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Derivation of experimental coefficients for cutting equations in orthogonal cutting

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Abstract
This study derives a cutting experiment formula that accurately determines the shear angle Ø from cutting conditions and the hardness of the workpiece. The experimental formula derived in this study is an exponential function with fractional terms. Orthogonal cutting experiments were conducted using carbon steel with varying hardness owing to heat treatment, and the constant terms of the formula were derived. When the experimental values were presented in a logarithmic graph, it was clear that the shear angle exhibits a linear relationship with the product of the cutting speed and depth of cut, as well as with the hardness of the workpiece. Using these results, the constant terms of the formula were calculated, and an experimental formula was derived to determine the shear angle based on the cutting conditions and workpiece hardness.

Orthogonal cutting, cutting theory, cutting equation, shear surface theory

1. Introduction

In the field of cutting, numerous machining experiments are required to optimize cutting conditions and improve productivity. Therefore, high-precision simulations using numerical analysis without requiring cutting experiments are in demand. Therefore, various studies on FEM analysis technology for cutting processing have been conducted. However, the FEM analysis of cutting processing requires a substantial number of experiments to determine the flow stress equation for each material, and various issues such as the low accuracy of the obtained analysis results, are prevalent. For cutting processing, the formulation of machining phenomena in orthogonal models has been attempted for a long time, and various cutting equations have been proposed [1]. The derivation of the shear angle is emphasized in the cutting equations, primarily because once the shear angle is determined, the cutting resistance, friction coefficient, stress, energy, and cutting temperature can be calculated. However, the cutting equations proposed thus far only qualitatively match the experimental values, but not quantitatively [2]. This is because the shear plane theory oversimplifies the cutting model. In addition, several formulas derived from cutting experiments have been proposed. Araki et al. and Nakayama et al. reported that the shear angle Ø can be expressed by an exponential function of the cutting speed and depth of cut [3]. The experimental formulae represented by these equations enable high-precision calculations of the cutting forces if the material parameters are accurately determined through precise orthogonal cutting experiments. Yamane et al. studied the correlation between tensile strength and material hardness and proposed an experimental formula with high practicality and versatility by replacing the tensile strength of the workpiece used as a material parameter in the experimental formula with hardness [3]. Therefore, this study attempts to derive the material parameters in the experimental formula reported by Yamane et al. using a newly developed orthogonal cutting machine. By employing this approach, the influence of workpiece hardness and cutting conditions on the shear angle was investigated, and an attempt was made to formulate a cutting process with high practicality and low error.

2. Experimental Equipment and Methods

In this study, the desired exponential function was set as Equation (1), and the purpose was to experimentally determine the coefficients n, ν, and standard values $\emptyset_s$, HV_s, and Vh_s of the exponential experimental formula. The fractional part of the exponent in the equation aligns with the dimensions of the experimental equation. The standard values $\emptyset_s$, HV_s, and Vh_s are the values that best match the experimental results with the experimental formula. In addition, standard values have the advantage of easy understanding of the experimental conditions used to derive the experimental formula. The parameters n andν in the equation indicate the slopes in the logarithmic graph of Ø and HV or vh.

$$\varphi = \varphi_s \left(\frac{HV}{HV_s}\right)^n \left(\frac{Vh}{Vh_s}\right)^\nu \quad (1)$$

The orthogonal cutting machine shown in figure 1 was used to conduct the orthogonal cutting experiments. A non-coated carbide K-type tool was used as the cutting tool. Carbon steel C50 (ICO) was used as the workpiece, and the Vickers hardness HV of the workpiece in Equation (1) was varied by changing the heat treatment method. Four types of heat treatments were used: normalization (HV201), quenching and tempering (HV333), Spheroidizing (HV154), raw material (HV282), and spheroidizing annealing. To vary the product Vh in Equation (1), orthogonal cutting experiments were conducted by varying the cutting speed and depth of cut, as listed in Table 1. The cutting conditions were set such that the built-up edge did not affect the experimental values. The chip thickness h_c discharged by the cutting experiment was measured using a micrometer, and the shear angle Ø was calculated using Equation (2) from the depth of cut h and rake angle γ. The cutting resistance was measured using a cutting dynamometer (Kistler Type9629AA).

