euspen's 25th International Conference & Exhibition, Zaragoza, ES, June 2025
www.euspen.eu

Reduction of the bending-torsion-stiffness ratio of flexure hinges based on stress-strain state transition

Matthias Wolf, Mario André Torres Melgarejo, Martin Wittke, René Theska

Technische Universität Ilmenau, Department of Mechanical Engineering
Institute of Design and Precision Engineering, Precision Engineering Group

matthias.wolf@tu-ilmenau.de

Abstract

Their numerous advantages motivate the use of flexure hinges in high-precision motion systems with the highest requirements for precision and reproducibility. It is desirable to have low bending stiffness and high torsional stiffness to enable reproducible motion with the lowest actuating forces. A reduction in the bending stiffness of a flexure hinge can be achieved by reducing the thickness of the hinge. However, the reduction of the hinge thickness is limited by the manufacturing processes. A further reduction in bending stiffness can be achieved by reducing the width of the hinge. However, as the width has a significant influence on the torsional stiffness, width-to-thickness ratios of approximately 200 are usually aimed for. An investigation has shown that reducing the width-to-thickness ratio leads to a non-linear reduction in bending stiffness. This is due to the transition from plane strain state to plane stress state as a function of the hinge width-to-thickness ratio. Based on that, a novel flexure hinge was designed with separated sections in the lateral direction, thereby no longer exhibiting the characteristics of a plate but rather those of parallel bending beams. The effective bending width is kept constant to provide high torsional stiffness, but the width-to-thickness ratio of the individual beams is significantly reduced to benefit from the stress-strain state transition and reduce the bending stiffness. The investigation concludes that, dependent on the width-to-thickness ratio, a reduction in bending stiffness of up to 12% can be achieved while the torsional stiffness was increased by more than 270 %. This reduces the bending-torsion-stiffness ratio by about 75 %

Keywords: Flexure hinge, bending stiffness, bending-torsion-stiffness ratio, plane strain, plane stress, stress-strain state transition

1. Introduction

High-precision systems require mechanical components to allow motions with the highest demand on accuracy and reproducibility. Due to numerous advantages like negligible backlash and friction, negligible wear, and no need for lubrication, in precision engineering conventional rotational bearings and linear guidings are frequently replaced by flexures often in form of bending beam elements [1]. A single flexure hinge allows a predictable rotational movement for small angular deflections and is used in mechanisms to allow mostly planar movements. To improve the performance of high-precision systems like high-precision force sensors presented in [2] flexure hinges with very low bending stiffness are required to minimize actuation forces. However, to avoid parasitic out-of-plane motions low bending-torsion- stiffness ratios are aimed for.

2. State of the art

Several contributions have investigated the influence of the hinge geometry parameters and various notch contours on the bending-torsion-stiffness ratio [3]. The hinge thickness has a favorable effect on the bending-torsion-stiffness ratio, thus it has been reduced down to the technological limit of about 50 µm. A further reduction of the bending stiffness can be achieved by reducing the width of the hinge as this has a linear influence on the bending stiffness.

Therefore, this work contributes to the reduction in bending stiffness of flexure hinges by reducing their width. The study was carried out on joints with a rectangular cross-section.

3. Assumptions and limitations of the investigation

Flexure hinges have been modeled as prismatic bending beams with length *l*, width *w*, and thickness *t* as shown in Figure 2a. To ensure low bending-torsion-stiffness ratios length-to-thickness ratios of around 10…40 and width-to-thickness ratios of approximately 200 are usually aimed for.

This investigation considers a systematic reduction of the bending width *w* from several millimeters down to the manufacturing limit of about 80 µm. With the minimum hinge thickness of 50 µm width-to-thickness ratios from about one to 200 were considered. For all calculations, a material with elastic behavior is assumed and the hinge geometry parameters are: E = 69 GPa, t = 0.05 mm, l = 0.5 mm, $\nu = 0.33$. The investigation is based on an analytical and a 3D finite-element model.

3.1. Analytical bending beam model

According to commonly used width-to-thickness ratios of approximately 200, the plate theory proposed by Kirchhoffs-law [4] is valid. With the geometrical parameters *l*, *w* and *t*, the Young's modulus *E* and the Poisson's radio ν the bending stiffness $K_{\mathrm{b\_KB}}$ of Kirchhoffs bending beam equals:

$$K_{\mathrm{b\_KB}} = \frac{E \cdot w \cdot t^3}{12 \cdot l} \cdot \frac{1}{1-\nu^2} \qquad (1)$$

3.2. 3D finite-element model

In addition to the analytical model, a parametric 3D finite-element model was developed to validate the analytical results. The bending stiffness investigation has been implemented by Ansys Workbench. Therefore, the 3D models with different widths were deflected with an angle of φ_z = 0.1° around the zaxis as shown in Figure 1a, and the reaction moment M was observed. The bending stiffness $K_{\mathrm{b\_sim}}$ results in:

$$K_{\mathrm{b\_sim}} = \frac{M}{\varphi_{\mathrm{z}}} \tag{2}$$

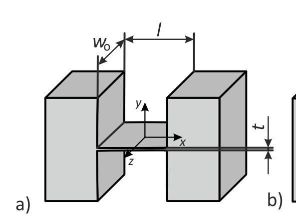

Figure 1. a) model of a rectangular flexure hinge; **b)** model of the splitted hinge design with two bending beams

3.3. Investigation results

The reduction of the width-to-thickness ratio leads to the obvious linear reduction in bending stiffness but also shows a difference between the analytical and the 3D finite-element results. The difference between the analytical and the simulated results depending on the beam width-to-thickness ratio is shown in the diagram in Figure 2.

