euspen's 22nd International Conference & Exhibition, Geneva, CH, May/June 2022

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High-precision parallel kinematic tip/tilt mirror with ± 30° motion range

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Abstract

For precise optical applications, tip/tilt mirrors are widely used. Consequently, they are designed to perform high-dynamic motions with high precision that usually leads to a comparably small motion range. In order to overcome this limitation, a high-precision parallel kinematic tip/tilt mirror is presented that allows a motion range of ± 30°. The presented parallel kinematic tip/tilt mirror is based on the spherical 5R parallel mechanism, where R denotes revolute joints, and is driven by two U-628.03 PILine® miniature rotation stages (by Physik Instrumente (PI) GmbH & Co. KG) allowing a high accuracy of the input shaft at a high speed. In this paper, the mechanism's design and kinematics is discussed and experimental results about the mechanism's accuracy are presented.

Parallel Kinematics, Tip/Tilt Mirror, High-Precision

1. Introduction

High-precision tip/tilt mirrors are widely used in numerous optical applications such as image processing and stabilization, optical filters, switches and traps, beam stabilization, laser micromachining, scanning, and many more. They are designed to rotate around two perpendicular axes and, in particular, to perform highly dynamic movements with high precision. This, however, is usually accompanied with a reduced motion range. In fact, the tip/tilt angles generally lie between ± 2 mrad for typical piezo mirrors and up to ± 35 mrad for special piezo mirrors and voice coil tip/tilt mirrors [1].

In order to overcome this limitation, a special type of parallel kinematic tip/tilt mirror can be used that allows large motion ranges, positioning payloads such as mirrors or lasers in the pivot point, has a simple inverse kinematics and a high stiffness to enable high angular velocities and accelerations. The proposed tip/tilt mirror is based on the spherical 5R parallel mechanism, a two-degrees-of-freedom device consisting of five revolute joints (denoted as R) where two of them are actuated and the others are passive. The joints' rotation axes intersect in a common point – the pivot point. They are connected by arc-shaped limbs. The limb in the middle contains the mirror whose tip/tilt angle can be controlled by the two actuated revolute joints.

The spherical 5R parallel mechanism was first mentioned by Ouerfelli and Kumar [2] and was patented as an orienting device by Gosselin [3]. It is a spherical version of the planar 5R parallel mechanism that also consists of five revolute joints that each are connected by passive links. One link represents the fixed base whose attached joints are actuated while the others are passive joints. The planar 5R parallel mechanism is very interesting as it can be used for high speed applications due to the lightweight and parallel kinematic structure but on the other hand has singularities within the workspace that need to be crossed or avoided for proper applications, see e.g. [4]. Due to the parallel structure, its direct kinematic description is quite complicated and raised interest of numerous researches, see e.g. [5-9]. As a further development of the spherical 5R parallel mechanism, in 1994, Gosselin and Hamel presented the agile eye, a three-degrees-of-freedom camera-orienting device [10].

As the spherical 5R parallel mechanism is very interesting in terms of its large motion range and high stiffness, in this paper, the mechanism is investigated concerning its usability as a high-precision tip/tilt mirror. As drives, two U-628.03 PILine® rotation stages (by Physik Instrumente (PI) GmbH & Co. KG) are used that allow a high accuracy of the input shaft (minimum incremental motion of 51 µrad) at a high maximum velocity of 720°/s. In contrast to the device proposed by Gosselin [3], the presented high-precision parallel kinematic tip/tilt mirror uses special u-shaped limbs to increase the mechanism's stiffness, see Figure 1.

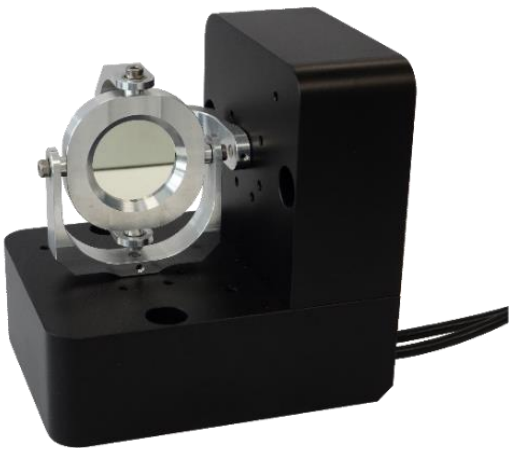

Figure 1. Photography of the proposed high-precision parallel kinematic tip/tilt mirror with ± 30° motion range.

