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Practical method for checking of self-locking in kinematic couplings with linear equations

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Abstract
In precision mechanics, kinematic couplings are well known for positioning two parts with respect to each other with high accuracy and high repeatability. These kinematic couplings consist of a fixed base and a movable part with six defined contact points between both parts, where each contact point is constraining one degree of freedom to the relative motion between the both parts. Under real assembly conditions with friction and gravity, self-locking can occur and thus preventing both parts to get into contact at all six contact points.
The presented method shows, how to check whether a match in all six contact points is possible for a given geometry of the kinematic coupling, taking into account friction, nesting force and possible external force such as gravity. The method can be used in the early design process for adapting the geometry or the nesting process, so that the kinematic coupling works properly.

kinematic coupling, self-locking, instantaneous screw axis, statically indetermined, lift-off of contact

1. Introduction

Kinematic couplings are often used for positioning two parts with respect to each other with high accuracy and high repeatability [1]. This accurate and repeatable alignement of the both parts is realised by six defined contact points, each constraining one degree of freedom of the relative motion between both parts. Since the reliable match of all six contact points is a necessary precondition for reaching a good repeatability of the kinematic coupling, which is treated and examined in the Ref. [2-5], this article is focusing on possible self-locking preventing a match in all six contact points. Patti and Vogels [6] present two methods for determing the limiting coefficient of friction, which indicates self-locking, if the real coefficient of friction is higher than the limiting one. In both methods presented in Ref. [6], the limiting coefficient of friction is estimated by a system of nonlinear equations based on static equilibrium. For practical application, it is favourable to avoid a system of nonlinear equations, because standard software is often not suitable for its solution.

In this article, the system of nonlinear equations is changed into a system of six easily solvable linear equations by presetting the coefficient of friction and implementing an accelerating load, which drives the movable part of the kinematic coupling into the nested position. For implementing the accelerating load, the nesting motion is determined, which is generally a combination of translation and rotation. This motion is described as screw motion with position, orientation and pitch of the screw axis (see Ref. [7]). It can be argued, that the ratio of translation to rotation – also understood as the pitch of the instantaneous screw axis – is the same as the ratio of force to torque of the accelerating load at every point on this screw axis. The system of equations is completed with the position, orientation and the “pitch” of the accelerating load, so that the values of the accelerating load and all six contact forces can be determined. The self-locking is detected by a negative sign of the acceleration load. The method in this article allows also to detect a possible lift-off in a contact point, indicated by a negative sign of a contact force, because on the contrary to the methods of Ref. [6], this method is no more based on static equilibrium.

In the second method of Ref. [6] a possible effect of deformation of the kinematic coupling to self-locking is considered. An example for this deformation effect is a ball sticking in an acute-angled v-groove by friction and an internal force loop through ball and v-groove (wedge effect). It is assumed in this article, that the geometry of the kinematic coupling is free from wedge effects and no internal force loops are stored by deformation from previous matching steps.

2. Self-locking and accelerating load

The situation with five matched contact points and only one degree of freedom left for matching the sixth one is considered as most critical for self-locking (see Ref. [2], [6]).

The self-locking in a kinematic coupling is indicated by a negative sign of the accelerating load, because the friction forces can only reduce the size of the accelerating load but cannot change its direction. Thus, a negative sign means no motion.

In this article, the accelerating load is the remaining combination of force and torque, which drives the kinematic coupling into the nested position, after subtraction of all contact forces and friction forces from the external nesting force. The accelerating load must not be confused with the inertial load, which results from the product of the inertia of a body with its acceleration. This inertial load can impact the contact forces, whereas the accelerating load has no impact on the contact forces per definition.

Since the inertia of the movable part of the kinematic coupling is not known in an early design state, the accelerating load is used in this article with the assumption, that the acceleration is too small for influencing on the contact forces.