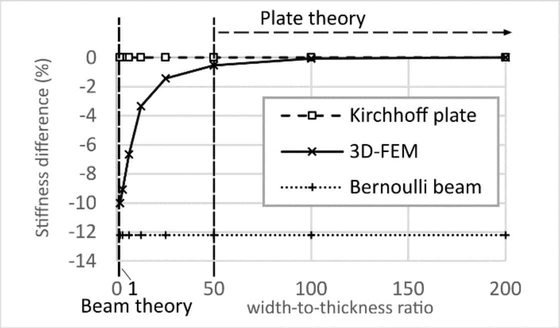

Figure 2. Bending stiffness difference to width-to-thickness ratio

The deviation results in the transition from plane strain state to plane stress state as a function of the hinge width-to-thickness ratio since the hinge behaves in width-to-thickness ratios around one as a beam instead of a thin plate. For the aspect ratio of one, the **beam theory from Euler/Bernoulli** [3] is valid and yields:

$$K_{\mathrm{b\_EBB}} = \frac{E \cdot w \cdot t^3}{12 \cdot l} \tag{2}$$

Consequently, the maximum stiffness deviation yields:

$$\Delta K_{\max} = \frac{1}{1 - \nu^2} \tag{3}$$

However, reducing the hinge width also significantly decreases the torsional stiffness.

4. Novel hinge design using stress-strain state transition

To avoid the decrease in torsional stiffness the advantageous stress-strain state transition is used in a novel design for a flexure hinge. For this purpose, a hinge was designed consisting of several parallel beam elements with low width-to-thickness ratios but in total, corresponds to the original bending width w_{o} as shown in Figure 1b. This approach is intended to reduce the bending stiffness by the material dependent stress-strain state transition factor ΔK from (3) while increasing the effective torsion loaded width w to provide high torsion stiffness.

4.1. Design and investigation parameters

The compact design is related to the manufacturing limits resulting from a high-precision etching process with a minimal beam width w_{B} of 0.08 mm and a slot width w_{slot} of about 0.12 mm, according to [5]. The hinge geometry is again based on the previously determined parameters. For the 3D finite-element analysis, the effective bending width w_{o} remains constant and the individual beam width-to-thickness ratio is modified by the number of beams from 1 to 128 to ensure width-to-thickness ratios from 200 down to about one. In addition to the bending stiffness Kz_φ_{z} and the torsional stiffness Kx_φ_{x} also the cross stiffness Kz_uz has been observed. For this purpose, a deflection u of about 1 µm was applied and the reaction force in the direction was observed.

4.2. Investigation results

The results of the 3D finite-element analysis are shown in the diagram in Figure 3. The bending stiffness Kz_φ_{z} could be reduced by around 12 % while at the same time the torsional stiffness Kx_φ_{x} were increased by approximated 270 % since the width w increase with the numbers of slots. However, the cross stiffness Kz_uz exhibits a high sensitivity in the area where the width-to-thickness ratios are of interest.

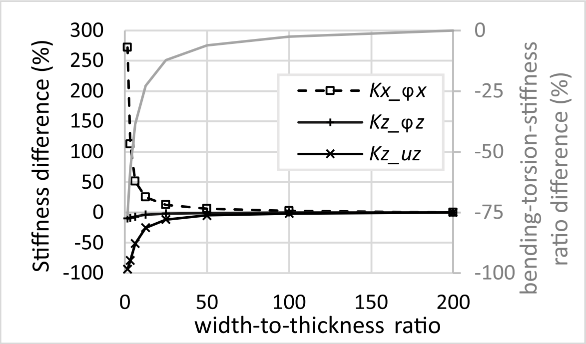

Figure 3. Results of the stiffness investigation

5. Conclusion

A novel flexure hinge was modelled to reduce the beding stiffness as an effect of the stress-strain state transition. The Investigation has shown that a further reduction in the bending stiffness is possible which depends on the width-to-thickness ratio. The novel compact flexure hinge design allows reducing the bending stiffness of up to 12 % while increasing torsional stiffness to 270 %. This reduces the bending-torsion-stiffness ratio by over 75 %. However, since stress-strain state transition also effects the cross stiffnesses the effect of the lower cross stiffness must be compensated by design modifications e.g. using other hinge orientation ore higher slot widths. In future work, the effect of the lower cross stiffness needs to be further investigated. In addition, the presented simulation results should be confirmed by experimental investigations.

Acknowlodgement

The authors gratefully acknowledge the support by the Deutsche Forschungsgemeinschaft (DFG) in the framework of Research Training Group “Tip- and laser-based 3D-Nanofabrication in extended macroscopic working areas” (GRK 2182) at the Technische Universiät Ilmenau, Germany.

References

[1] Krause W., 2018. Konstruktionselemente der Feinmechanik

[2] Wolf M., Torres M., Wittke M., Theska R., 2024. Orientation-dependent behavior of miniaturized compliant mechanism for high-precision force sensors. *Proceedings of the 24th International Conference EUSPEN Dublin*

[3] Linß S., Henning S., Zhao C., Erbe T., 2021. The bending-torsion-stiffness ratio of flexure hinges with common and polynomial notch shapes. *Duisburg-Essen Publications Online*

[4] Mittelstedt C., 2022. Flächentragwerke

[5] Herz Ätztechnik, https://www.aetztechnik-herz.de/