eu**spen'**s 23rd International Conference &

Exhibition, Copenhagen, DK, June 2023

www.euspen.eu



Modeling of machine structure based on multiple reduced-order flexible bodies for successive update of boundary parameters

Issei Ota¹, Shuntaro Yamato¹

¹ Kyoto University, Nishikyo-ku Kyotodaigaku Katsura C3, Kyoto, 615-8530, Japan

E-mail: yamato@prec.kyoto-u.ac.jp

Abstract

The essence of the digital twins is to reflect real-world conditions in real-time through a combination of sensor feedback from the IoT. In machine tools, the machine dynamics are greatly affected by boundary conditions (BCs) of machine interfaces. It is well known that BCs identified in a static state by a preliminary experiment differ from actual ones during real process states. Therefore, successive and iterative update of boundary parameters using the real-time response is an interesting task for digital-twin machine tools. In this study, a novel model structure is proposed based on reduced-order multi-flexible body models. In the proposed model structure, each reduced-order flexible body intuitively connects each other through the action-reaction feedback principle representing BCs, and boundary parameters can be changed flexibly without reconducting reduced-order operations. For verification, the proposed model structure for a simple test bench structure is developed and investigated by being compared with an experiment and another modeling approach such as multi-rigid-body model. Responses in an excitation test are reproduced in the proposed system with the proper BCs of the contact surfaces by the data assimilation based on time response.

Reduced-order multi-flexible body model, Multi-rigid body model, FEM, Boundary condition estimation, Digital twin

1. Introduction

Multibody modeling is widely used for a digital twin because of its less calculation time [1]. Numerous research uses rigid bodies for the multibody modeling, but they cannot consider the flexibility of the structure, so the model with consideration of elasticity is desired [2]. Reduced-order FEM model connected with each other can reproduce the dynamics of machine tools more precisely [3, 4].

The boundary conditions (BCs) of its contact surfaces that affect the structural dynamics cannot be theoretically given in analysis, and preliminary experiments are currently required. Also, such BCs may change depending on various factors such as force, temperature, and aging conditions during machining. Therefore, the quasi-real-time estimation/update of the BCs from measured time-response signals such as vibration and cutting force during process is quite useful for the purpose of machine condition monitoring or reliable digital-twin simulation. In this study, a novel model is proposed based on reducedorder multi-flexible body models (ROMFBM). In the proposed structure, each reduced-order flexible body intuitively connects each other, and BC parameters can be changed flexibly in a timedomain simulation, which enables data assimilations of BCs.

As a verification of ROMFBM, the excitation experiment of a simple test bench structure is conducted. From the result, the BCs are estimated by ROMFBM, and the validity of the model is discussed through the comparison with the result of the estimation by MRBM.

2. Data assimilation of contact surface boundary conditions

2.1. Reduced-order multi-flexible body models

Reduced-order state space model (ROM) can be created by model order reduction of FEM that expressed as:

 $E\ddot{\boldsymbol{u}} + D\dot{\boldsymbol{u}} + K\boldsymbol{u} = \mathbf{L},\tag{1}$

where E, D, and K are the mass, damping, and stiffness matrices, respectively, and L is the load vector. By solving the eigenvalue problem, projection matrix Φ whose columns are the eigenvectors is formed and an approximation u_r is written as:

$$\boldsymbol{u}_r = \Phi \boldsymbol{q}. \tag{2}$$

Replacing u by u_r and premultiplying by Φ^H yield the equation as the following by the reduced-order matrices projected onto the subspace:

$$E_r \ddot{\boldsymbol{q}} + D_r \dot{\boldsymbol{q}} + K_r \boldsymbol{q} = \Phi^H \mathbf{L}, \qquad (3)$$

where $E_r = \Phi^H E \Phi$, $D_r = \Phi^H D \Phi$, and $K_r = \Phi^H K \Phi$.

The ROMFBM is created by coupling these ROMs with virtual springs and dampers based on the feedback principle of action-reaction of multiple ROM inputs and outputs (MIMO). Figure 1 shows a representative structure of the coupling system when 2 ROMs are connected. In Figure 1, the arrangement of the virtual springs and dampers is defined at the 4 corners of each contact surface. These virtual connections enable to reproduce the BCs by action-reaction principle and can be changed flexibly in a time-domain simulation without reconducting reduced-order operations, which enables data assimilations of BC parameters.





