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Multi-probe roundness measurement and harmonic content of Reuleaux polygons

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Abstract
This paper aims to clarify misconceptions present in some literature on roundness measurement related to diameter measurement and the harmonic content of Reuleaux triangles, which are sometimes referred to as shapes of constant diameter and thus implied to exhibit odd lobing. To demonstrate, the paper investigates regular Reuleaux polygons in the angular frequency domain by evaluating their harmonic components numerically. A pattern is observed in the frequency domain representation of the Reuleaux polygons, with harmonic components repeating at intervals corresponding to the number of angles in the polygon. Finally, the paper discusses the implications of mechanical filtration effects and the existence of such geometries for roundness measurement in practice, discussing the limitations of two-point diameter measurement in roundness measurement.

Roundness, Harmonics, Reuleaux

1. Introduction

In roundness measurement as well as machine design and manufacturing, it is important to be aware of the existence of geometries, which have uniform diameter or uniform width, but are not circles. One example of such a geometry is the Reuleaux triangle, a commonly known shape of constant width.

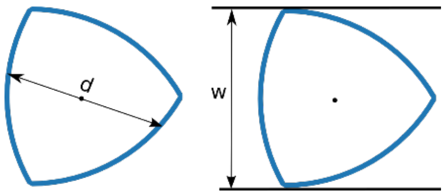

Figure 1. Two-point diameter (*d*) and width (*w*)

Confusion related to the subject, particularly to the measurement of diameters and the harmonic content of Reuleaux polygons is apparent in literature related to roundness measurement. Examples exist of Reuleaux triangles being referred to as shapes of constant diameter [1], implying that Reuleaux triangles exhibit constant diameter and thus a number of "odd-lobing". Along with a figure of tangential line contacting diameter measurement Whitehouse claims that "even lobing is measurable by diametrical assessment but odd lobing is not" and that "coins to be used in vending machines have to be odd-lobed otherwhise they run the risk of jamming in the slot" [2]. Essentially, the misconception relates to missing knowledge of the harmonic content of regular Reuleaux polygons as well as confusion between the different definitions of diameter and their measurement. Furthermore, in some publications constant diameter shapes are incorrectly referred to as Reuleaux triangles [2, 3, 4].

To clarify these misconceptions, this paper presents numerical calculations of the harmonic content of several Reuleaux polygons, commonly known shapes with a constant width, which are examples of geometries which can result in a mechanical filtration effect when supported between parallel surfaces. Furthermore, this paper discusses different definitions of diameter and their measurement.

Firstly, it is paramount to define diameter and width. In mathematical papers, the diameter and the width can be defined as the maximum and minimum width of a profile respectively [5], with the width of a profile in general defined as the distance between two parallel tangent lines of the profile [6]. However, in the context of roundness metrology and multi-probe roundness measurement [3, 12], the workpiece diameter can be treated corresponding to the *two-point size* defined in ISO 17450-3 and ISO 14405-1, which can also be called the *two-point diameter* for cylinders [7, 8]. By this definition, the diameter can be regarded as the length of a cord through the midpoint of the profile, which also corresponds to the diameters of circles and ellipses i.e. the lengths of the chords through their midpoints. Often, the used definition of the diameter is not explicitly stated in literature related to roundness measurement.

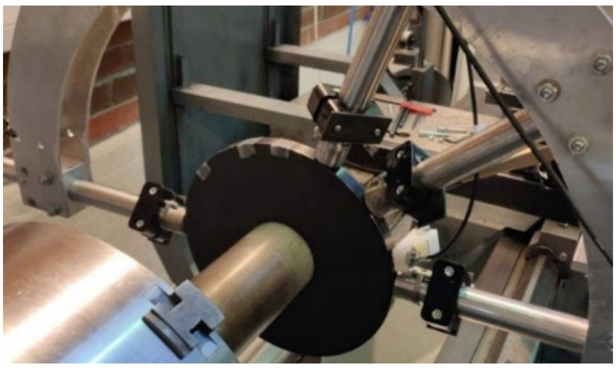
Figure 2. Multi-probe roundness measurement with line-contacting probes

This distinction has practical relevance. If line contacting probes are used in a roundness measurement instrument (such as the one shown in Figure 2), a *two-point diameter* measurement setup might actually be measuring width. This mechanical filtration effect may also relevant in other applications, such as in the design and manufacturing of machine elements where rollers are supported between two parallel surfaces. If a line contact occurs, constant width shapes such as Reuleaux polygons can cause the point of contact to vary during rotation, effectively hiding the variation of the two-point diameter. It should be noted, that in precision metrology, the situation will not be very common: especially when measuring large and nominally round workpieces, measurement even with line-contacting probes yields a very similar result for width and two-point diameter.

