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## High-accuracy projector calibration method by reducing perspective transformation error

Jin Yu<sup>1,2</sup>, Zonghua Zhang<sup>1</sup>, Feng Gao<sup>2</sup>, and Xiangqian Jiang<sup>2</sup>

<sup>1</sup>School of Mechanical Engineering, Hebei University of Technology, Tianjin, 300130, China

<sup>2</sup>EPSRC Future Metrology Hub, University of Huddersfield, Huddersfield, HD1 3DH, UK

[J.Yu@hud.ac.uk](mailto:J.Yu@hud.ac.uk); 363782772@qq.com

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### Abstract

This paper proposes a novel projector calibration method to improve the calibration accuracy of digital light processing (DLP) projector. The existing projector calibration methods usually fix the position of the camera and projector without considering the effects caused by the perspective projection error of the captured images. By fixing the position of the camera image plane parallel to the calibration board and the camera image centre cocentre to the calibration board centre, the proposed method essentially reduces the perspective transformation error and effectively reduces the distortion of the extracted marker points. The proposed projector calibration procedures are given as follows: Firstly, the optical axis of the camera is adjusted parallel to the normal of the hollow ring calibration board and cocentre to the calibration board, and a texture image is captured by the camera; Secondly, the horizontal and vertical fringe patterns with nine different positions and directions are projected onto the calibration board, and nine sets of projected images are taken; Finally, a one-to-one correspondence between the camera and the projector is established, and the projector is accurately calibrated by using the phase equivalence. The experimental results show that the proposed projector calibration method is feasible and easy to operate, which can essentially reduce the perspective transformation error to improve the calibration and measurement accuracy.

Structured light; projector calibration; perspective transformation; 3-D measurement

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### 1. Introduction

Structured light three-dimensional (3-D) sensing technology based on fringe projection profilometry (FPP) has become the mainstream optical method on 3-D shape measurement due to its advantages of non-contact, high accuracy, full-field measurement, and efficient point cloud reconstruction [1-3]. It has been widely used in industrial inspection, reverse engineering, biomedical, architecture and heritage conservation, etc. [4-6]. The calibration accuracy of the camera and projector plays an important role in the reconstruction procedure. The projector, unlike a camera, cannot capture images, so the relative relationship between projector pixels and object points cannot be obtained directly [7]. To address this problem, the projector is usually regarded as a reverse camera [8], and its high-accuracy calibration is still a remaining challenge.

The plane calibration board with multiple marker points has been widely used in computer vision fields such as system calibration and 3-D measurement because of its high positioning accuracy, easy identification, and good stability [8-10]. The positioning accuracy of the marker points directly affects the calibration accuracy of the camera and projector as well as the overall measurement system. Locating the center of the hollow ring coded marker point is mainly performed by extracting the innermost ring of the marker point. Many extraction methods have been studied, such as the ellipse fitting method based on image edges [11], the Hough transform method [12,13], and the grayscale center-of-mass method based on image grayscale [14].

However, the ring marker points are greatly affected by the perspective projection transformation, and the points obtained

from the traditional ellipse fitting center have some deviation from the real ring center. To address these issues, this paper proposes a simple and flexible method to avoid perspective projection transformation, and then accurately calibrate a DLP projector. The procedures are given as follows: Firstly, the position of the camera and the calibration board is fixed to ensure that the optical axis of the camera is parallel to the normal of the calibration board. Then, the projector is moved to several different positions, and the camera captures the fringe patterns projected from different perspectives. Finally, the corresponding relationship between the camera and the projector is established, and the intrinsic and extrinsic parameters of the projector are calculated. By contrast with the conventional methods, the proposed method improves the calibration accuracy of the projector by ensuring that the camera is aligned with the calibration plate at each position, which also avoids elliptical deformation of the ring caused by the perspective phenomenon directly. The validation experiments demonstrated that the proposed method is simple and effective.

### 2. High-accuracy calibration of the projector

In FPP, a projector, mostly DLP projector, projects sinusoidal fringe patterns onto the surface of the measured object. Then the camera captures the distorted fringe patterns by depth modulation of the object surface. Finally, the depth information of the object surface is obtained by demodulating the phase information of the deformed fringe patterns. The relationship between absolute phase and 3-D data is established by system calibration. The DLP projector is an important device for structured light-based 3-D imaging system.