2.1. Determing position, orientation and pitch of the instantaneous screw axis

Assuming small motions, a system of linear equations is presented in Ref. [6] for determing the nesting motion described by a translation vector $\Delta\vec{s}_O$ and a rotation vector $\Delta\vec{\theta}$ in the origin of the describing coordinate system. Necessary for the calculation are the position vectors $\vec{r}_i$ and $\vec{r}_j$ of the contact points and the unit normals $\vec{n}_i$ and $\vec{n}_j$ of the tangent planes of both surfaces in the contact points. The index i expresses, that the point is in contact, whereas the index j designates the single point, which is not in contact.

$$\begin{cases} \left(\Delta\vec{s}_O + \Delta\vec{\theta} \times \vec{r}_i\right) \cdot \vec{n}_i = 0 \quad i = 1,2,\dots,6 \; except \; j \\ \left(\Delta\vec{s}_O + \Delta\vec{\theta} \times \vec{r}_j\right) \cdot \vec{n}_j = -1 \end{cases} \tag{1}$$

For the further calculation the translation vector $\Delta\vec{s}_O$ and rotation vector $\Delta\vec{\theta}$ are transformed to the standardised translation vector $\Delta\vec{s}_O$ and the standardised rotation vector $\Delta\vec{\theta}_s$ by dividing both by the value $\left|\Delta\vec{\theta}\right|$ of the rotation vector. The standardised rotation vector $\Delta\vec{\theta}_s$ represents also the orientation of the instantaneous screw axis.

If the value of the rotation vector $\Delta\vec{\theta}$ is zero, the nesting motion is a pure translation and the standardised translation vector is obtained by dividing the translation vector $\Delta\vec{s}_O$ by its value. In this case, the pitch and the position of the instantaneous screw axis have not to be determined.

$$\begin{cases} \Delta\vec{s}_{Os} = \frac{\Delta\vec{s}_O}{|\Delta\vec{\theta}|} \qquad \Delta\vec{\theta}_s = \frac{\Delta\vec{\theta}}{|\Delta\vec{\theta}|} \qquad if \; \left|\Delta\vec{\theta}\right| \neq 0 \\ \Delta\vec{s}_{Os} = \frac{\Delta\vec{s}_O}{|\Delta\vec{s}_O|} \qquad\qquad\qquad if \; \left|\Delta\vec{\theta}\right| = 0 \end{cases} \tag{2}$$

The scalar product of the standardised rotation vector $\Delta\vec{\theta}_s$ with the standardised translation vector $\Delta\vec{s}_{Os}$ describes the value of the component of the translation vector, which is parallel to the rotation vector and represents the pitch p_a of the instantaneous screw axis.

$$p_a = \Delta\vec{\theta}_s \cdot \Delta\vec{s}_{Os} \tag{3}$$

The position of the instantaneous screw axis is described by he perpendicular $\vec{r}_a$ from the origin of the coordinate system to the instantaneous screw axis. The direction of the perpendicular $\vec{r}_a$ can be expressed with the cross product of the rotation vector $\Delta\vec{\theta}_s$ and the translation vector $\Delta\vec{s}_{Os}$ because of its perpendicularity to both vectors (see Figure 1.).

$$\frac{\vec{r}_a}{|\vec{r}_a|}\sin\alpha = \Delta\vec{\theta}_s \times \frac{\Delta\vec{s}_{Os}}{|\Delta\vec{s}_{Os}|} \tag{4}$$

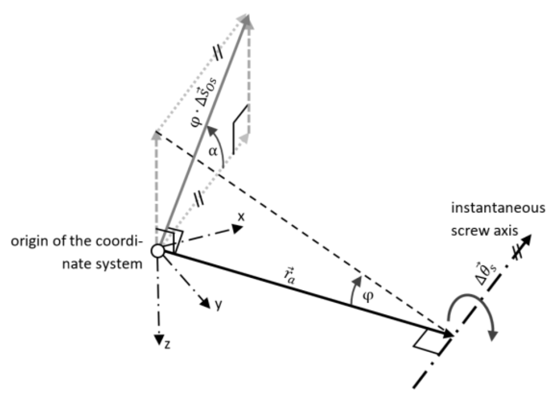

Figure 1. Position and orientation of the screw axis

According to Figure 1., the relation between the values of the perpendicular $\vec{r}_a$ and the translation vector $\Delta\vec{s}_{Os}$ can be obtained for small rotation angles φ around the screw axis as shown in equation (5).