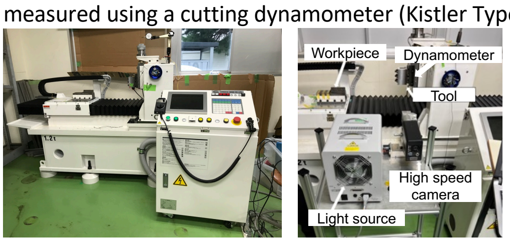

Figure 1. Orthogonal cutting machine

$$\tan\varphi = \frac{(h/h_c)\cos\gamma}{1-(h/h_c)\sin\gamma} \quad (2)$$

3. Results and discussion

The relationship between the product Vh of cutting speed and depth of cut and the shear angle $\emptyset$ was summarized in a logarithmic graph for each heat treatment, as shown in Figure 2. Evidently, the experimental results align along a straight line on the logarithmic graph for any heat-treated workpiece. Additionally, the slope shown in the graph does not show significant differences owing to heat treatment differences. Therefore, the value of the exponent v in the fractional term of Vh in Equation (1) was set to the average value of the slopes, 0.194. Although differences in the intercept of the straight line are observed in the graph, this can be attributed to the influence of workpiece hardness, which will be determined in the next section, and does not pose a problem for determining the exponent n. Figure 3 depicts the relationship between the hardness HV of the workpiece and the shear angle at cut depths of 0.1 mm and 0.14 mm in a logarithmic graph. A linear relationship can be observed between the hardness HV and the shear angle under any condition. In addition, the value of the raw material deviated significantly from that of the others. Therefore, the value of the raw material was excluded as an outlier, and the average slope of the experimental results for the remaining conditions (0.432) was set as the value of the exponent n in Equation (1). Subsequently, the experimental constants $\emptyset_s$, HV_s, and Vh_s in Equation (1) were set to HV_s=333, $\emptyset_s$=22.9, and Vh_s=0.0150, as the experimental values and the approximate straight line matched well in the graph. The reason for the significantly different results for the raw material compared to the other materials was examined. The hardness test results for each workpiece are shown in Figure 4. It can be observed that the raw material has a larger variation than the other materials. Furthermore, Figure 5 presents the results of the hardness test conducted in the cutting direction of the test piece, and it can be observed that the hardness varies in the cutting direction. Thus, it is believed that the experimental results for the raw material showed a different trend from those for the other materials because the hardness of the workpiece was not constant.

4. Conclusion

In this study, orthogonal cutting was conducted using carbon steel with varying hardness due to heat treatment, and an experimental formula that could accurately determine the shear angle from the cutting conditions and workpiece hardness was derived. It was confirmed that the relationship between the product Vh of cutting speed and depth of cut and the shear angle $\emptyset$, as well as the relationship between hardness HV and the shear angle $\emptyset$, are linear on a logarithmic graph. Furthermore, the exponents *n, v* in the fractional terms of Vh and HV, and the standard values $\emptyset_s$, HV_s, and Vh_s were determined from orthogonal cutting experiments, and the following exponential experimental formula was derived to determine the shear angle $\emptyset$.

$$\varphi = 22.9\left(\frac{HV}{333}\right)^{0.432}\left(\frac{Vh}{0.0150}\right)^{0.194} \quad (3)$$

References

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Acknowledgement
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Table 1 Cutting conditions in orthogonal cutting

Work material	C50 (ISO)
Cutting speed V [m/min]	100, 125, 150
Depth of cut [mm]	2
Atmosphere	Dry
Cutting distance [mm]	100
Rake angle [deg]	0

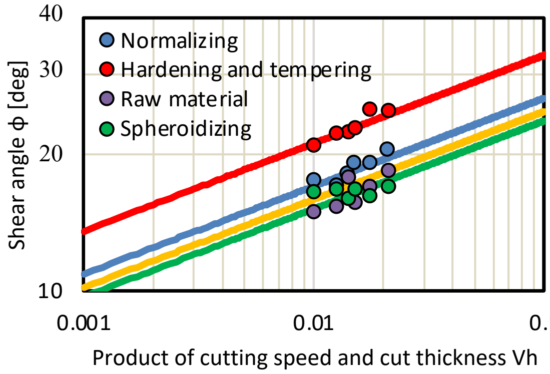

Figure 2. Relationship between shear angle $\emptyset$ and Vh

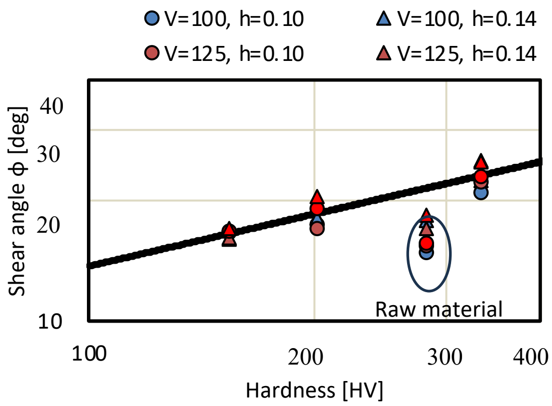

Figure 3. Relationship between shear angle $\emptyset$ and hardness HV

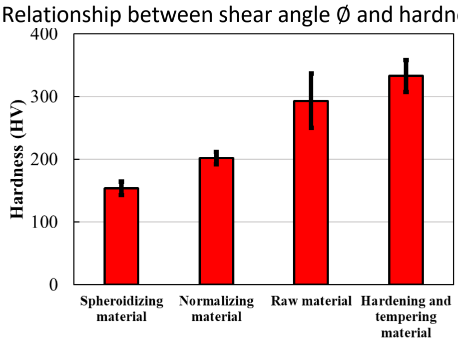

Figure 4. Hardness of different materials

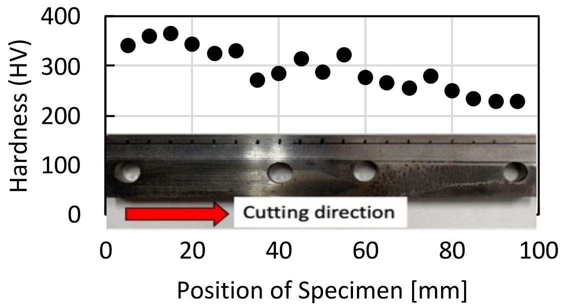

Figure 5. Distribution of hardness according to position of workpiece