

## Model-based determination of the reproducibility of kinematic couplings

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### Abstract

Nanofabrication machines such as those developed at Technische Universität Ilmenau require an improved reproducibility of automated tool changing systems under vacuum conditions. Common designs based on kinematic couplings are limited by the sliding friction at the contact points. For the estimation of the reproducibility of the kinematic coupling, a simplified model is created. This model is based on the approach that the coefficient of friction varies at the point contacts and changes over coupling cycles due to the acting forces and further numerous influencing parameters. Since the load on the kinematic coupling is comparatively small these further frictional effects are expected to be most influential on reproducibility. During the coupling process, a force equilibrium is established at the final position of the coupling element between the forces resulting from the preload force, the centering force, and the frictional force. The analysis is performed utilizing a FE model to also consider the correlations between frictional effects and elastic deformations at the contact points. By varying the coefficient of friction at the individual contacts within defined limits, the reproducibility can be estimated.

For the evaluation of the results obtained with this model, reproducibility tests will be performed experimentally. The vacuum-compatible, automated measurement setup used for this purpose allows the measurement of the spacial position of the movable coupling half with a measurement uncertainty of less than 10 nm in length and less than 11 nrad in rotation. Based on the obtained reproducibilities for individual material pairings, the FE model can be adjusted accordingly. To refine the accuracy of the model further experimental investigation of different arrangements, material pairings, and different preload forces of the kinematic coupling is necessary.

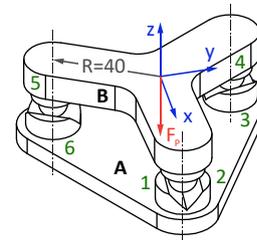
**Keywords:** kinematic coupling, nanofabrication, tool changing system, highest reproducibility, modeling approach

### 1. Introduction

Nanofabrication machines such as those developed at the Technische Universität Ilmenau require a highly reproducible tool-changing system, that is based on kinematic couplings. In this paper, a model for estimating the reproducibility of kinematic coupling is presented. By this, different design variants of the kinematic coupling can be investigated and, thus, the experimental effort can be reduced since only promising variants have to be considered further. To determine the reproducibility for those experimentally, a measurement setup with measurement uncertainties of less than 10 nm in each coordinate direction and 11 nrad around two coordinate axes was developed [1].

### 2. Modeling approach

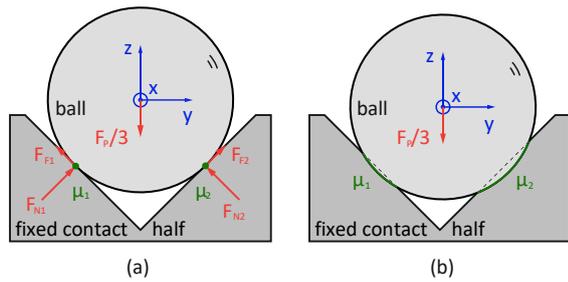
The kinematic couplings used for the tool changing system in the nanofabrication machines consist of six contact points arranged in a way that each DOF is blocked exactly once (see Figure 1). Since this is an open paired situation, a preload force is necessary, which is assumed to be applied in negative z-direction exactly at the geometric center and center of gravity of the kinematic coupling, without any deviations in amount and angle. Since the operating forces in the nanofabrication machines are negligible, a low preload force of about 20 N in sum is sufficient. Due to this, friction at the contact points between the movable and fixed contact halves has the most significant influence on the reproducibility of the coupling [2].



**Figure 1.** An exemplary design of a kinematic coupling with A) fixed coupling half consisting of three fixed contact halves and B) moving coupling half consisting of three balls. The preload force is applied in the negative z-direction.

