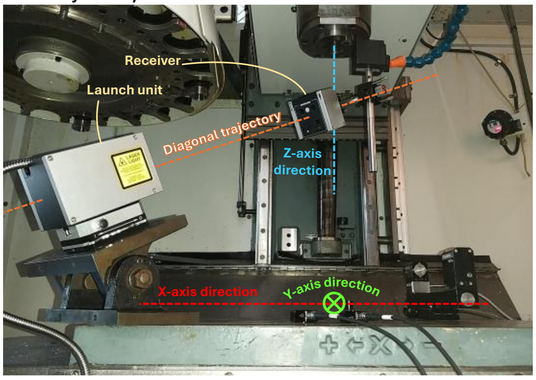


**Figure 5. Experimental setup with Renishaw XM-60 laser system for body diagonal test on a VMC, Pythagorean quadruple: 16-15-12-25**

Laser alignment along the body diagonal was achieved using a simple but rigid mechanical angle plate. The launch unit was mounted on this plate, positioned on the machine's table, while the laser receiver was aligned with two dial gauge arms. No specialised fixtures or manipulators were required beyond the standard Renishaw XM-60 kit.

The Renishaw software CARTO 4.12 does not support diagonal tests with the appropriate notation by default. However, by defining the diagonal trajectory as a linear axis, e.g., X-axis, the test can be implemented. The correct positive directions for $e_{S1D}$ and $e_{S2D}$ must be checked manually. Once confirmed for one trajectory, the same convention can be applied to others, eliminating the need for repeated adjustments.

## 7. Results of the tests and simulations

**Figure *6*** presents positioning deviations, $e_{DD}$, predicted by HTM and those directly measured by the laser system along a body diagonal trajectory defined by Pythagorean quadruple 6-9-2-11. Additionally, this figure includes the Type A uncertainty for both HTM predictions and the laser measurements. The uncertainty band for the direct laser measurements is derived from the repeatability of the results at each target position. Specifically, the upper envelope represents +2S of the 10 data points (5 bidirectional runs) recorded at each target position, while the lower envelope corresponds to −2S of these values.

The uncertainty envelope derived from the HTM equations is obtained using a guided Monte Carlo uncertainty approach. To estimate this uncertainty, only the boundary values of all the 21 error motions involved in the HTM equations were used. For instance, instead of randomly choosing a value for $e_{XX}$ (positioning deviation of X-axis at a given target position) within $e_{XX}$±2S$e_{XX}$, only the extreme values were used in the equations. This approach allowed for the calculation of the maximum and minimum diagonal deviations. In this analysis, 5000 permutations were used to estimate the uncertainty. Increasing the number of permutations to 100,000 with incorporating extreme values of $e_{NN}$±2S$e_{NN}$ (where $e_{NN}$ represents any of the machine's error motions) did not reveal significant changes in the results. Therefore, 5000 permutations were considered sufficient for the guided Monte Carlo simulation used to estimate the uncertainty band for all remaining trajectories.

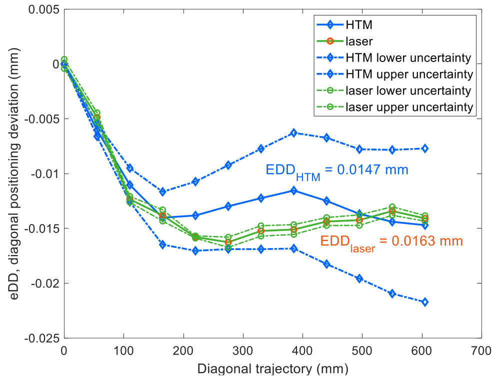


**Figure 6. Body diagonal positioning deviations (laser and HTM) with Type A uncertainty along Pythagorean quadruple 6-9-2-11 (PPP)**

**Figure *7*** and **Figure 8** show the body diagonal straightness 1 (in vertical plane) and straightness 2 for the same trajectory, respectively.

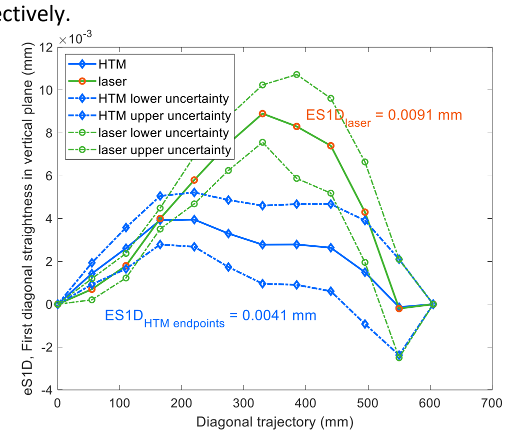


**Figure 7. Body diagonal straightness 1 deviations (laser and HTM) with Type A uncertainty along Pythagorean quadruple 6-9-2-11 (PPP)**

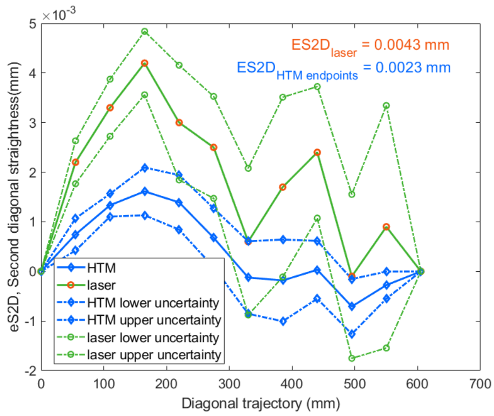


**Figure 8. Body diagonal straightness 2 deviations (laser and HTM) with Type A uncertainty along Pythagorean quadruple 6-9-2-11 (PPP)**

