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A grouped random assembly method for precision and yield rate

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Abstract

Precision is one of the most important indicators of mechanical products. While product precision is mainly affected by geometric errors, this paper presents a method to improve assembly precision and estimate yield rate of the product by grouped random assembly (GRA). In this method, components are first sorted and grouped. Error analysis and genetic algorithm (GA) are combined to achieve the best assembly precision. An dual-axis rotary index table (DRIT) example, with 100 parts of each component, is used to illustrate this GRA method. The 100 parts are sorted and divided into 4 groups at the beginning. The combination of different sub-groups are re-grouped for minimum error stack-up by GA. The three-dimensional(3D) statistical errors of GRA are reduced by 19% to 52%, compared with assembly ungrouped assembly. When the number of GRA groups increase to 20 groups, the yield rate will reach 100%. The main contribution of this research is to develop the method which quickly and effectively allocates the best GRA grouping, with minimum 3D assembly errors. It also calculate the yield rate before the parts are sent to assembly line. This method provides a solution for the best GRA grouping and production planning of precision products.

Keywords: grouped random assembly (GRA), assembly precision, error stack-up analysis, genetic algorithm (GA).

1. Introduction

The precision of a product in an assembly is mainly affected by geometric errors of components that are generated from variations in fabrication process. Although product precision can be evaluated by tolerance analysis in design, final precision is determined in the assembly process when components are fabricated. While it is time-consuming and not cost-effective to achieve product precision by selective assembly or rework, it is an crucial issue to develop a systematic method that ensures good assembly precision with good quality.

Error stack-up analysis was mainly based on tolerance analysis in single degree-of-freedom (DOF) dimensional chain in the early literature. Various methods, including tolerance chart and dimensional chain, have proposed for dimensional error analysis [1,2]. Error analysis employed in this paper is based on the error model proposed by Tsai[3]. This model represents an ideal geometric constraint *i* as a homogeneous transformation matrix (HTM) $\mathbf{T}_{\text{ideal}}(i)$. Geometric error caused by the geometric variation (tolerance) associated with this constraint is represented as a differential HTM $\Delta\mathbf{T}(i)$. Thus, the real geometric constraint is calculated as $\mathbf{T}_{\text{real}}(i) = \mathbf{T}_{\text{ideal}}(i)\cdot\Delta\mathbf{T}(i)$ considering the influence of the tolerance[3]. When components with geometric tolerances are assembled, tolerances will be stacked up thus the accumulated errors are shown as in (1).

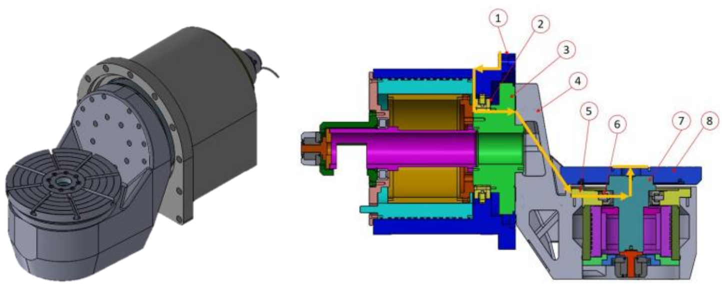
$$\boldsymbol{T}_{ideal}(P)=\prod_{n}^{i=1}\boldsymbol{T}_{ideal}(i)\ ;\ \boldsymbol{T}_{real}(\mathrm{P})=\prod_{n}^{i=1}\boldsymbol{T}_{real}(i)$$

$$\Delta\boldsymbol{T}(P)=\boldsymbol{T}_{ideal}(P)^{-1}\cdot\boldsymbol{T}_{real}(P) \tag{1}$$

where $\Delta\mathbf{T}(\mathrm{P})$ represents the three-dimensional translational and angular errors accumulated along a tolerance stack-up route P[3]. Computation of error stack-up can use either the worst-case analysis or statistical analysis.

To improve the precision of a product in assembly when components are fabricated, different parts are sometimes assembled by selective assembly. Pugh[4] proposed a method to group parts according to their size and then select appropriate groups for combination to reduce assembly errors. Tsai et al[5] summed up the grouped random assembly (GRA) of different number of groups with mathematical models. The method is to randomly assemble the large-size component group with small-size group such that errors are compensated.

This paper employs the method for calculating geometric error accumulation in three DOFs[3], as well as genetic algorithm (GA) to search for the best combination of GRA. As an illustrative example, a dual-axis rotary index table (DRIT), an important module for multi-axis machine tools[6], as shown in Figure 1 is used in this paper. While this DRIT is composed of eight major parts, errors of the rotary table relative to the fixing seat due to geometric tolerances are stacked-up through the eight components, as the route P shown in yellow arrows.