The simplified modeling approach for an exemplary ball V-groove contact can be seen in Figure 2. This approach is based on a variation of the friction coefficient over several coupling cycles within predefined limits [3]. This variation is caused by local surface properties since the contact between the ball and the fixed contact element deviates slightly in its position during each coupling cycle. However, a position deviation of the moving coupling half, as well as any wear and run-in effects are not modeled. In the final position of the ball within the fixed contact half, a force equilibrium is established between preload force  $F_p$ , elastic counterforce  $F_N$ , and frictional force  $F_f$ . For this reason, only the part of the movement in which the ball is in contact with the fixed contact half at both points that solely results from the elastic deformation is considered in the modeling approach. The position of the balls, which deviates due to the variation of the friction, is then evaluated for the entire kinematic coupling. By

variations of the friction coefficient within the defined limits, the reproducibility of the kinematic coupling can be determined.



**Figure 2.** Modeling approach based on the variation of the friction coefficients (a) Initial state with the acting forces; (b) exemplary state for  $\mu_1 < \mu_2$  resulting in different elastic deformations due to different friction forces, which are counter-directed to the respective part of the preload force

While resulting displacements were calculated based on just one value for the coefficient of friction in previous works [4], the modeling approach presented here allows conclusions on the reproducibility under variation of the friction coefficients.

### 3. Modeling

The calculation of the reproducibility can be done analytically or numerically.

#### 3.1. Analytical model

An analytical model is a simplified approach in which a force and moment equilibrium is established for the whole kinematic coupling. The friction force  $F_F$  acting against the centering is also taken into account. The resulting deformations are determined based on the calculated acting forces according to Hertzian theory. The displacement of the moving coupling half of the kinematic coupling is then specified.

A complete consideration of a contact point including friction and tangential contact stiffness is possible, but very complex [5]. Furthermore, the interactions between the total of six contact points cannot be taken into account without difficulty.

#### 3.2. Numerical model

In contrast to previous works, the numerical model is based on a transient, structural-mechanical analysis, which is realized using the FE software ANSYS. The specific control of small time increments of the transient analysis is crucial to allow the contact initiation between the ball and fixed contact half. In analogy to the measurement setup, a preload force is introduced in the center of gravity of the moving half of the kinematic coupling, starting at zero and increasing with every time increment up to the final value of 20 N.

A frictional contact is used, whereby the transition between static and sliding friction is considered by a sudden change [6]. By this, even stick-slip effects are modeled. The contact formulation is carried out using the Augmented Lagrange Method since this is recommended for frictional contacts [7]. In comparison to the Pure Penalty method, it is less sensitive to the selection of the normal contact stiffness. In addition, unlike the Normal Lagrange method, symmetrical treatment of the contact is also possible, which means that deformations of both the ball and the fixed contact half are considered [7].

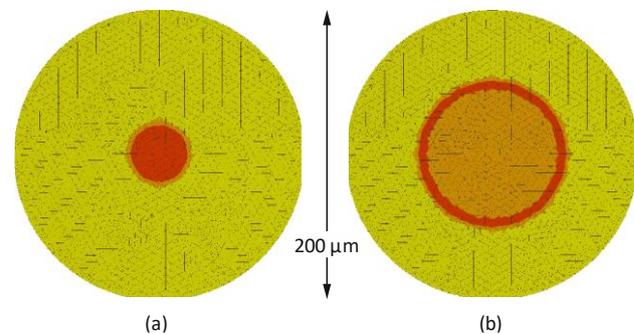
Furthermore, integration point detection is used instead of nodal detection, which increases the number of detection points and improves contact finding [7]. The contact is adjusted for

touch, which means the FE program controls the contact offset in such a way that a possible initial gap due to geometric or numerical inaccuracies of the CAD model is closed. A mesh refinement at the contact surfaces using tetrahedral elements is selected. Since the size of the elements at the contact surface has a significant influence on the results, a mesh convergence analysis is carried out in chapter 3.3.

The results of the displacements are evaluated at an external point located in the center of the moving coupling half in the plane of the three balls' centers. Because the center of rotation of the kinematic coupling is close to this point, the translatory displacements of the moving coupling half are not influenced by rotations.