## 2.1. Generation of DMD images

The DLP projector can be used as a reverse camera. Digital micromirror device (DMD) of the projector emits light while the CCD camera receives light. The DMD image is converted from the CCD image pixel by pixel, which is called the "captured" image of the projector. Based on an ideal pinhole model of the projector, the mathematical transformation between the world coordinates  $(x_w, y_w, z_w)$  and the pixel coordinates  $(u, v)$  is as follows

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = s \begin{bmatrix} f_u^p & 0 & u_0^p & 0 \\ 0 & f_v^p & v_0^p & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R^p & T^p \\ 0_3^T & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} = s I^p E^p \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \quad (1)$$

where parameter  $s$  is an arbitrary scale factor.  $I^p$  represents the intrinsic parameter matrix of the projector, which consists of focal length coordinates  $(f_u^p, f_v^p)$  and main point coordinates  $(u_0^p, v_0^p)$ .  $E^p$  represents the extrinsic parameter matrix of the projector, which consist of  $3 \times 3$  rotation matrix  $R^p$  and  $3 \times 1$  translation vector  $T^p$  between the world coordinate system and the projector coordinate system. It can be seen from Eq. (1), the projector can be calibrated by the correspondence between world coordinates and pixel coordinates.

The most important step in projector calibration is the conversion of the pixel points in the camera pixel coordinate system to the projector pixel coordinate system. The corresponding relationship between camera pixel coordinate system and projector pixel coordinate system is established by using a series of horizontal and vertical fringe patterns. The camera captures the image reflected by the projector on the calibration board. Assuming that a point in the camera pixel coordinate system is  $p(u^c, v^c)$ , and its absolute phase values in horizontal and vertical directions are  $\phi_h(u^c, v^c)$  and  $\phi_v(u^c, v^c)$ , respectively, the corresponding point  $p'(u^p, v^p)$  in the projector pixel coordinate system is calculated by the following equations

$$\begin{cases} u_i^p = \frac{V \phi_v(u_i^c, v_i^c)}{2\pi N_v} + \frac{V}{2} \\ v_i^p = \frac{H \phi_h(u_i^c, v_i^c)}{2\pi N_h} + \frac{H}{2} \end{cases} \quad (2)$$

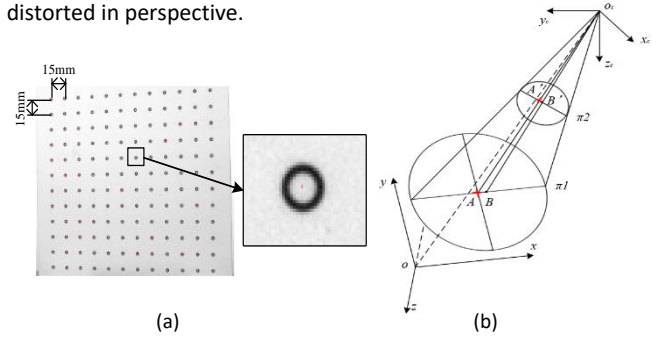
where  $N_h$  and  $N_v$  are the maximum number of fringes in the horizontal and vertical fringe patterns projected on the calibration board,  $V$  and  $H$  are the width and height of the fringe pattern, respectively. Through Eq. (2), the corresponding relationship between camera and projector imaging plane at each center of marker point can be established. The pixel coordinates of the center of all marker points under the projector pixel coordinate system are generated using the above conversion method, and then a set of DMD images is generated. Once the DMD images of the projector are obtained, the calibration procedure of the projector is similar to the calibration of the camera. The projector can be calibrated by the correspondence between world coordinates and pixel coordinates, and then the intrinsic and extrinsic parameters of the projector can be obtained.

## 2.2. Reduction of perspective error of the projector

The use of ring object targets is very common in spatial object calibration and reconstruction. The existing projector calibration methods usually fix the position of the camera and projector, and provide spatial point pixel coordinates by moving the ring object targets to different positions. The plane target at different positions are captured by the camera in sequence. The image of the calibration board at any one of these positions is shown in figure 1(a), and then the pixel coordinates of the center points of all image rings are extracted.

