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Effects of geometric and assembly errors of angular contact ball bearing spindles on its kinematic motion error

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Abstract
In machine-tools spindles are responsible of the geometric precision of the machined parts, as they transmit rotation to the workpiece and the tool. The experimental tests show that the rotational error of a spindle with bearings shows a predominant 2X (an error that appears twice per revolution), the current literature does not define the cause of this behavior, but it says that a pair of bearings under a load and a moment has a stiffness in its perimeter with a 2X shape.
In this work, this predominant 2X was been observed experimentally, and it was been verified that it is not due to dynamic behavior. Later, a simplified structural analytical model of the head has been developed, and it has been validated with the results offered by the Schaeffler BEARINX program. Finally, rotational error results similar to those visualized experimentally have been obtained, introducing certain assembly errors into the model.

Keywords: Spindle, ball bearing, modelling, misalignment, kinematics, stiffness matrix, rotational error

1. Introduction

In machine-tools, the precision of the bearing itself as well as the geometric and assembling precision of the surrounding elements directly affects the motion error of spindles (responsible for the rotational movement of the workpiece and the tool), and thus the geometric precision of the machined parts. In precision applications, such as precision grinding and hard turning, hydrostatic journal bearings are more widely used due to their excellent dynamic and thermal behaviour, and life expectancy. In machine-tool applications, rolling bearings (from now on referred to as "bearings") are more common due to their low cost and good performance.

As [1] and [2] mention, the predominant motion error appearing twice per revolution when angular contact ball bearings are used is due to the misalignment between front and rear bearings.

2. Experimental measurements

An angular contact ball bearing based spindle has been tested in a test bench, as shown in Figure 1, in order to measure and analyze the spindle error. The measurements are carried out with capacitive displacement sensors with nanometric resolution, measuring on spheres with a sphericity tolerance of 50nm, as advised by [3].

Figure 1. Test bench for the measurement of rotational error of spindles.

5 displacement sensors are used (Figure 2):

- 2 for the horizontal radial displacement (X1, X2)
- 2 for the vertical radial displacement (Y1, Y2)
- 1 for the axial displacement (Z)

This way, the 5DoF of the mechanism were measured while the spindle was rotating.

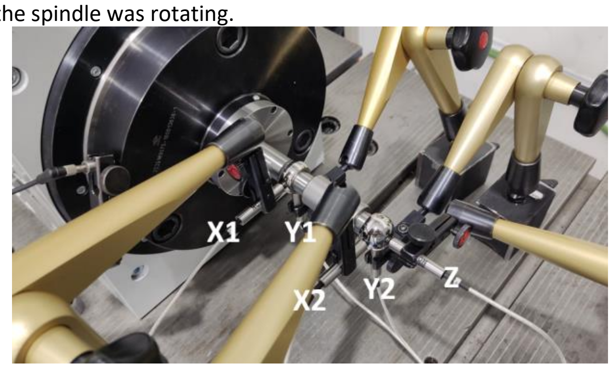


Figure 2. Set-up of the displacement transducers for the measurement of the 5DoF.

Figure 3 depicts the error motion of a spindle with ball bearings, and it is clear that the 2X frequency predominates.

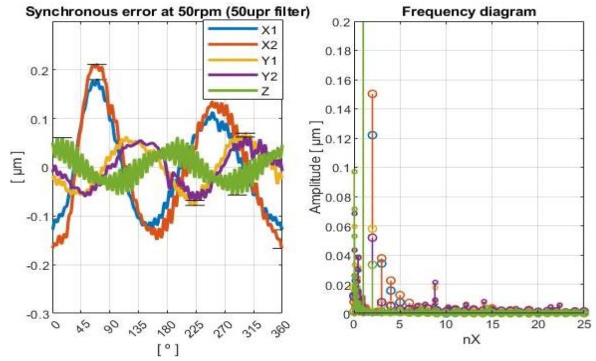


Figure 3. Synchronous error in X2 direction (as defined by ISO 230-7) of the spindle from experimental measurements (a), and their frequency response (b) The signals have been filtered at 50upr (units per revolution).

In order to see if this behavior is due to dynamic effects, the test was carried out at different speeds:

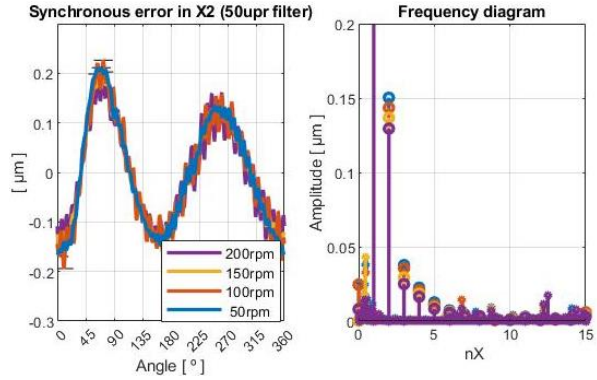


Figure 4. Synchronous error of the spindle at 200, 150, 100 and 50rpm (a), and their frequency response (b).