#### 3.3. Comparison and validation of the models

In the analytical model, the tangential stiffness of the contact can be taken into account, but the sliding motion within the contact cannot be modeled without further effort. Thus, the distinction between sticking and sliding motion is not possible. Furthermore, initial calculations with the analytical model also show a significantly varying reproducibility in the x- and the y-direction, which is not expected for the symmetrical design of the kinematic coupling. The transient FE model offers the possibility to represent sticking and sliding. For each of the program-controlled time increments of the transient FE analysis, the contact status is calculated and it is evaluated whether a sticking or a sliding motion is occurring. Figure 3 shows the contact modeled with the FE software. At the beginning of the preload force application ( $F = 0.1$  N), the contact has a small area of sticking in the center, and sliding occurs around that. With increasing preload, the sliding region spreads inside towards the center of the contact, as described in [5]. For the reasons mentioned, the analytical model is not investigated furthermore.



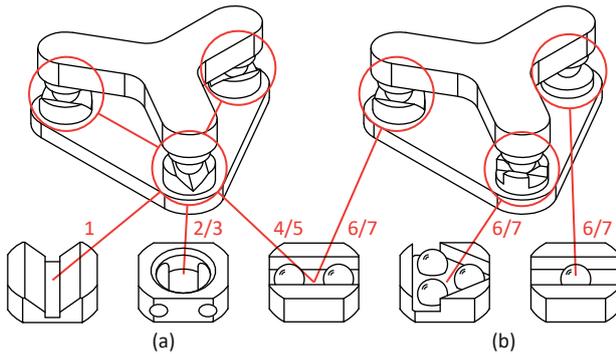
**Figure 3.** One of the ball-V-groove contacts of a kinematic coupling with a fine-meshed area from 200  $\mu\text{m}$  in diameter. Sticking (red) and sliding (orange) are shown for (a)  $F = 0.1$  N; (b)  $F = 20$  N

To exclude the influence of the meshing on the displacement of the moving coupling half, a convergence analysis is performed. The element size at the contact points is varied incrementally from 100  $\mu\text{m}$  to 2  $\mu\text{m}$  and the stresses and deformations at the contact points are evaluated. It can be proven that the results for stress and deformation are nearly independent of the meshing when the element size is 5  $\mu\text{m}$  or less. The maximum stress had to be evaluated within the body since stress singularities occur at the contact surfaces. These are no further relevant since convergence could be shown and deformations at the locations of the stress singularities are still correct. Thus, these singularities do not influence the calculation of reproducibility [8]. As a result, an element size of 5  $\mu\text{m}$  is chosen as a good compromise between mesh independence and computing time.

To prove that the targeted reproducibilities of the kinematic coupling in the range of 50 nm can be simulated with this FE model, the numerical uncertainty of the model is investigated. For this purpose, a friction coefficient of zero is set at all six contact points. In doing so, the displacement of the moving half of the coupling is expected to be zero in five directions except for the z-direction. The result of the simulation is a displacement of the moving coupling half in the order of  $10^{-11}$  m, which represents the numerical uncertainty of the model and is significantly below the targeted reproducibility.

#### 4. Results for the reproducibility

The design of the kinematic coupling has been taken from the developed measurement setup for the determination of reproducibility [1]. This results in a coupling diameter of 80 mm, whereby the kinematic coupling can be designed in form of a Maxwell or Kelvin clamp (see Figure 4). Furthermore, the V-grooves can be realized via planes, cylinders, or balls. The default opening angle of the V-groove is specified at  $90^\circ$  within this work since this results in a uniform stiffness of the kinematic coupling in all directions [9]. Using the same opening angle for all arrangement variants also provides the best comparability of the results between the individual variants. The materials of the V-groove and balls can also be varied. For each of the variants resulting in Table 1 and Figure 4, a separate simulation model was created.