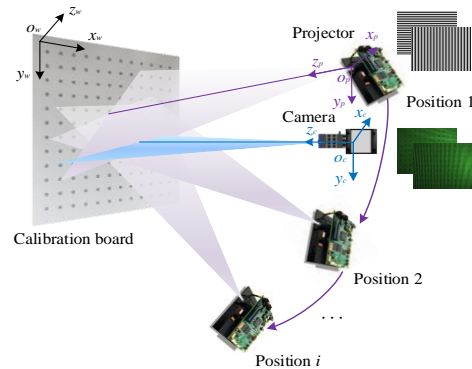
The projector is calibrated by establishing the conversion relationship between the pixel coordinates of the camera and the projector at each position. However, due to the perspective transformation of the camera, an object ring on the camera imaging plane will be elliptical deformed if the calibration board is not parallel to the camera imaging plane, resulting in the deviation of the pixel coordinates of the center of the extracted mark point, as shown in figure 1(b). The center of the ellipse does not coincide with the projection point of the center of the ring. The center of the ring plane  $\pi_1$  is the red cross A. After being photographed by the camera, it is elliptical on the camera imaging plane  $\pi_2$  with the projection point A', but the center B' of the ellipse should be extracted as the marker point. This pixel deviation of the extracted marker points directly affects the accuracy of the camera calibration, and will accumulate effect on the calibration accuracy of the projector.

The accuracy of the projector calibration is largely limited by the positioning accuracy of the ring center, because the pixel accuracy of the extracted center of the ring marker will directly affect the pixel conversion relationship between the camera and the projector. Therefore, the accuracy of projector calibration can be guaranteed by ensuring that the captured image is less distorted in perspective.



**Figure 1.** Ceramic hollow ring calibration board. (a) extracted ring center point image of the manufactured board; (b) perspective projection transformation of space ring

A novel projector calibration method is proposed to solve the problem of inaccurate extraction of hollow ring center point, which reduces the perspective error and does not require software correction for elliptical distortion. The calibration configuration of the proposed method is illustrated in figure 2. It ensures that the camera is toward the calibration board, that is, the optical axis of the camera is always parallel to the normal direction of the board and image centre is cocentre to the calibration board. The whole calibration procedure requires only the movement of the projector. In order to adapt to different working distances and orientations, the projector should be placed in several different positions and orientations within the camera's field of view.



**Figure 2.** The calibration configuration of the proposed method

In order to ensure that the captured image with minimum perspective distortion, it needs to adjust the camera toward the

calibration board. The specific steps are as follows: The first step is to make a rough adjustment to the height of the camera. The shooting viewpoint, that is, the height of the camera position, should be adjusted to have the same height as the object's midpoint. The second step is to make a fine adjustment to the relative position of the camera and the object.

### 2.3. Optimized projector calibration

In order to achieve better results in projector calibration, it is necessary to optimize each parameter to achieve the minimum value of re-projection error in the calibration results, where the objective function  $F$  is

$$F = \min \sum_{i=1}^N \sum_{j=1}^M \left\| p_{ij} - \tilde{p}_{ij}(I^p, R_j^p, T_j^p, P_i) \right\|^2 \quad (3)$$

where  $N$  is the number of reference points,  $M$  is the number of images for each position of the projector,  $p_{ij}$  denotes the pixel coordinates of the  $i$ -th marker point on the  $j$ -th DMD image,  $\tilde{p}_{ij}$  denotes the coordinates that are re-projected to the DMD image through the parameters to be optimized,  $I^p$  denotes the intrinsic parameters of the projector,  $R_j^p$  and  $T_j^p$  are the rotation matrix and translation vector of the  $j$ -th image,  $P_i$  is the input 3-D world coordinate. The objective function  $F$  is optimized by the nonlinear least square optimization algorithm [15], and the high-accuracy calibration results are obtained.

Procedure of the proposed projector calibration method mainly includes the following eight steps.