The paper is structured as follows. In Section 2, the direct and inverse kinematics of the spherical 5R parallel mechanism is reviewed and, in Section 3, the mechanical design of the high-precision tip/tilt mirror with ± 30° motion range is presented. In Section 4, simulation results for estimating the mechanism's Eigenfrequencies are presented. Furthermore, experimental results are presented that include measurements on the minimal incremental motion, bidirectional repeatability, and linearity. In Section 5, the results are summarized and discussed.

2. Kinematics of the Spherical 5R Parallel Mechanism

Consider a parallel mechanism as shown in Figure 2 where the base holds two perpendicular axes that intersect in a common point. Limb 1 is driven by an active revolute joint and rotates around axis 1. Similarly, limb 2 is driven by an active revolute joint and rotates around axis 2. While limb 1 is directly linked to the mirror via a passive revolute joint, limb 2 is connected in series to limb 3 and then to the mirror (both via passive revolute joints). In consequence, the mirror can only rotate around the intersection point of the two axes by rotating the active revolute joints.

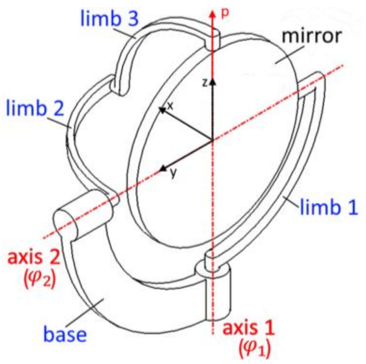

Figure 2. Kinematic structure of a spherical 5R parallel mechanism.

In the following, the inverse and direct kinematics of such a mechanism is revisited for arc-shaped limbs with an angle of 90°. While the inverse kinematics describes the coordinates of the active joints as a function of the mirror's pose and is comparably easy to solve, the direct kinematics describes the pose of the mirror with respect to the active joints' coordinates and is significantly more complicated to solve.

2.1. Inverse Kinematics of the Spherical 5R Parallel Mechanism

Knowing the two degrees of freedom of the parallel kinematic spherical 5R mechanism, i. e., the tip/tilt angles of the mirror, the input angles of the active revolute joints can be calculated. Consider α as the angle between the x-axis and the projection of the vector $\boldsymbol{p}$ in the $\boldsymbol{O}_{xy}$ plane and β as the angle between the vector $\boldsymbol{p}$ and the z-axis. The vector $\boldsymbol{p}$ represents the direction vector of the mirror as shown in Figure 2 and is given by

$$\boldsymbol{p} = [p_x \quad p_y \quad p_z]^T = [-\sin\beta\cos\alpha \quad \sin\beta\sin\alpha \quad \cos\beta]^T .$$

The angles for the active joints φ_1 and φ_2 can then be calculated as follows:

$$\varphi_1 = \begin{cases} -\arccos(\sin\alpha) + 90° \\ -\arccos(\sin\alpha) - 90° \end{cases},$$

$$\varphi_2 = \begin{cases} 180° - \arccos\left(\dfrac{\cos\beta}{\sin(\arccos(\sin\alpha\sin\beta))}\right) \\ -\arccos\left(\dfrac{\cos\beta}{\sin(\arccos(\sin\alpha\sin\beta))}\right) \end{cases}.$$

It can be seen that two angles are possible for each active joint, resulting in four solutions for the inverse kinematics – the four corresponding working modes of the mechanism. If the angle of the passive revolute joint φ_3 is required in addition, its four solutions can be calculated as follows:

$$\varphi_3 = \begin{cases} -\arccos(\sin\beta\sin\alpha) + 180° \\ \arccos(\sin\beta\sin\alpha) \\ \arccos(\sin\beta\sin\alpha) - 180° \\ -\arccos(\sin\beta\sin\alpha) \end{cases}.$$