In roundness measurement, the harmonic domain $\frac{1}{rev}$ (synchronous angular frequency domain) can be used to describe the roundness deviation of a workpiece cross-section, with the frequency domain after a discrete Fourier transform selected so that the first component corresponds to a small eccentric motion, the second to a two-lobed shape, the third to a three-lobed shape etc. [3]

For small and rigid workpieces, the roundness profile of the workpiece cross-sections can be directly measured on roundness measurement instruments with precision spindles. For large and flexible workpieces where it is not possible to eliminate the center-point motion of the cross-section, multi-probe methods can be used, where at least three probe signals are used to calculate the roundness error and the center point location for each relative orientation of the workpiece and the measuring instrument [3, 9, 10, 12]. To reduce the effects of harmonic suppression, redundant methods with more than three probes can be used. These methods can use subsets of three probes [11], least squares minimization [9] or alternatively directly calculate the even components from the two-point diameter variation profile [12] to reduce uncertainty in the evaluation of the Fourier coefficients of the roundness profile. An example of a multi-probe setup using line contacting probes for diameter measurement is shown in Figure 2.

2. Methods

An equation for the polar form of Reulaux polygons has recently been published online, where the equation for the boundary of the polygon was derived from a parametric presentation using the distance formula [13] to obtain the following equation for an *n*-sided regular Reuleaux polygon with unit radius:

$$r = \cos\left(\theta - \frac{2\pi}{n}\left\lfloor\frac{n(\theta-\pi)}{2\pi}\right\rfloor + \frac{1}{2}\right) + \sqrt{1 + 2cos\frac{\pi}{n} + cos^2\left(\theta - \frac{2\pi}{n}\left\lfloor\frac{n(\theta-\pi)}{2\pi}\right\rfloor + \frac{1}{2}\right)}$$

Using the equation, boundaries of Reuleaux polygons with n from 3 and up to 11 were numerically evaluated at 250 points spread evenly between the interval θ = 0,...,2π and the discrete Fourier transform was used to calculate the harmonic components of the profiles.

3. Results

The results of the calculations for the Reuleaux triangle and pentagon are shown in Figures 3 to 7, where it can be seen that the harmonic components of Reuleaux polygons correspond to the number of sides in the Reauleaux polygon, with the components repeating at frequencies corresponding to the number of sides in the Reuleaux polygon. As the number of sides in a Reuleaux polygon is always odd, every second harmonic component will be even. It is known that shapes with a constant two-point width only consist of odd components. Reauleaux polygons are only of constant width. Furthermore, a similar pattern was observed for the phases of the harmonic components, as can be seen in Figures 3 to 7. Reuleaux polygons with higher numbers of sides were observed to follow a similar pattern.

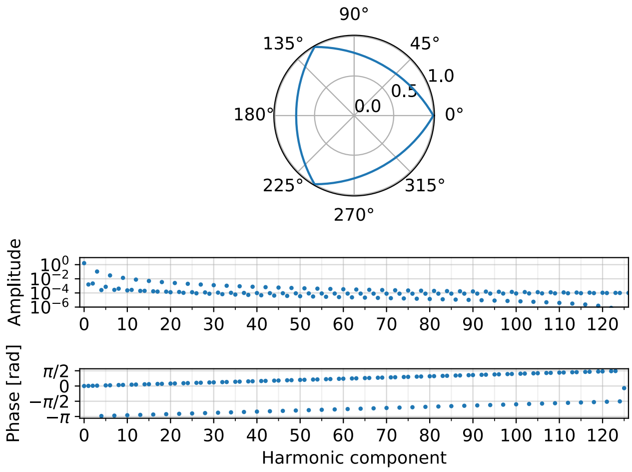

Figure 3. Harmonic components of a regular Reuleaux triangle. Note the logarithmic scale on the amplitudes.

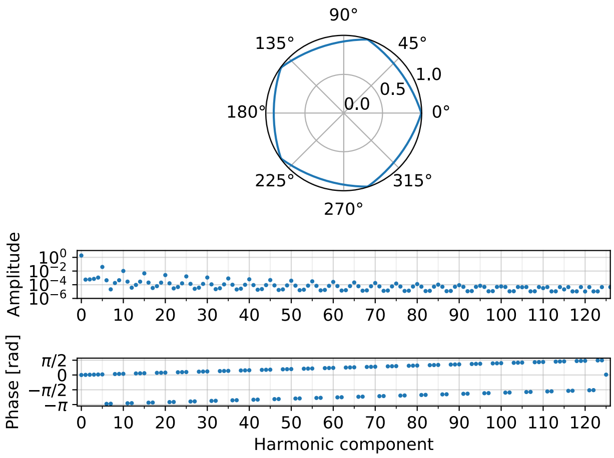

Figure 4. Harmonic components of a regular Reuleaux pentagon. Note the logarithmic scale on the amplitudes.

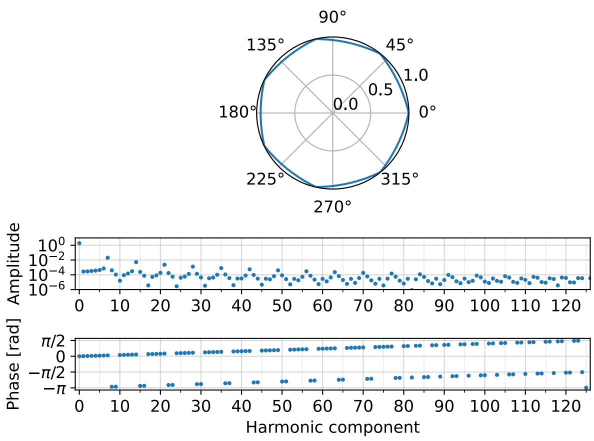

Figure 5. Harmonic components of a regular Reuleaux heptagon. Note the logarithmic scale on the amplitudes.