**Figure 4.** A kinematic coupling as (a) Maxwell Clamp and (b) Kelvin Clamp with the possible designs of the fixed contact half (the numbers refer to the variants presented in Table 1)

**Table 1.** Investigated combinations

#	arrangement	material pairing; diameters	coefficient of friction
1	Maxwell clamp (V-groove)	steel/steel ball = 10 mm; V-groove = $\infty$	$0.5 \pm 0.05$
2	Maxwell clamp (cylindrical pins)	steel/steel ball = 10 mm; pin = 4 mm	$0.5 \pm 0.05$
3	Maxwell clamp (cylindrical pins)	steel/silicon carbide ball = 10 mm; pin = 4 mm	$0.6 \pm 0.05$
4	Maxwell clamp (balls)	steel/steel ball = 8 mm; ball <sub>V-groove</sub> = 6 mm	$0.5 \pm 0.05$
5	Maxwell clamp (balls)	steel/silicon carbide ball = 8 mm; ball <sub>V-groove</sub> = 6 mm	$0.6 \pm 0.05$
6	Kelvin clamp	steel/steel ball = 8 mm; ball <sub>fixed contact half</sub> = 6 mm	$0.5 \pm 0.05$
7	Kelvin clamp	steel/silicon carbide ball = 8 mm; ball <sub>fixed contact half</sub> = 6 mm	$0.6 \pm 0.05$

The coefficient of static friction for the steel-steel pairing is given in the literature as a value of approximately 0.5 for a dry pairing [10]. For the dynamic coefficient of friction, 80% of the static coefficient of friction is assumed. Since the kinematic coupling in the nanofabrication machine is operated in a rough vacuum, the liquid lubricating film of the air is absent, resulting in a dry pairing. Due to the vacuum environment and required clean environment for nanofabrication, no use of lubricants is intended. A value of  $\pm 0.05$  with a rectangular distribution is assumed as the variation of the coefficient of friction for the simulations. For the ceramic/steel pairing, a coefficient of static friction of 0.6 is specified in a datasheet of a ball manufacturer, and the dynamic coefficient of friction is again estimated at 80% [11]. A value of  $\pm 0.05$  with a rectangular distribution is specified as the variation of the coefficient of friction here as well.

#### 4.1. Calculation of the reproducibility

To perform the simulations, the six friction coefficients at the V-grooves must be varied in such a way that the maximum displacement of the moving coupling half can be determined. This is realized using the design of experiments based on an Advanced Latin Hypercube experimental plan [12]. A total of 25 simulation cycles are performed for each investigated combination. Analogous to the measurement setup for determining the reproducibility, the reproducibility of the kinematic coupling around the tool axis, which is the z-axis, is not considered any further [1].

#### 4.2. Analysis of the obtained results

Based on the simulations performed, the standard deviations shown in Table 2 are generated. The reproducibility can be calculated for a confidence interval of  $\pm 3\sigma$ . In general, the minimum and maximum values of the displacement of the moving coupling half are symmetric around the zero position for all five analyzed freedoms. Likewise, the simulated standard deviations for  $\sigma_x$  and  $\sigma_y$ , and  $\sigma_{\phi_x}$  and  $\sigma_{\phi_y}$ , respectively, are almost identical for all variants. In addition, the mean value between the maximum and minimum displacement of about  $10^{-10}$  m again almost corresponds to the numerical accuracy of  $10^{-11}$  m specified in chapter 3.3. The minimal degradation caused by the used experimental plan based on statistical methods is accepted to reduce the number of simulations significantly. Based on these results, the fundamental capability of the simulation to determine the reproducibility can be confirmed.

**Table 2** Results of the reproducibility simulations (the numbers refer to the variants presented in Table 1). For #6 and #7, the values in the brackets correspond to an evaluation directly in the sphere-tetrahedron pairing.