$$\varphi \; |\vec{s}_{Os}| \sin\alpha = \varphi \; |\vec{r}_a| \tag{5}$$

The combination of equations (4) and (5) results in the cross product of the rotation vector $\Delta\vec{\theta}_s$ and the translation vector $\Delta\vec{s}_{Os}$ for calculating the perpendicular $\vec{r}_a$.

$$\vec{r}_a = \Delta\vec{\theta}_s \times \Delta\vec{s}_{Os} \tag{6}$$

After determing all parameters of the instantaneous screw axis the accelerating load can be defined.

2.2. Implementation of the accelerating load

In Ref. [6], six equations of static equilibrium are presented with the values N_i of the forces in the five contact points, the displacement directions $\vec{s}_i$ in the contact points, the external nesting force $\vec{F}_e$, the position $\vec{r}_e$ of its force application point and the coefficient of friction μ.

$$\begin{cases} \vec{F}_e + \sum_{i \neq j}\left(N_i\vec{n}_i - \mu N_i \frac{\Delta\vec{s}_i}{|\Delta\vec{s}_i|}\right) = 0 \\ \vec{r}_e \times \vec{F}_e + \sum_{i \neq j}\vec{r}_i \times \left(N_i\vec{n}_i - \mu N_i \frac{\Delta\vec{s}_i}{|\Delta\vec{s}_i|}\right) = 0 \end{cases} \tag{7}$$

The displacement directions $\vec{s}_i$ at the five contact points are obtained equivalent to Ref. [6] with equation (8).

$$\Delta\vec{s}_i = \Delta\vec{s}_{Os} + \Delta\vec{\theta}_s \times \vec{r}_i \tag{8}$$

A body guided in one degree of freedom is accelerated only by forces parallel to the displacement direction of their force application point or by torques parallel to the rotation axis of the guided motion. The accelerating load for a screw motion is a combination of force and torque both parallel to the screw axis and with a force application point on the screw axis. For being parallel to the screw motion, the ratio p_l of force to torque of the accelerating load is the same as the ratio of translation to rotation, the pitch p_a of the screw axis, only regarding the absolut values and not the units.

$$|p_l| = |p_a| \qquad absolute \; values \; without \; units \tag{9}$$

Equation (9) allows to express the accelerating force $\vec{F}_{acc}$ and the accelerating torque $\vec{M}_{acc}$ with the value L_{acc} of the accelerating load according equations (10) with the ratio p_l of the screw axis.

The pitch p_a of the screw motion describes also the ratio of the displacement, which is parallel to the screw axis, to the displacement, which is perpendicular to the screw axis, on a unit circle around the screw axis. For the compatibility between different length units such as meter and millimeter, all length dimensions inclusive the ratio p_l are divided in the following equations (10), (11) and (12) by the value $|p_a|$ of the pitch, which is a characteristic length dimension of the screw axis for standardisation.

If the value of the rotation vector is zero, the accelerating force is parallel to the standardised translation vector according equation (2) and is applied in the origin of the coordinate system, whereas the accelerating torque is zero. If the value $|p_a|$ of the pitch is zero, the motion is a pure rotation, so that the accelerating force is zero and the accelerating torque is parallel to the standardised rotation vector.

$$\begin{cases} \vec{F}_{acc} = L_{acc}\,\Delta\vec{s}_{Os} & \vec{M}_{acc} = 0 & if\ |\Delta\vec{\theta}| = 0 \\ \vec{F}_{acc} = 0 & \vec{M}_{acc} = L_{acc}\,\Delta\vec{\theta}_s & if\ |p_a| = 0 \\ \vec{F}_{acc} = L_{acc}\,\frac{p_l}{|p_a|}\,\Delta\vec{\theta}_s & \vec{M}_{acc} = L_{acc}\,\Delta\vec{\theta}_s & otherwise \end{cases} \quad (10)$$