3. Modelling

A MATLAB-based analytical model of spindles with angular contact ball bearings was created to predict the run-out error due to geometric imperfections of the shaft and the internal and external housings of the bearing.

The work of Lim [4] and Rijnberg [5] have been largely followed, as Lim obtained the stiffness matrix for a single bearing and Rijnberg was able to get it for two bearings placed a distance apart. The terms established by Lim were used to form the stiffness matrix and the terms of each bearing were organized in accordance with Rijnberg's expressions. To solve it, it was assumed the system had five degrees of freedom (displacement along the X, Y, and Z axes, and rotation around the Y and Z axes). Based on Hertz's contact law, which is thoroughly described in [6] and [7], both authors added their own assumptions to solve the problem.

3.1. Ball bearing modelling

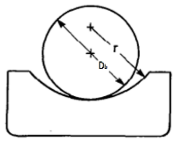
The authors' development is based on the Hertz contact theory, which makes it possible to ensure that the force applied to a bearing varies proportionally to a non-linear displacement, as indicated in the following equation. Additionally, it should be noted that when using balls as rolling elements, this nonlinearity will be an exponent of value 1.5.

$$F = K \cdot \delta^{1.5} \qquad\qquad K = \frac{1}{\left[(1/K_o)^{1/1.5}+(1/K_i)^{1/1.5}\right]^{1.5}};$$

where,

- F: Force generated in the bearing (radial or axial). $[N]$
- K: Stiffness of each rolling element. $[N/mm^{1.5}]$
- δ: Displacement of the bearing (radial or axial). $[mm]$
- K_o: Stiffness of contact between outer ring and ball. $[N/mm^{1.5}]$
- K_i: Stiffness of contact between inner ring and ball. $[N/mm^{1.5}]$

In addition to the bearing's geometric data, such as the number of balls (z), ball diameter (D_b), and contact angle (α), it is also necessary to define the osculation ratio of the inner (*f1*) and outer (*f2*) raceway shoulders, which can also be referred to as the curvature coefficients of the inner and outer raceway shoulders. These coefficients are defined as the ratio between the radius of curvature of the shoulders (r_i and r_o) and the diameter of the balls, and indicate how well the balls fit into the gap defined by these two shoulders.



$$f_1 = \frac{r_i}{D_b}, \qquad f_2 = \frac{r_o}{D_b}$$

Figure 5. Osculation ratio representation.

Manufacturers do not provide these coefficients, but according to the literature, in ball bearings, they are often approximately 52% or 0.52. Additionally, this value is usually assumed to be the same for both the inner and outer raceway shoulders (although in reality, they are independent of each other and their values differ slightly). If the value of this coefficient is lower than the indicated range, the friction will be high, and therefore, the generated heat will be high as well. However, if the value is above the range of values, it will imply that the friction is low but the contact stresses will be higher, thus reducing the fatigue life of the bearing.

It is also necessary to define the mean diameter (D_m) and the centre-to-centre distance (D_c). The first determines the distance between the centres of two opposite balls (with the radius measuring the distance from the centre of the bearing to the centre of each ball), and the second is the distance between the centres that define the inner and outer raceways.

Initially, it is necessary to define how the contact between each ball and the two raceway shoulders occurs. According to the Hertz theory, when balls are used as rolling elements, the two ball-raceway contacts will have an elliptical shape, with diameters R_{xi} and R_{yi} for the inner raceway shoulder and R_{xo} and R_{yo} for the outer one, as seen in figure XX. From these parameters, an equivalent diameter R will be calculated for each raceway shoulder. In addition, by dividing R_y by R_x, the parameter α will be calculated.

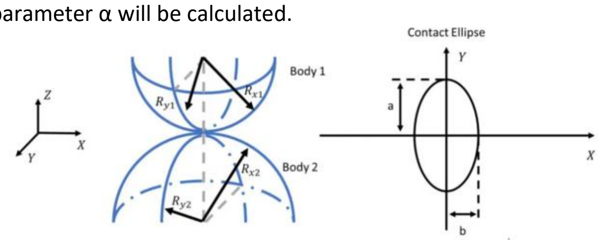


Figure 6. Elliptical contact surface between each ball and the shoulders.

Afterwards, the dimensionless parameters $\bar{k}$, $\bar{\varepsilon}$ and $\bar{\mathcal{F}}$ are also calculated for the inner and outer raceway shoulders, which are the parameter of ellipticity, the first-order elliptic integral and the second-order elliptic integral, respectively.