1) Fixing the position of the camera and the calibration board. In this position, ensuring that the optical axis of the camera is always parallel to the normal of the calibration board, and then taking a texture image of the hollow ring calibration board.

2) Placing the projector in the camera's field of view. In an arbitrary position, 24 images of the projected fringe patterns, 12 patterns horizontal and 12 patterns vertical, are captured by the camera.

3) Calculating the absolute phase value. The horizontal and vertical absolute phase values are obtained using the four-step phase-shift algorithm and the optimum three-fringe number selection method [16, 17].

4) Changing the positions and orientations of the projector  $n$  times. If  $n \leq 9$ , repeating step 2 and step 3 in different projecting positions.

5) Saving the horizontal and vertical absolute phases obtained from nine different projector positions, and extracting all hollow ring center points of 1 ring calibration board texture image.

6) Establishing the correspondence. The one-to-one mapping relationship between the camera and projector is established, allowing the conversion of camera pixel coordinates to projector pixel coordinates and generating a set of DMD images.

7) Calibrating the projector. The intrinsic and extrinsic parameters of the projector are obtained by substituting the known projector world coordinates and pixel coordinates into the camera calibration method.

8) Optimizing the projector calibration parameters. A nonlinear least square optimization algorithm is used to optimize the objective function  $F$  and reduce the re-projection error.

## 3. Experiments

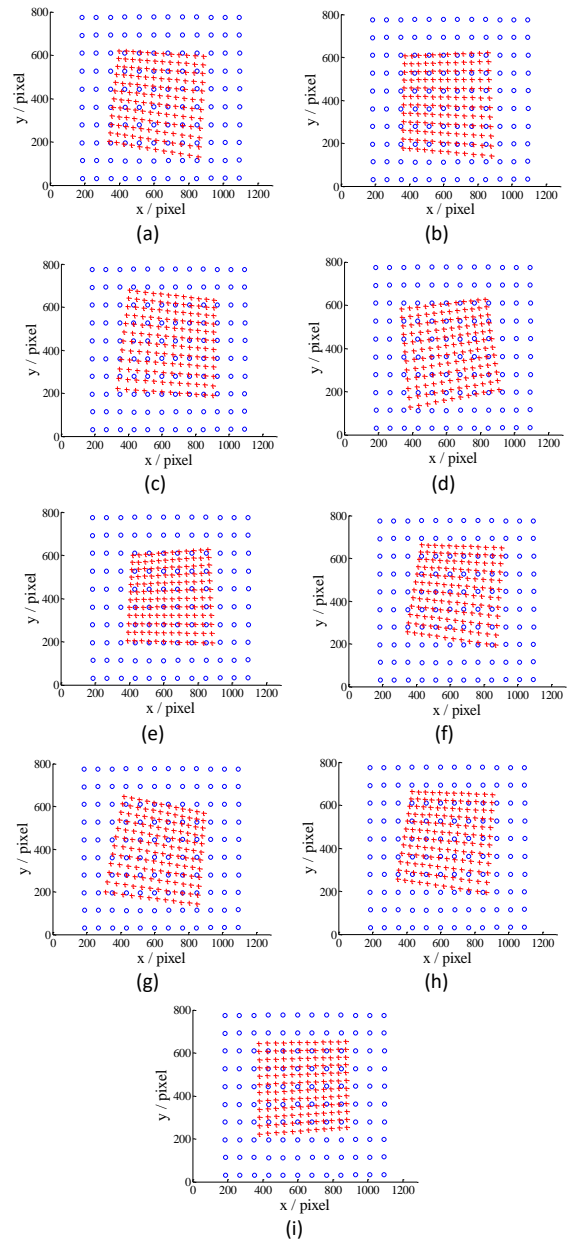
### 3.1. System setup

The hardware system mainly consists of a digital projector with a resolution of  $1280 \times 800$  pixels, a CCD camera and a planar ceramic calibration board (with a hollow ring pattern). The resolution of the camera is  $1296 \times 964$  pixels with a nominal focal length of 12-36 mm. There are  $12 \times 12$  discrete black hollow ring markers on the surface of the calibration board. The

separation of neighbor markers along X and Y direction has the same value of 15 mm with an accuracy of  $5 \mu\text{m}$ .