The system of six equations (7) are complemented with the accelerating force $\vec{F}_{acc}$ and the accelerating torque $\vec{M}_{acc}$ on the right side. For the extension of the torque equations, the additional torque resulting from the accelerating force $\vec{F}_{acc}$ and the distance between origin of the coordinate system and the screw axis, which is the perpendicular $\vec{r}_a$, has to be considered. Also all length dimensions are standardised by dividing them by the value $|p_a|$ of the pitch.

$$\begin{cases} \vec{F}_e + \sum_{i\neq j}\left(N_i\,\vec{n}_i - \mu\,N_i\,\frac{\Delta\vec{s}_i}{|\Delta\vec{s}_i|}\right) = \vec{F}_{acc} \\ \frac{\vec{r}_e}{|p_a|}\times\vec{F}_e + \sum_{i\neq j}\frac{\vec{r}_i}{|p_a|}\times\left(N_i\,\vec{n}_i - \mu\,N_i\,\frac{\Delta\vec{s}_i}{|\Delta\vec{s}_i|}\right) = \\ = \frac{\vec{r}_a}{|p_a|}\times\vec{F}_{acc} + \vec{M}_{acc} \end{cases} \quad (11)$$

For a screw motion, the equations (11) are transformed into a system of six linear equations for calculating the values N_i of the five contact forces and the value L_{acc} of the accelerating load by replacing the accelerating force $\vec{F}_{acc}$ and the accelerating torque $\vec{M}_{acc}$ with the equations (10). These equations can be complemented with additional external forces $\vec{F}_k$ for example a force due to gravity by considering the torque resulting form forces $\vec{F}_k$ and the distances between the origin of the coordinate system and the positions $\vec{r}_k$ of application points of the forces $\vec{F}_k$.

$$\begin{cases} \vec{F}_e + \sum_k\vec{F}_k + \sum_{i\neq j}N_i\left(\vec{n}_i - \mu\,\frac{\Delta\vec{s}_i}{|\Delta\vec{s}_i|}\right) = L_{acc}\,\frac{p_l}{|p_a|}\,\vec{\theta}_s \\ \frac{\vec{r}_e}{|p_a|}\times\vec{F}_e + \sum_k\frac{\vec{r}_k}{|p_a|}\times\vec{F}_k + \sum_{i\neq j}\frac{\vec{r}_i}{|p_a|}\times N_i\left(\vec{n}_i - \mu\,\frac{\Delta\vec{s}_i}{|\Delta\vec{s}_i|}\right) = \\ = L_{acc}\left(\frac{\vec{r}_a}{|p_a|}\times\frac{p_l}{|p_a|}\,\vec{\theta}_s + \vec{\theta}_s\right) \end{cases} \quad (12)$$

A negative sign of the value L_{acc} indicates self-locking, because the friction, which works against the accelerating load, cannot change the direction of the acceleration. Thus, a negative sign of the accelerating load means no motion at all.

3. Verification of the method

With the presented method, the limiting coefficient of friction can be determined approximately by varying the friction until the accelerating load is reduced to zero.

For verifying the method, the limiting coefficients of friction were estimated for a standard kinematic coupling (see Figure 2.) and compared with the known limiting coefficients of friction without deformation effect in Ref. [6] (see Table 1).

This kinematic coupling has three v-grooves equidistant arranged on the circumference of a circle and three balls matching to the v-grooves. The nesting force is applied on the center of the circle. The limiting coefficients of friction are estimated for several v-groove angles β, which is the angle between the contact plane and the vertical. A change in the diameter of the circle or the balls has no influence to the limiting coefficient of friction.