Part NO.	Description
1	Fixing seat
2	Bearing-A axis
3	Mandrel -A axis
4	Saddle
5	Bearing seat
6	Bearing-C axis
7	Mandrel -C axis
8	Roatry table

Figure 1. A DRIT assembly and the route of error stack-up

Each component has corresponding tolerances as listed in Table 1, where flat, dis, part and per means flatness, distance tolerance, parallelism and perpendicularity respectively. With the assigned tolerancing specifications shown in table 1, statistical error stack-up analysis results in ±0.014mm, ±0.018mm and -0.021mm/0.018mm in x, y and z directions.

Table 1. tolerance parameter and value of the DRIT components

Component	Tolerance parameter and value (mm)	
Fixing seat (A)	Fla1	0.010
	Dis1	±0.005
	Par1	0.008
Bearing-A axis (B)	Dis2	±0.004
	Par2	0.008
Mandrel -A axis (C)	Fla2	0.007
	Dis3	±0.006
	Par3	0.007
Saddle (D)	Dis4	±0.018
	Dis5	±0.009
	Per1	0.008

Component	Tolerance parameter and value (mm)	
Bearing seat (E)	Dis6	±0.003
	Par4	0.010
Bearing-C axis (F)	Dis7	±0.005
	Par5	0.008
Mandrel -C axis (G)	Fla3	0.005
	Dis8	±0.005
	Par6	0.005
Roatry table (H)	Fla4	0.003
	Dis9	±0.005
	Par7	0.003

2. Grouping and regrouping of Components

Components are inspected when they are fabricated. The measured data, including dimensions and geometric tolerances, are sorted and grouped before assembly. These data can be simulated as a normal distribution or a uniform distribution by the Monte Carlo method if the analysis is conducted before components are fabricated. For components with more than one tolerance, they are sorted based on the item with highest contribution to the accumulated error. Taking the fixing seat as an example, Par1 is more important than the other two items and is selected as the item for grouping. The 100 fixing seats are sorted based on the measured tolerances Par1. In the DRIT example, parts are divided into four groups with 25 components in each subgroup.

In the GRA method, the first step is to assemble subgroups of each component randomly. While the randomly assembled group usually does not give best result, generic algorithm (GA) is then employed for regrouping the assembly. The procedure is the same to many GA methods[7,8]. The cumulative variation of each combination are calculated as the fitness. If the individual in group does not reach the target fitness, three steps of GA selection, crossover and mutation were further iterated and the fitness of the new group is re-calculated. If resultant fitness does not reach target, the three steps iterates again until the target fitness is reached or the number of iterations is reached.

The fitness in this paper is set to the cumulative error of individual grouping combinations and the goal is to make the fitness function as small as possible. The median of error Em_i, calculated from the upper limit and the lower limit, is used as the index to evaluate how close all sub-combinations to the target value through the root mean square error (RMSE) as in (2). The fitness function counts RMSEs of 3D errors as in (3).

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(obj - Em_i)^2} \quad (2)$$

$$fitness = w_x \cdot RMSE_x + w_y \cdot RMSE_y + w_z \cdot RMSE_z \quad (3)$$

The results of the best four sub-group combination is shown in Table 2 that shows the 3D errors of the DRIT example are reduced by 19% to 52% with GRA.

The assembly error can be further reduced when the number of groups increased. Figure 2 shows the change of yield rate of the DRIT with respect to the number of groups if the acceptable specification of the 3D errors are ±0.01mm. It showed the yield rate reaches 100% when the 100 components are divided into 20 sub-groups.

Table 2. 3D error reduction by four sub-group GRA

directional error	without grouping	with four groups	reduction of error
x (mm)	±0.014	±0.007	52%
y (mm)	±0.018	±0.015	19%
z (mm)	-0.021/0.018	-0.013/0.010	41%

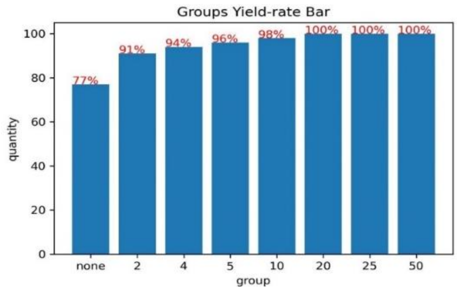


Figure 2. Change of yield rate of the DRIT example with different number of groups

3. Conclusion

This paper presented a method to improve assembly precision and yield rate of a product by GRA. In the method, components are first sorted based on crucial dimensional and geometric tolerances of parts and then divided into groups. Error analysis and GA are employed to search for the best grouping with minimum assembly precision. A DRIT example with eight crucial components is used to demonstrate this method. 100 parts of each component of the DRIT are sorted and divided into four groups in the beginning. The combination of sub-groups are re-grouped for minimum error stack-up by GA. The 3D statistical errors of GRA are reduced by 19% to 52% after regrouping. As the number of GRA groups increases to twenty, the yield rate reaches 100%.

The main contribution of this research is to develop a method which can quickly and effectively allocate the best GRA grouping, with minimum assembly errors. The yield rate can be also calculated before the parts are sent to assembly line. It provides a solution for the best GRA grouping and production planning of precision products.

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