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Dynamic estimation of the point of interest based on sensor positions using an observer

Anna-Carina Kurth[1], Viviane Bauch[1], Martin Glück[1], Jakob Köhler-Baumann[1]

[1]*Carl Zeiss SMT GmbH, Oberkochen, Baden-Württemberg, Germany*

{anna-carina.kurth, viviane.bauch, martin.glueck, jakob.koehler-baumann}@zeiss.com

Abstract

For the precise positioning of a flexible object using control, a specific point of interest is usually used as a reference point. If the position of this point cannot be detected directly by sensors, it is reconstructed from the positions of the sensors using a rigid body assumption. However, as soon as the object deforms, this assumption is no longer correct, so that the control system calculates the input of the actuators based on an incorrect position. This can lead to unstable system behaviour and performance losses. In this contribution, an observer-based approach is proposed that dynamically estimates the point of interest based on the signals from the sensors and the control signal from the actuators. This allows the control system to obtain correct position information even at higher frequencies and hence leads to an increase in performance and robustness.

Position Estimation, Luenberger Observer, Flexible bodies

1. Introduction

In high-precision applications a very common task is the precise positioning of an object with a feedback controller. Due to the elasticity of the object the position of a special point of interest (POI) is controlled.

The POI position is measured by sensors at locations distributed over the object because of the use of highly sensitive unidirectional sensors. Hence, the system is not collocated, since the sensors and actuators are not at the same position [1,2]. The POI is reconstructed from the sensor positions by using a rigid body transformation, which leads to a collocated layout if the assumption of a rigid body holds true. As soon as first flexible resonances occur, the calculated POI does not match the actual POI. Therefore, the control cannot position the right POI and servo errors or even instability can occur [1,2].

The commonly used approach is to use a lowpass-filter or notch filters that turn off the control as soon as flexible resonances occur [2]. Hence, the bandwidth of the controller is limited by the flexible resonance frequencies. This contribution focuses on an alternative approach to dynamically estimate the POI even in frequency regions, where the object has flexible resonances. Thus, a controller with a higher bandwidth can be used and the positioning of the object as well as disturbance rejection is improved.

The dynamic estimation is based on a Luenberger observer, which is a basic observer suitable for systems without noise, but typically not used for virtual collocation. The POI is estimated by the observer using the sensor and actuator signals and a mechanical model. A model order reduction is needed to achieve real-time applicability.

The observer output is used as an input for the controller, which can be either a full state feedback or output feedback, which uses only part of the estimated positions namely the POI. The advantage of using the whole state for feedback instead of just the output is that the object can be positioned with respect to the POI by a reduced occurrence of flexible mode shapes.

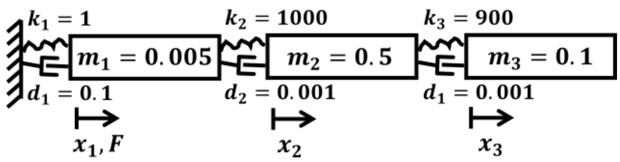

Figure 1. Considered system with three masses connected via springs and dampers and their respective values.

2. Problem statement

This section describes the basics needed to do model order reduction and design a Luenberger observer for the original as well as the reduced system to estimate the POI of a flexible object.

2.1. Modelling & analysis of the plant

An elastic object can be interpreted as an interconnection of several spring-mass-damper systems, as is also done in modelling using FEM. For proof of concept, a system with three masses is chosen in this contribution. The masses m_1, m_2, m_3 with their respective positions x_1, x_2, x_3 are coupled by springs with stiffness k_1, k_2, k_3 and dampers with damping d_1, d_2, d_3. The lowest mass m_1 is the POI, which is actuated and coupled to the fixed world. The third mass m_3 is measured by sensors, see Fig. 1. Therefore, the system is not collocated. The equations of motion result in the system dynamics Σ_y with the measured output $y \in \mathbb{R}^1$, which is the position of mass m_3 resp. Σ_z with the POI output $z \in \mathbb{R}^1$, which is the position of mass m_1

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= C_1 x(t) \\ z(t) &= C_2 x(t) \end{aligned} \tag{1}$$

Hereby, the states $x \in \mathbb{R}^6$ represent the positions and velocities of the masses and the input $u \in \mathbb{R}^1$ displays the force acting on mass m_1. The dimensions of the dynamic matrix A, the input matrix B and the output matrices C_1 and C_2 are accordingly.

If looking at system Σ_z, alternating resonances and antiresonances arise in the Bode plot. Hence, the phase of the input-output behaviour will always be between 0 degree and -180 degree. From a control point of view, this is particularly advantageous, since in the Nyquist diagram no further encirclement of the critical point can occur with any amount of amplification by a controller [1]. Therefore, the gain margin (GM) is infinity. However, this holds only true for academic examples without sensor or actuator dynamics or delay for example. If these parasitic effects are considered bounded phase shifts can occur, but not phase jumps like for the case with two resonances directly after each other.