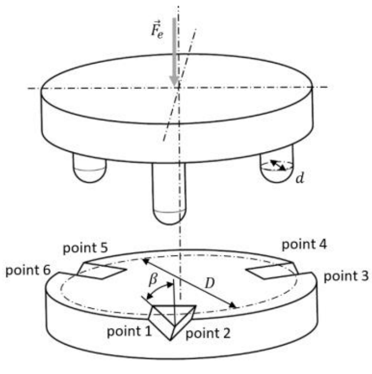

Figure 2. Standard kinematic coupling with central nesting force, contact point diameter, ball diameter and v-groove angle

The limiting coefficients of friction of Ref. [6] are extracted from 'Fig. 6. : Limiting coefficient of a standard kinematic coupling.' in the reference and are not calculated in this article. That is why the values are indicated only approximately.

Comparison of limiting coefficient of friction		
v-groove angle β	limiting coefficient of friction without deformation effect in Ref. [6]	limiting coefficient of friction estimated with the presented method
10°	~0.3	0.298
20°	~0.355	0.354
30°	~0.365	0.361
40°	~0.34	0.337
50°	~0.29	0.291
60°	~0.23	0.230

Table 1 Comparison of limiting coefficient of friction determined by two different methods

The limiting coefficients of friction estimated by the method in this article show a good congruence with those of Ref. [6], so that the method seems to offer a reliable tool for examinating kinematic couplings in regard to the limiting coefficient of friction. This comparison does not allow a statement about a correct calculation of the accelerating load L_{acc} and the values N_i of the contact forces, because the accelerating load is reduced to zero by varying the coefficient of friction.

Thus, the calculation of the contact forces according to equations (12) is tested by a comparison with contact forces calculated by using the static equilibrium. In the following example, the friction is set to zero and an additional lateral force $\vec{F}_{lat}$ is applied to a kinematic coupling parallel to a v-groove in the plane, which is defined by the centre points of the three balls (see Figure 3.), so that the accelerating load L_{acc} vanishes and the equations (12) as well as the equations (13) of static equilibrium can be applied.

$$\begin{cases} \vec{F}_e + \vec{F}_{lat} + \sum_i N_i\,\vec{n}_i = 0 & i = 1,2,\dots,6 \\ \vec{r}_e\times\vec{F}_e + \vec{r}_{lat}\times\vec{F}_{lat} + \sum_i\vec{r}_i\times N_i\,\vec{n}_i = 0 \end{cases} \quad (13)$$

The accelerating load and the contact forces in point 4 and 5 vanish for a ratio of the lateral force $\vec{F}_{lat}$ to the nesting force $\vec{F}_e$ of nearly 0.577, which corresponds to an inclination of the central axis of the kinematic coupling of nearly 30° to the gravitational direction, if nesting force and lateral force only result from gravity. In Table 2 are shown the calculated values of the accelerating load and the contact forces for the load cases 'no contact in point 4' and 'no contact in point 5' and for the static equilibrium.

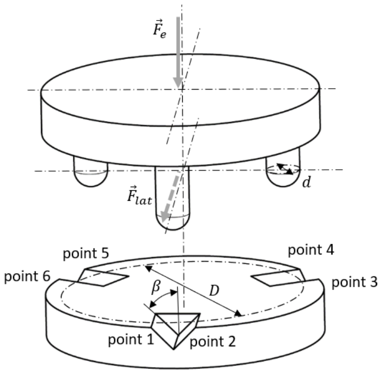

Figure 3. Standard kinematic coupling with central nesting force, lateral force and the position of the contact points

Comparison of contact forces

diameter of contact points	$D = 100\,mm$
diameter of balls	$d = 10\,mm$
v-groove angle	$\beta = 45°$
nesting force	$F_e = 10\,N$
lateral force	$F_{lat} = 5.774\,N$
coefficient of friction	$\mu = 0$

no contact in	$\frac{L_{acc}}{N}$	$\frac{N_1}{N}$	$\frac{N_2}{N}$	$\frac{N_3}{N}$	$\frac{N_4}{N}$	$\frac{N_5}{N}$	$\frac{N_6}{N}$
point 4	0.0	2.4	2.4	4.7	--	0.0	4.7
point 5	0.0	2.4	2.4	4.7	0.0	--	4.7
static equi.	--	2.4	2.4	4.7	0.0	0.0	4.7

Table 2 Comparison of contact forces

The comparison between the calculation of the contact forces according to the equations (12) and (13) show a good congruence, so that the equations (12) seem to be correct.