2.2. Successive update of boundary conditions

This section proposes a method to estimate both state values and BCs at the same time by integrating particle filtering (PF) algorithm [5] into the developed system structure. Each particle has extended state space vector that has state variables and BCs of each contact surface as:

$$\boldsymbol{x}_{t}^{p} = \{x_{1}, x_{2}, \cdots, k_{1}, k_{2}, \cdots\}^{T}, (p = 1, \dots, N),$$
(4)

where N is the number of particles, x_i and k_i represent state variables and BCs such as stiffness and damping, respectively. The state space model of the time-domain simulation is described as the following.

$$\begin{cases} x_t^{(p)} = f_t \left(x_{t-1}^{(p)}, v_t \right) \\ y_t^{(p)} = h_t \left(x_t^{(p)} \right) + w_t \end{cases}$$
(5)

where $y_t^{(p)}$ represents the observation vector (displacements), and v_t , w_t represent system noise and observation noise, respectively. Particles are resampled based on the likelihoods calculated through the comparison between the sample data and predicted data, in that, the parameters of each particle are updated successively based on actual time responses. As time increases, the probability distribution of the state variables is converged, and the plausible BCs are estimated probabilistically.

3. Experimental verification

For a verification of ROMFBM and data assimilation algorithm, a simple test bench structure is developed (see Figure 2), and an excitation experiment is conducted. The contact surface of the 2 bodies fixed with 4 bolts and the bottom surface where body 2 is fixed with 4 bolts are defined surface 1 and surface 2 respectively. A sinusoidal force is inputted in y-direction by a piezoelectric actuator and the acceleration at the corner of the body is observed. The data assimilation is conducted by the method mentioned in section 2. In the simulation, at the 2 contact surfaces, it is assumed that 4 identical virtual springs connect the bodies, and their BCs are respectively defined for the vertical and horizontal direction to each contact surface as the following equations describe.

$$\begin{cases} k_{iv} = \theta_{kiv}/4 \times 10^9, & c_{iv} = \theta_{civ}/4 \times 10^4 \\ k_{ih} = \theta_{kih}/4 \times 10^9, & c_{ih} = \theta_{cih}/4 \times 10^4 \end{cases}$$
(6)

where *i* represents the index of the contact surfaces (i = 1,2), and *k* and *c* represent the spring constant and damping constant, respectively. 8 BC parameters in total and the response of the object are estimated by PF. Table 1 shows the condition of the experiment and simulation. The signal generator outputs a single-frequency voltage, which is applied to the piezo actuator, and the object is vibrated.



Figure 2. Excitation experiment with a simple test bench structure (a)Appearance of developed structure,(b)Setup of excitation experiment

Table 1 Conditions of experiment and simulation

Excitation frequency [Hz]	200, 500, 900
Time step [s]	$1.0 imes 10^{-4}$

The data assimilation is conducted by using the force observed by the force sensor and the time response of the acceleration by the accelerometer as the input and the sample data, respectively. Using the results at each frequency in turn, data assimilation was performed to estimate the parameters that make the frequency response match the experiment. As a comparison, parameter estimation by MRBM that is composed of the 2 rigid bodies instead of flexible bodies was performed using the same method. Figure 3 shows the results of FRFs in the y-direction estimated by each model. As a result, ROMFBM could obtain closer FRF to the measured one than MRBM. A tortional deformation occurs in the body 2 in the mode shape around 900Hz. (see Figure 4 (a)) On the other hand, the tortional mode could not be seen in the 2nd eigen mode. (see Figure 4 (b)) Therefore, MRBM could not consider the elasticity of the model and it is necessary to consider the elasticity for the proper estimation of BCs.



Figure 3. Frequency response function of experiment and simulations



Figure 4. Mode shapes at the second eigen frequency (a) ROMFBM at 888Hz (b) MRBM at 577Hz

4. Conclusion

This study presented the system architecture for estimation of BCs based on actual time responses of object by successive update of the BCs, and compared the flexible body and rigid one. As a result, the FRF obtained by ROMFBM is closer to the measured one than MRBM. That clarified that the consideration of the elasticity enables to estimate the BCs more precisely. Future work will estimate BCs using the cutting data and apply this system to a machine tool and investigate how it works in case where the model consists of more bodies and joints.

References

- [1] C. Wang, K. Erkorkmaz, J. McPhee and S.Engin 2020 CIRP Ann. 69 321-24
- [2] D. Kono, T. Lorenzer, S. weikert and K. Wegener 2010 Prec. Eng. 34 399-407
- [3] H.N. Huynh and Y. Altintas 2021 J. Manuf. Sci. eng. 143 021012-1
- [4] H.N. Huynh and Y. Altintas 2022 CIRP Ann. 71 325-28
- [5] M. S. Arulampalam, S.Maskell, N. Gordon and T. Clapp 2002 IEEE Trans. Signal Processing 50 174-88