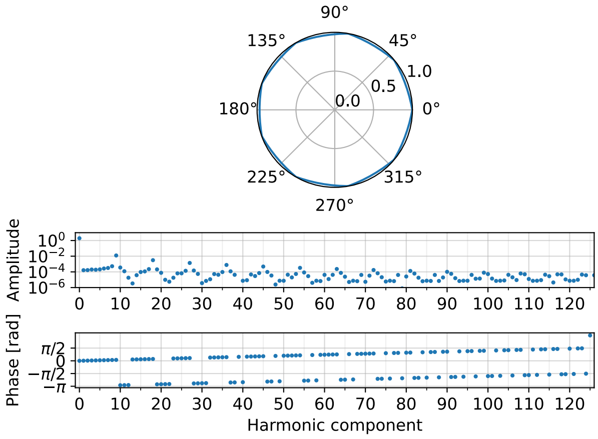

Figure 6. Harmonic components of a regular Reuleaux nonagon. Note the logarithmic scale on the amplitudes.

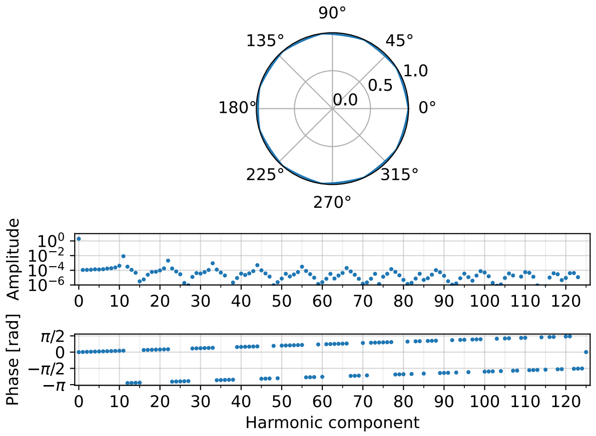

Figure 7. Harmonic components of a regular Reuleaux hendecagon. Note the logarithmic scale on the amplitudes.

4. Discussion

With the calculations, it was shown that Reuleaux polygons have harmonic content repeating at a pattern corresponding to the number of angles in the polygon. Contrary to how the subject is presented in the referenced literature [1, 2], Reuleaux triangles (as well as other Reuleaux polygons) feature both odd and even lobe numbers.

Reuleaux polygons are constructed with circle arcs drawn from the sides of polygons. It must be noted that it is possible to extend the sides of the polygons and use the extended sides as supports when constructing the Reuleaux rotors [14]. The resulting polygon will exhibit a similar constant width but will not have sharp angles. The authors are not aware of a published parametric equation for this type of Reuleaux rotors. It would be an an interesting study to to perform a similar analysis of harmonic components for this type of geometries as well.

When considering two-point diameter, a constant diameter will occur in a workpiece which only consists of odd harmonic components. Essentially, this results from the parity of the harmonic components sine waves, with the opposite phase wave repeating at the other side of the workpiece. The situation becomes more complex when the diameter is understood as the distance between parallel lines. In this case, mechanical filtration effects and the changing of the contact point can distort the measurement and the workpiece cross-section may also contain even harmonic components, as was shown by the calculations of the Reuleaux polygons.

Non-circular geometries exist with both a constant width and two-point diameter (for example a geometry with a three-lobed roundness error with a large enough radius to make the profile convex) as well as geometries with constant two-point diameter but not constant width (for example a geometry with a three-lobed roundness error with a smaller radius resulting in a convex profile). The Reuleaux polygons investigated in this paper are rather extreme examples of the mechanical filtering effects which will occur in all types of measurements and machine elements.

5. Conclusion

To summarize, it is important to make the distinction between the different definitions of the diameter, whether the diameter is defined as the distance between two parallel tanget lines of a profile [5, 6] or as the two-point diameter [7, 8] understood to correspond to the diameter of a circle or ellipse as a chord through the midpoint. Profiles with only odd harmonic components will yield a constant two-point diameter. A cross-section with a constant parallel tangent line diameter may also contain even components, which was was here shown to be the case with Reuleaux polygons. Contrary to how odd-lobing in roundness metrology is presented in the referenced books [1, 2], regular Reuleaux polygons also contain even harmonic components with non-zero amplitudes.

It is important to be aware that typical roundness measurement methods, instruments and analyses are restricted to nominally round workpieces with minor roundness deviations and non-convex geometries. Due to mechanical filtering effects, using line contact probes can actually result in a measurement of an incorrect quantity. The authors acknowledge that this may not occur often in practice, but it is something one should be aware of. Furthermore, the subject may be more relevant for other machine design and manufacturing problems.

Acknowledgement

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