2.2. Direct Kinematics of the Spherical 5R Parallel Mechanism

The solution for the direct kinematics problem of the spherical 5R parallel mechanisms was presented in [2]. Using the following abbreviations:

$$\begin{aligned} a_1 &= \sin\varphi_1 , & a_2 &= 0 , \\ b_1 &= -\cos\varphi_1 , & b_2 &= -\cos\varphi_2 \\ c_1 &= 0 , & c_2 &= -\sin\varphi_2 , \\ d_1 &= 0 , & d_2 &= 0 , \end{aligned}$$

the mechanism's closure equation can be written as a quadratic expression for the z-component of the mirror's pose $\boldsymbol{p}$:

$$Ap_z^2 + Bp_z + C = 0 ,$$

with

$$A = (a_1b_2 - a_2b_1)^2 + (a_1c_2 - a_2c_1)^2 + (b_1c_2 - b_2c_1)^2 ,$$
$$B = -2\left((a_1c_2 - a_2c_1)(a_1d_2 - a_2d_1) + (b_1c_2 - b_2c_1)(b_1d_2 - b_2d_1)\right) ,$$
$$C = (a_1d_2 - a_2d_1)^2 + (a_1b_2 - a_2b_1)^2 + (b_1d_2 - b_2d_1)^2 .$$

From this, the components of the mirror's direction vector $\boldsymbol{p}$ can be written as

$$p_z = -\frac{B}{2A} \pm \sqrt{\left(\frac{B}{2A}\right)^2 - \frac{C}{A}},$$
$$p_y = -p_z\tan\varphi_2 ,$$
$$p_x = -\sqrt{1 - p_y^2 - p_z^2} .$$

Note that p_z (and consequently the mirror's pose $\boldsymbol{p}$) has two solutions that corresponds to the two assembly modes of the parallel mechanism.

3. Mechanical Design of the High-Precision Tip/Tilt Mirror

Knowing the mechanism's kinematics, the mechanical design of the high-precision tip/tilt mirror is presented in this section.

From Figure 2 and the kinematic description in Section 2 it can be seen that the mechanism in total consists of a base, three limbs, a mirror as well as passive and active revolute joints. As mirror, a float glass mirror with a diameter of 22 mm, a surface flatness of 4-6 λ (i.e., a deviation from a flat surface measured in values of wavelengths), and a recommended wavelength range of 400-700 nm is used. The mirror is held in an outer platform so that the surface of the mirror exactly lies at the pivot point of the mechanism. The outer platform also allows to connect the limbs' axes. Here, miniature ball bearings (by SBN) with an outer diameter of 6 mm are used to address the little space.

Arc-shaped limbs with an angle of 90° are used to connect the outer platform with the base platform. In order to increase the mechanism's stiffness and to counterbalance the miniature ball bearings, the first and third limb are u-shaped, see Figure 1. In consequence, two additional passive joints are required. This increases the mechanism's stiffness while the range of motion of the limbs and therewith of the mirror is simultaneously reduced. In fact, with a mechanism consisting solely of 90° arc-shaped limbs, full rotability around both axes would be possible, see [1-2]. As singularities limit the workspace down to ± 90°, this however would not be practically applicable. By using u-shape limbs instead, only rotation angles of ± 30° are possible (in theory ± 45°, but further limited by the non-zero limbs' width).

As active joints, two U-628 PILine® rotation stages are used. These are miniature stages with ultrasonic piezo motors and a drive torque of 25 mNm. They are connected to the limbs via a motor shaft. The rotation stages have an unlimited range of motion and a maximum velocity of 720°/s. Furthermore, they are very accurate with a minimal incremental motion of 51 µrad and a bidirectional positioning repeatability of 102 µrad [11]. The two rotation stages are mounted inside of the housing that is closed by a cover plate. The housing itself consists of two identical parts that are mounted perpendicularly. Holes in both parts of the housing allow multiple mounting possibilities.

The two rotation stages can be driven by a C-867.2U2 motion controller where the kinematics of the parallel mechanism can be implemented. The parts and assembly of the high-precision tip/tilt mirror are shown in Figure 3 in a sectional view.