The subscript "p" refers to "I" (inner raceway shoulder) and "o" (outer raceway shoulder). As can be seen in the following equation, these dimensionless parameters depend on α, which implies that the behaviour of a bearing will also depend on the contact between surfaces.

$$\bar{k}_p = \frac{(\alpha_p)^2}{\pi}, \qquad \bar{\varepsilon}_p = 1 + \frac{\frac{\pi}{2} - 1}{\alpha_p}, \qquad \bar{\mathcal{F}}_p = \frac{\pi}{2} + \left(\frac{\pi}{2} - 1\right) \cdot ln(\alpha_p)$$

Before calculating the rigidity of the bearing, it is necessary to obtain the equivalent Young's modulus for the inner and outer raceways.

$$E_p{}' = \frac{2}{\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}}$$

The rigidity of the contact between the two raceways and the balls can be calculated using the following formula based on the parameters defined so far:

$$K_p = \pi \cdot k_p \cdot E_p{}' \cdot \sqrt{(2 \cdot \varepsilon_p \cdot R_p)/(9 \cdot \mathcal{F}_p^3)}$$

In this way, the stiffness of both contacts of each ball (K_i and K_o) can be calculated, and thus the stiffness in each rolling element of the bearing.

Knowing the stiffness in each ball of the bearing, a displacement can be introduced in the centre of the bearing, the load on each ball can be calculated, and by vector summing the forces, the resultant force on the bearing can be obtained. This way, the bearing stiffness curve can be obtained.

This model was validated with results from the commercial software BEARINX from Schaeffler, and the osculation ratios (very close to 0.52) were adjusted to make the curve even more accurate.

3.2. Spindle modelling

As previously mentioned, the works carried out by Lim and Rijnberg will be mainly followed, since by combining both, a stiffness matrix with five degrees of freedom (δ_z^S, δ_y^S, δ_x^S, $\delta_{\theta y}^S$, $\delta_{\theta x}^S$) can be obtained for two bearings connected by an infinitely rigid shaft. Therefore, to solve the problem, different external forces and moments (F_z, F_y, F_x, M_y, M_x) will be applied. To do this, Lim's single bearing model with five degrees of freedom will be combined with Hernot's two bearing model with two degrees of freedom separated by a distance. Finally, the expressions defined by Rijnberg will be used to obtain a stiffness matrix with five degrees of freedom for two bearings separated by a distance.

In Figure 7, the positioning and orientation of the coordinate system for each bearing (I1, I2) and their midpoint (0) can be observed. In this figure, the bearings are arranged in an "O" shape (<>), but it can easily be arranged in an "X" shape (><) by correctly orienting the coordinate systems.

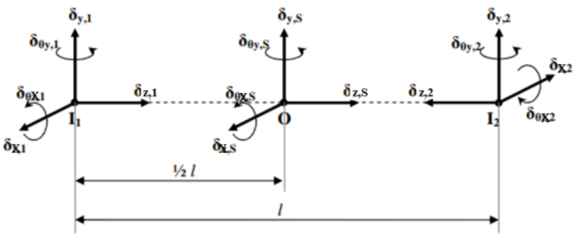


Figure 7. Positioning and orientation of the coordinate system of each bearing.

As they are joined by an infinitely rigid shaft, the displacements in the bearings can be related to each other through the displacement of the midpoint (δ^S), the axial preload applied to each of the bearings (δ_z^0), and the distance between the coordinate systems. The objective of this model is to see how much the midpoint between bearings displaces (both in the front and rear package) and then relate them to the tip of the headstock, the point of interest of the headstock.

Without going into further detail, a specific bearing configuration, typically used in workheads of cylindrical grinding machines, with two B71910 bearings at the back and three B7016 at the front (as showed in Figure 8) was chosen in order to validate this initial model. These bearing references are from the Schaeffler brand:

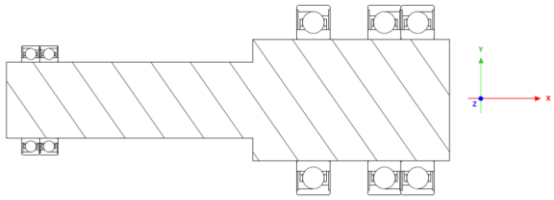


Figure 8. A representation of the workhead section of a cylindrical grinding machine.

The radial stiffness curve was calculated by introducing different external forces as input parameters in the model and compared to the results of Schaeffler's BEARINX commercial software, with a maximum 0.5% deviation between each other.