### 3.2. Experimental results

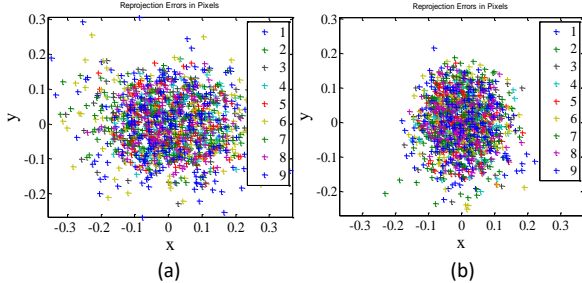
The experiments have been carried out to demonstrate the validity and feasibility of the proposed method. Totally one image of the ring calibration board and nine sets of projected fringe patterns are captured for the projector calibration. Each set of fringe patterns contains 12 horizontal and 12 vertical fringe patterns, and ensures that the camera is always toward the calibration board. The horizontal and vertical absolute phase values of nine positions are calculated to establish the corresponding relationship between the camera and the projector, and then nine sets of DMD images are generated. As shown in figure 3, one extracted ring calibration point image and nine generated DMD images are displayed on the same image. The blue dots indicate the pixel coordinates of the marker points on the camera imaging plane, and the red crosses indicate the pixel coordinates of the marker points on the projector imaging plane after coordinate transformation.



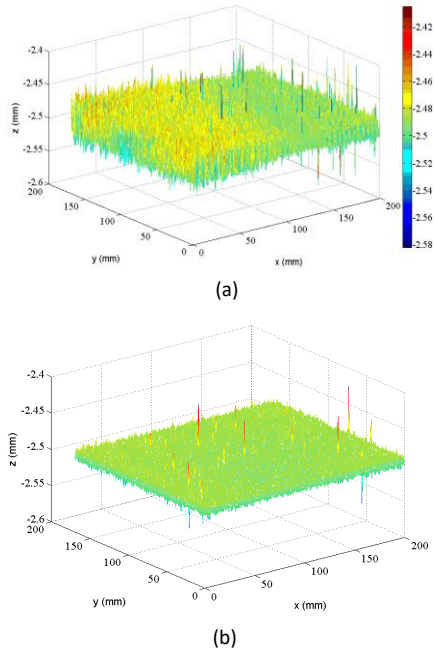
**Figure 3.** Generation of the DMD images. (a)-(i) are the generated DMD images of the nine projector positions

### 3.3. Experimental analysis

Figure 4 shows the re-projection error images of the projector with and without perspective projection transformation error method. The average re-projection errors of the two methods are [0.0950 0.0904] and [0.0683 0.0662] pixels. It can be seen that the re-projection error values in figure 3(b) are more aggregated, which verifies that the proposed method can effectively avoid elliptical deformation of the ring calibration board, reduce perspective errors, and further improve the center extraction accuracy.



**Figure 4.** Re-projection error images of the projector. (a) With and (b) without perspective projection transformation error



**Figure 5.** Distance difference images of the reconstructed white board between -2.5 mm position and the reference position. (a) With and (b) without perspective projection transformation error

To quantitatively evaluate the performance of the proposed projector calibration method, the white board is placed on an accurately translating stage with an accuracy of  $1\ \mu\text{m}$  and moved to the positions of -2.5 mm and 0 mm (chosen as the reference). 3-D shape is reconstructed by using the methods with and without the perspective projection transformation error. Figures 5(a) and (b) show the distance difference images of the white board of the two methods. The shape irregularity is significant when the perspective error is not eliminated and is greatly reduced by applying the proposed method. The comparison experiment confirms the effectiveness of reducing the perspective error to improve the measurement accuracy.

#### 4. Conclusion

A novel projector calibration method is proposed to provide higher measurement accuracy compared to the existing methods. It essentially reduces the influence of the geometric-optical perspective phenomenon on the calibration results by ensuring that the camera's optical axis is always parallel to the normal of the calibration board, and only the position of the

projector needs to be moved. It has the advantages of easy operation, simple equipment and high accuracy.

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