4. Application and limitations of the method

The limiting coefficient of friction is not a pure property of the geometry of a kinematic coupling, it depends also from the applied nesting force or direction of gravity, which is a well known phenomenon in the matching process of a kinematic coupling. Besides the limiting coefficient and self-locking, a possible lift-off in an already matched contact point is also counterproductive for the matching process. A lift-off is indicated by a negative sign of a contact force.

In the following example of Figure 3, the friction is set to 0.1 and the ratio of lateral force $\vec{F}_{lat}$ to the nesting force $\vec{F}_e$ is changed to 0.521, which corresponds to an inclination of the kinematic coupling of nearly 27.5°. In Table 3, the values of the accelerating load L_{acc} and the contact forces N_i for all six possible constellations are listed for showing a matching step with lift off resulting in a matching step with self-locking.

For the load case 'no contact in point 1' in the first line of the table, the point 4 lifts off indicated by the negative value for N_4 ($N_4/N = -0.1$). This leads to the load case 'no contact in point 4' in the fourth line, where the values of the accelerating load is negative and no aligning is possible. Also the load case 'no contact in point 2' with lift off in point 5 results in load case 'no contact in point 5' with self-locking. Only in the load cases 'no contact in point 3' and 'no contact in point 6' an alignment of the kinematic coupling is possible.

In Ref. [6] a method is presented, which includes the effect of deformation for self-locking and differs significantly in the results for small v-groove angles from the methods excluding deformation. Thus, the method described in this article cannot be applied for kinematic couplings with small v-groove angles.

Accelerating load and contact forces

diameter of contact points	$D = 100\,mm$
diameter of balls	$d = 10\,mm$
v-groove angle	$\beta = 45°$
nesting force	$F_e = 10\,N$
lateral force	$F_{lat} = 5.210\,N$
coefficient of friction	$\mu = 0.1$

no contact in	$\frac{L_{acc}}{N}$	$\frac{N_1}{N}$	$\frac{N_2}{N}$	$\frac{N_3}{N}$	$\frac{N_4}{N}$	$\frac{N_5}{N}$	$\frac{N_6}{N}$
point 1	2.4	--	2.9	6.2	**-0.1**	2.0	4.3
point 2	2.4	2.9	--	4.3	2.0	**-0.1**	6.2
point 3	5.9	6.4	2.1	--	0.9	4.3	4.0
point 4	**-0.9**	2.1	1.9	4.8	--	0.1	4.1
point 5	**-0.9**	1.9	2.1	4.1	0.1	--	4.8
point 6	5.9	2.1	6.4	4.0	4.3	0.9	--

Table 3 Values of accelerating load and contact forces with lift-off and self locking

An example for such a possible deformation is a ball sticking similar to a wedge in an acute-angled v-groove held by friction and an internal force loop through ball and v-groove due to elasticity. This means otherwise that the two parts of the kinematic coupling stick together and have to be pulled apart with force for separating them. This behaviour is unlikely for a common kinematic coupling, so that the presented method should be suitable for most kinematic couplings.

5. Summary

The presented method allows to check a kinematic coupling in regard to self-locking and lift-off without solving a system of nonlinear equations. This enables an unsophisticated application of the method using standard software available nearly for everybody in the design process and not only for simulation specialists, which is especially important for a short development time.

The approach enables the examination of self-locking as well as the examination of a lift-off of an already matched contact point, which is also important for designing the matching process.

For the future, a better verification of the range of validity for the method should be better examined concerning small v-groove angles

At the moment the presented method treats only the nesting process. But with the information obtained from the nesting process, such as the variation of the contact forces, a better prediction of the repeatability of the kinematic coupling could be possible.

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