Type A uncertainty was estimated for all target positions where experimental data were collected as shown in **Figure *6*** to **Figure *8***. For each body diagonal trajectory, the reported uncertainty represents the maximum value across all the targets. The experiments were carried out along 12 different trajectories, half of them with PPP combination of motions and half of them with NPP. ***Figure 9*** shows the direct laser measurements for $E_{DD}$, $E_{S1D}$ and $E_{S2D}$, for these trajectories, along with the maximum Type A uncertainty over the full travel for each diagonal error. ISO 230-6 recommends conducting the tests along main body diagonals. Although the trajectories with Pythagorean quadruples 15-16-12-25 and 16-15-12-25 are close to the main diagonal of the machine, some other trajectories exhibit larger diagonal positioning error, $E_{DD}$. Therefore, testing along multiple trajectories can reveal larger deviations than those found only along the main diagonal. In terms of sensitivity of diagonal errors, the results show that in some trajectories, one of the diagonal straightness errors exceeds the positioning error. Therefore, measuring both diagonal straightness and positioning deviations provides a more comprehensive understanding of a machine's volumetric errors.

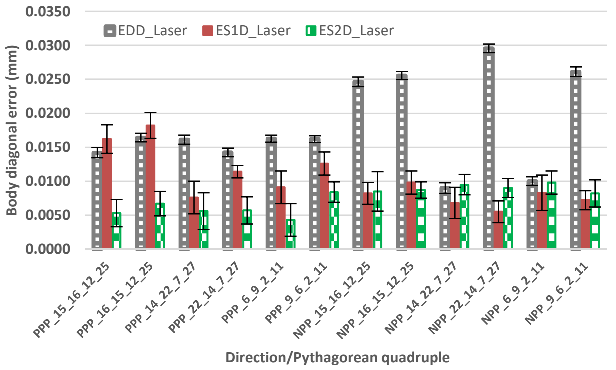


**Figure 9. Laser results for body diagonal errors ($E_{DD}$, $E_{S1D}$, and $E_{S2D}$) with Type A uncertainty for all trajectories**

## 8. Discussion on the application of diagonal tests

As shown in **Figure 6** to **Figure 8**, HTM does not always perfectly predict the machine's performance. Since HTM output is based on rigid body theory, deviations may suggest that the machine does not exhibit fully rigid behaviour along certain trajectories. Furthermore, the machine may demonstrates a higher degree of rigidity along some trajectories but deviates from this behaviour along others. Therefore, it is a good practice to conduct diagonal tests at multiple angles. While all machine tools display some level of non-rigidity depending on their design and fabrication, HTM can still serve as a general and efficient predictor of volumetric performance. However, for a more accurate evaluation of volumetric performance, direct measurement of diagonal positioning deviations alongside diagonal straightness deviations provides not only a clearer picture of machine volumetric performance but also insights into the machine's non-rigid behaviour. In other words, body and face diagonal test results can be used to assess non-rigid body behaviour in machine tools within a fairly short timeframe. This capability makes them a useful tool not only for research studies but also for specific industrial applications. Since testing along all possible Pythagorean triples/quadruples is impractical, mathematical models like HTM efficiently estimate volumetric performance across the entire working volume.

Direct measurement of diagonal errors using laser systems can be conducted with smaller uncertainty compared to the mathematical models such as HTM where the uncertaintaties of error motions measured individually on each machine axis propagate to the final estimation of deviations. Moreover, diagonal measurements can serve as a valuable index for verifying whether the compensation of the machine's linear axes has been correctly implemented.

A common industrial health check method is the circular test using a ballbar, which is fast and informative for diagnostics. Comparing the results obtained over time can give a powerful tool for prognosis purposes too. However, since circular tests typically cover a short axis travel (e.g., 300 mm), they do not fully reveal a machine's planar/volumetric performance, particularly in large machines. In contrast, face and body diagonal tests efficiently capture cumulative error effects over full axis travels in under 30 minutes per test, as examined during more than 1000 conducted face and diagonal tests. For a more precise health check, diagonal tests should be performed at consistent X, Y, and Z positions and directions.

## 9. Summary and conclusion

This paper evaluated volumetric performance of a VMC using direct laser measurements along different 12 body diagonal trajectories. Additionally, it compared the HTM modelling outputs with the laser measurement along with their Type A uncertainty. Compared to mathematical predictions based on measurements of all error motions at one snapshot, direct diagonal measurements with less uncertainty can give insight on volumetric performance of any machine tool, especially for machines affected by non-rigidity and thermal changes. For effective health check of an individual machine tool, repeating diagonal tests over time using the same Pythagorean triple/quadruple and identical start and end points in machine X, Y, Z coordinates is recommended. In the near future, the quantification of non-rigidity and thermal changes in machine tools with different kinematic chains based on body diagonal test results will be addressed.

## Acknowledgement

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