3.3. Modelling of the 2X behaviour

Experiments have shown that the spindles have a radial run-out error while rotating without any external load, which is caused by geometric and assembling errors of the surrounding components. This error has a dominant 2X shape, leading to the next hypothesis of the cause of this effect: As mentioned by [8], the perimetral radial stiffness of a pair of angular contact ball bearings under a moment exhibits a 2X shape due to certain assembling errors that are illustrated in Figure 9:

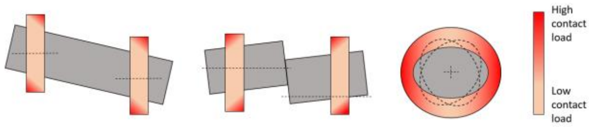


Figure 9. Assembling errors.

- Bearing outer housing parallel misalignment (Figure 9 left).
- Shaft housing parallel misalignment (Figure 9 middle).
- Shaft cylindricity error with 2X shape (Figure 9 right).

A static radial force is induced on each bearing package due to the parallel misalignment between the bearings outer housings, and the parallel misalignment of the shaft provokes a rotary 2X perimetral stiffness.

Consequently, the combination of these two phenomena (parallel misalignment between bearings and parallel misalignment of the shaft) would lead to a 2X run-out error, and the 2X shape error of a shaft would also cause a 2X perimetral stiffness on the bearings, thus creating a parallel misalignment between them and resulting in a 2X run-out error. Thus, the model took into account these geometric error options.

4. Results and discussion

The following combinations were evaluated in these calculations:

- The parallel misalignment of the bearing packages and a non-concentric parallel shaft.
- The parallel misalignment of the bearing packages and a shaft with a 2X shape error.

The first result was obtained with a 4μm misalignment error between bearings and a 2μm non-concentric error of the shaft, while the second was calculated with a 4μm misalignment error between bearings and a 1.5μm 2X shape error of the shaft; however, the flexibility of the shaft was not taken into account for these calculations.

As shown in Figure 10, the rotational error motion of the spindle had a predominant 2X shape similar to the experimental test results:

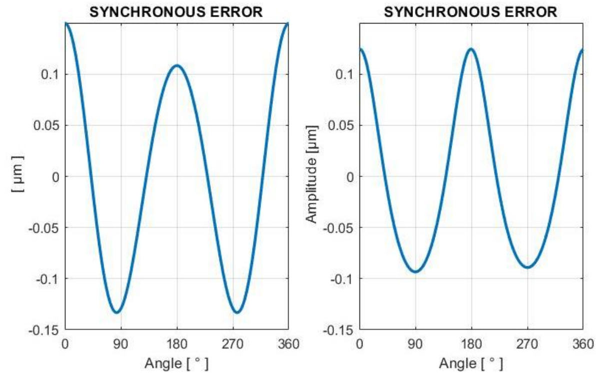


Figure 10. Modelling first results: (a) with outer housings and shaft parallel misalignment, (b) with outer housings misalignment and 2X shaft shape error

5. Conclusions and future work

Based on Hertz's law of contact, an analytical model of spindles with angular contact bearings was developed first of all. Secondly, its validation was carried out by comparing its radial stiffness curve to that provided by Shaeffler's BEARINX commercial software.

In order to predict the run-out error of the spindle and the cause of the 2X run-out error, the option of introducing assembling errors was included.

The rotational error motion of the spindle had a predominant 2X shape similar to the results of the experimental tests, which was verified to be caused by the combination of some geometric errors of the surrounding elements.

The geometric errors of the spindle components used for experimental testing are uncertain. In order to validate the developed mathematical model, a test bench with a spindle whose surrounding components' geometric errors are well known will be manufactured for future works. It is essential that the rear bearing package can be precisely aligned or misaligned with the front package using a sensitive mechanism on this bench.

References

[1] Patel T H and Darpe K 2009 Experimental investigations on vibration response of misaligned rotors, Mech. Syst. Signal Process., vol. 23, no. 7, pp. 2236–2252, Oct.

[2] Hariharan V and Srinivasan P S S 2009 Vibration analysis of misaligned shaft-ball bearing system, Indian J. Sci. Technol. 2, 45–50.

[3] Marsh E R 2010 Precision spindle metrology 2nded, DEStechPublications,Inc.

[4] Lim T C and Singh R 1990 Vibration Transmission Through Rolling Element Bearings, Part I: Bearing Stiffness Equationtion, Journal of Sound and Vibration, 139, pp. 179-199.

[5] Rijnberg J L 2007 Theoretical modeling of the stiffness of angular contact ball bearings using a two DOF and a five DOF approach. (DCT rapporten; Vol. 2007.129). Technische Universiteit Eindhoven.

[6] Hamrock B J and Anderson W J 1983 Rolling-element bearings, NASA Reference Publication 1105.

[7] Harris T A, Kotzalas M N 2007 Essential concepts of bearing technology, Rolling bearing analysis. Boca Raton, FL, USA: CRC Press.

[8] Zhang Y, Fang B, Kong L and Li Y 2020 Effect of the ring misalignment on the service characteristics of ball bearing and rotor system, Mechanism and Machine Theory.