#	$\sigma_x$ [nm]	$\sigma_y$ [nm]	$\sigma_z$ [nm]	$\sigma_{\phi_x}$ [nrad]	$\sigma_{\phi_y}$ [nrad]
1	6.2	6.2	3.8	139.7	139.7
2	8.2	8.2	4.9	179.5	178.9
3	6.6	6.6	5.1	184.1	183.3
4	9.5	9.4	5.5	194.6	198.3
5	6.1	6.2	4.2	161.2	158.5
6	7.1 (7.1)	5.3 (7.9)	3.9 (6.0)	142.6	130.5
7	4.3 (4.3)	3.2 (4.2)	2.2 (3.4)	81.6	76.2

It can be concluded that a higher stiffness, in general, leads to a higher reproducibility. This higher stiffness can be achieved either by the material pairing or by the design of the fixed contact half. The fixed contact half becomes stiffer as it approaches a plane. Due to the higher stiffness, elastic deformations and the variation of elastic deformation resulting from the variation of friction are smaller. The variant presented in [2] of realizing the target bodies with curved surfaces was not

investigated in this paper, since the manufacturing of this geometry is very difficult. In addition, when cylindrical pins and balls are used instead of V-grooves, due to the curvature in one or two directions, a movement of the ball in one direction leads to a movement in other directions. Furthermore, due to its design, the fixed contact half has always a higher stiffness in the z-direction than in the x- and y-direction. This leads to a slightly higher reproducibility in the z-direction for all investigated variants.

The Kelvin arrangement shows a non-uniform reproducibility in all spatial directions due to the non-uniform distribution of the stiffness of the fixed contact halves. When the evaluation is made in the sphere-tetrahedron pairing instead, it shows a uniformly distributed reproducibility.

The simulation results obtained can be used to derive general design guidelines and properties for highly reproducible kinematic couplings:

- The coefficient of friction and difference of the coefficient for sliding and sticking of the material pairing should be as low as possible since this also reduces the variation of the coefficient of friction [13]
- The stiffness of the used materials and design should be as high as possible for the highest reproducibility
- Curved V-grooves consisting of cylindrical pins or balls result in slightly lower reproducibility
- The reproducibility needs to be determined in the plane of the three ball-centers to ensure that the translatory reproducibility does not contain any rotatory components
- The reproducibility in the z-direction is slightly higher than in the x- and y-direction since friction effects have a higher influence in these directions
- The Kelvin clamp shows a non-uniform reproducibility in all special directions when the evaluation is carried out in the center of the moving coupling half

The next steps are the experimental verification of these design guidelines with the developed measurement setup [1] and an extension and adaptation of the model. Preselection of the combinations to be investigated experimentally can be done based on the results of the model. This allows a reduction of the experimental effort. Furthermore, the variation of the coefficient of friction in the model can be refined based on measurement results. This allows the model to be used to make predictions about the reproducibility of kinematic couplings with other arrangements, sizes, and loads. However, so far the modeling was based on an ideal coupling process without geometric deviations of the moving coupling half. In the next steps, it will be investigated how geometric deviations of the moving coupling half during the coupling process influence the reproducibility of the model and how they can be integrated into the model in a simplified way. This may allow further variants of the kinematic coupling to be simulated with the model.

## 5. Summary and outlook

In summary, the numerical FE-model created is very well qualified for estimating the reproducibility of the kinematic coupling. The analytical model, on the other hand, is not applicable because it does not show the symmetrical distribution of the deviations and does not represent the transition between sticking and sliding nor interactions between the contacts. The numerical model has been used to calculate and compare the reproducibility of a total of seven promising variants of the kinematic coupling. From this, initial design guidelines for highly reproducible couplings are derived. Due to

the large uncertainty in the friction coefficients, however, the model cannot be used to make any predictions to absolute values of reproducibility.

In the next steps, the results obtained must be investigated experimentally by use of the developed measurement setup [1]. This allows the overall number of experiments to be reduced. Furthermore, the model parameters can be adapted in such a way that predictions concerning the reproducibility of kinematic couplings with other arrangements, dimensions, and loads are possible. An extension of the model to include geometric deviations and displacements of the upper coupling half during the coupling process may be considered as well.

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