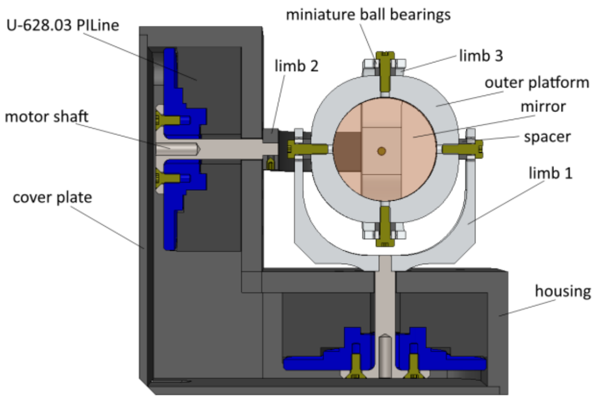

Figure 3. Components and assembly of the high-precision tip/tilt mirror.

4. Simulation and Experimental Results

Knowing the kinematics and the components of the parallel kinematic tip/tilt mirror, simulation and experimental results are presented in this section that helps evaluating the mechanism's performance. Main aspects in this section are the mechanism's Eigenfrequencies and its minimal incremental motion.

The Eigenfrequencies limit the control bandwidth and are mainly defined by the mechanical design (the shape, stiffness, material, and arrangement of the mechanism's components). In contrast to that, the achievable minimal incremental motion of the mechanism is largely determined by the accuracy of the active joints and the transfer function of the parallel kinematic mechanism.

4.1. Eigenfrequencies of the High-Precision Tip/Tilt Mirror

Knowing the mechanical design of the high-precision tip/tilt mirror, it is possible to estimate its Eigenfrequencies by using an FEM simulation tool. In order to reduce the complexity, the miniature ball bearings are modelled as 6d springs with the following stiffness matrix ***C***:

$$\boldsymbol{C} = \begin{bmatrix} 15{,}333 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15{,}333 & 0 & 0 & 0 & 0 \\ 0 & 0 & 15{,}241 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16{,}616 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16{,}616 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where ***C*** has the units N/mm and Nmm/°. The limbs and the outer platform are made from Alloy while the motor shafts are made of stainless steel.

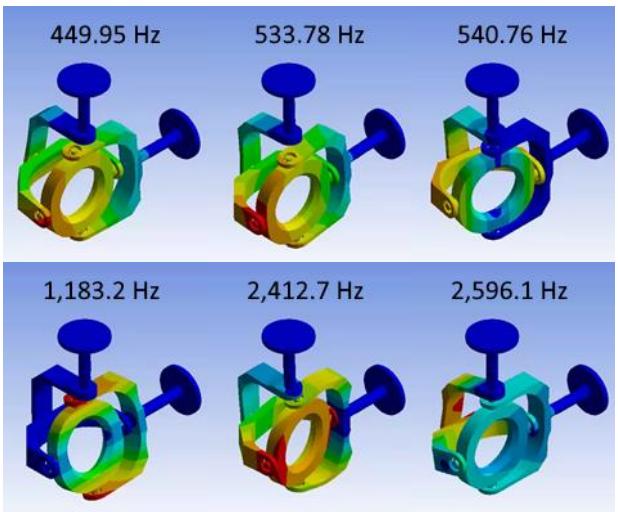

Figure 4. Eigenfrequencies and their corresponding Eigenmodes of the high-precision tip/tilt mirror.

Using those information, the mechanism's Eigenfrequencies can be computed. Here, the FEM software Ansys Mechanical is used. For the simulation, the active joints are considered to be fixed. Figure 4 shows the first six Eigenfrequencies and their corresponding Eigenmodes. It can be seen that the first Eigenfrequency lies at 449,95 Hz followed very closely by two others (533.78 Hz and 540.76 Hz). The mechanism's stiffness seems to be limited by limb 2, the only limb that is not u-shaped. If entirely arc-shaped limbs with an angle of 90° were used, the Eigenfrequencies would be significantly lower. In fact, according to FEM simulations, the Eigenfrequencies would instead lie at 254.76 Hz followed by 331.87 Hz and 477.18 Hz.