If looking at the input-output behaviour of system Σ_y, three resonances appear directly after each other. Therefore, phase jumps occur, and control design is more challenging regarding robust stability. With the observer, designed in the next section, the system is virtually collocated by estimating the position of the POI, which can then be used for control. However, normally FEM are high dimensional and cannot be used in the observer. Hence, system Σ_y is reduced by balanced truncation resulting in system Σ_r. Hereby, states that contribute only slightly to the transmission behaviour are neglected. The dimension of the reduced system is then four, such that one resonance is missing in the reduced system. Differences between the reduced and the original model occur only high-frequent.

2.2. Luenberger Observer

A Luenberger observer estimates not measurable states of a system based on a mechanical model. The observer structure is simple and suitable for systems without or little noise. Two approaches are considered in this contribution: In the first approach system Σ_y is used in the observer as model resulting in overall system Σ_{full}, in the second approach the reduced model Σ_r is used as basis for the observer design resulting in combined system Σ_{red}.

If the observer uses system Σ_y as model, the separation theorem can be used to design observer and control. Therefore, the controller is designed based on system Σ_z or with the whole state x as feedback.

If the reduced model is used, the separation theorem is not valid anymore. However, the differences due to the reduction only occur at high frequencies. Therefore, either a low-pass filter can be used, or the observer can be designed especially robust to take the differences between plant and observer model into account. Hereby, the reduction method via balanced truncation is beneficial, since it delivers also an error bound. In this contribution the assumption is made that the separation theorem is also valid for the case with the reduced model.

The observer gain is designed with an LQR approach to get an optimal gain. The input to the observer is the input u as well as the output y. The output of the observer is either the estimated position of the POI z or the estimated state x, whose dimension is dependent on the choice of the model in the observer.

2.3. Control design

The goal of the control is the positioning of mass m_1 by actuating it but using the measurement of mass m_3.

For proof of concept a simple proportional controller with gain k_P is used for output-feedback, which is aggressively tuned. For the state feedback a LQR control approach is used to show the potential of the presented approach [2]. In case without observer output y is fed back. In case with observer the estimated position of the POI z is used for the output feedback and the estimated states x for state feedback. The interesting output is in all cases the position of the POI z.

3. Simulations

The open loop of system Σ_y from the control error to the measured output has a GM of 58.3 dB and a phase margin of 10.4°, while the open loop of the observer with the full system as model from the estimated control error of the POI to the estimated position of the POI has an infinitely high GM and a phase margin of 12.8° and with the reduced system from the same input to output has a GM of 60.2dB and phase margin of 12.8°. Increasing the proportional gain k_P leads to a linearly increasing bandwidth while linearly decreasing the gain margin. Therefore, with the same robustness criterions a higher bandwidth can be achieved by using an observer. However, it must be considered that the observer shall be faster than the controller, which limits performance.

For comparison an impulse output disturbance is applied. As performance criterion, the peak amplitude (PA), which is the maximum position of the POI z, and the transient time (T), which is when the position of the POI z is converged, are chosen, see Table 1. As proportional gains 90% of the values are chosen for which one of the systems is close to instability. If a bar appears in the table, the respective system is unstable. For the LQR approach no special tuning was done. The Q- and R-matrix are chosen as simple unity matrices.

The higher the gain, the less high is the PA for the observer with the full model. However, from a certain gain decreasing PA comes with an increasing T. The observer Σ_{red} performs better for lower gains, which is due to the mismatch of the plant Σ_r used in the observer and system Σ_y. The best performance can be achieved by a state-feedback. The PA as well as the T are low compared to the output feedback.

Table 1 PA | T (s) of the position of the POI.

System	$k_P = 740$		$k_P = 920$		$k_P = 2000$		LQR	
	PA	T	PA	T	PA	T	PA	T
Σ_y	18	221	-		-			
Σ_{red}	15	163	15	910	-		1	3
Σ_{full}	16	313	15	192	12	427	1	3

4. Conclusion

In this contribution an approach to estimate the POI of an elastic object based on sensor positions and the applied control force was presented. Firstly, the problem was stated, and the resulting system was analysed with a special focus on collocation. Afterwards, the model was reduced with respect to its dimension and the basics for an observer design were presented. With the introduced control design, simulations were carried out. By using an observer concept, the control performance is increased. Using a reduced model in the observer leads to slightly decreasing performance. However, if a state-feedback is chosen, which is enabled due to the estimation of the full state, a huge performance increase is achieved even though the controller was not tuned especially. This translates directly into a more precise positioning of the object and therefore into increased system performance. In future work, the results will be transferred to more realistic models and finally validated in experiments.

References

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