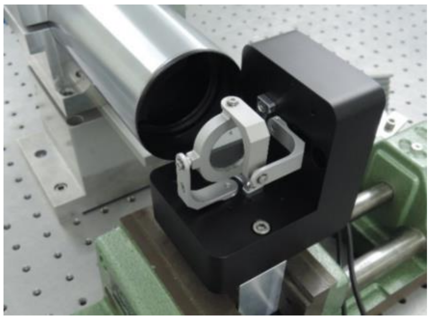

Figure 5. Measurement setup for the experimental investigation of the high-precision tip/tilt mirror.

4.2 Minimal Incremental Motion

In this section, the achievable accuracy of the high-precision tip/tilt mirror is experimental investigated. As a measurement device, the ZMI1000 interferometer is used. Figure 5 shows the measurement setup. The minimal incremental motion is tested for several input angles $\varphi_1$and φ_2, with

$$\varphi_i \in [-25°, 20°, \cdots, 20°, 25] \quad \forall i \text{ in } \{1,2\}.$$

This is done by setting the mirror at a designated pose and then by testing different incremental step sizes until the test criteria are broken. The test criteria are based on DIN ISO 230:2014-5 and ASME B5.54.2005. Figure 6 shows representative results for a minimum incremental motion measurement with a step size of 110 µrad. In total, 20 steps with the tested incremental step size are performed, ten in the one and ten in the other direction. Afterwards, the measurement data are evaluated regarding the average step size deviation, the variation of the step sizes, and the maximum value of noise. For comparison, Figure 6 shows both, successful and unsuccessful measurements results. It can be noticed that the unsuccessful result has a lower average step size deviations (1.2 % vs. 2.1 %), but also shows significantly higher variations in the step sizes (75.6 µrad vs. 25.5 µrad) and noise (0.4 µrad vs. 0.2 µrad) compared to the successful result.

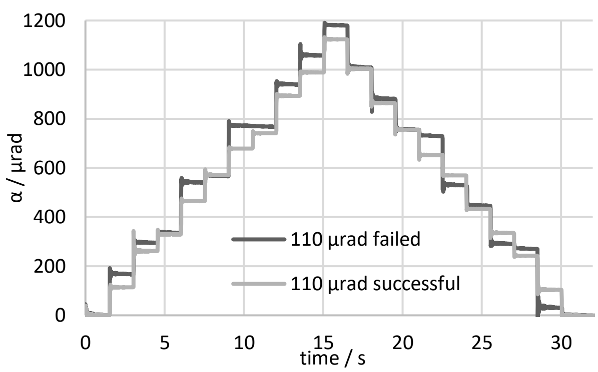

Figure 6. Measurement results for successful and unsuccessful minimum incremental motion tests. Here, steps with 110 µrad step sizes are made and the mechanism's response is measured.

Figure 7 shows the achievable minimal incremental motion of the high-precision tip/tilt mirror for the entire workspace. It can be seen that the minimal incremental motion varies between 90 µrad and 200 µrad. Especially for the minimal incremental motion of β, there is a minimum for $\varphi_1 = 10°$ that does not depend on φ_2. In contrast to that, α has a minimum for $\varphi_1 = \varphi_2 = -25°$. Surprisingly, β has a maximum for these angles.

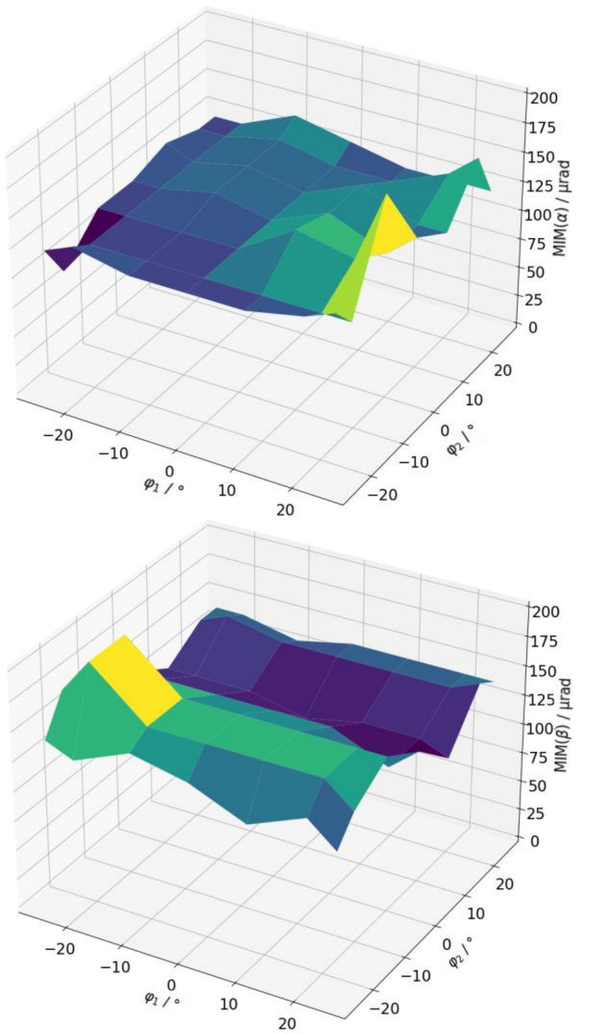

Figure 7. Achievable minimum incremental motion for the tip/tilt angles α and β for different input angles $\varphi_1$and φ_2.

Knowing that the minimum incremental motion of the active joints is 51 µrad, it can be assumed that the transfer function of the parallel kinematic mechanism amplifies errors in the input angles onto the output angles by the factor 1.7 to 3.9 (depending on the tip/tilt angle). For given application requirements, i. e., specific requirements in the minimum incremental motion, the active joints can be chosen accordingly.

4.3. Specifications of the High-Precision Tip/Tilt Mirror

In the experiments, further measurements were performed to evaluate the accuracy of the proposed parallel kinematic tip/tilt mirror including the mechanism's linearity, backlash, and the bidirectional positioning repeatability.

For the linearity, a rotation of 6,000 µrad is tested with small step sizes of 300 µrad and the deviation from the straight-line motion is evaluated. For α and β, mean linearity errors of 1-2.7 % and 1.5-2 % were observed, respectively.

The backlash limits the absolute positioning accuracy. For the proposed mechanism, a mean backlash of 52 µrad (for α) and 77 µrad (for β) was obtained.

The bidirectional positioning repeatability is a measure on how accurate a target pose approached from different directions is hit. It is investigated multiple times with various step sizes reaching the target pose and the measurements standard deviation is evaluated. Similar to the minimum incremental motion, it depends on the tip/tilt angle and ranges from 150-160 µrad (for α) and 100-260 µrad (for β). Compared to the bidirectional positioning repeatability of the active joints, only an amplification of the factor 1.0 to 2.5 can be recognized through the transfer function of the parallel kinematic mechanism. For β, partly even better bidirectional positioning repeatability values for the tip/tilt mirror are obtained compared to the active joints (102 µrad).

5. Conclusion and Discussion

In this paper, a high-precision parallel kinematic tip/tilt mirror was presented that allows a large motion range of ± 30°. The tip/tilt mirror is based on the spherical 5R parallel mechanism whose direct and inverse kinematics was revisited in this paper. In addition, the mechanical design was presented and simulation and experimental results were shown that prove the high-precision capabilities of the proposed tip/tilt mirror.

The mechanical design limits the motion range to ± 30° but helps preserving very high Eigenfrequencies of 450 Hz and more. The experiments furthermore show that the proposed parallel mechanism is an accurate positioning system. In fact, minimum incremental motion of 90 µrad to 200 µrad with a bidirectional positioning repeatability of 100 µrad to 260 µrad and a linearity error of 1-2.7 % can be achieved.

Due to the high stiffness, the mechanism can move very fast. The limits here are not investigated as the active joints, i. e., the two U-628.03 PILine® rotation stages, are fast but do not have a very high acceleration due to their drive concept. Here, further investigations are necessary. Furthermore, it seems that the accuracy of the active joints directly influences the achievable accuracy of the parallel mechanism (amplified by the transfer function of the parallel kinematic mechanism). Therewith, more accurate active joints would further improve the tip/tilt mirror.

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