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Multivariable stability analysis of position-controlled payloads with flexible eigenmodes

Luca Mettenleiter[1], Matthias Fetzer[1], Steffen Waimer[1]

[1]*Carl Zeiss SMT GmbH, Oberkochen, Baden-Württemberg, Germany*

luca.mettenleiter@zeiss.com

Abstract

In this paper we investigate the multivariable stability analysis of feedback control systems. First, stability for feedback control systems is defined and the Nyquist criterion with its extension to MIMO systems is presented. This is extended by introducing the concept of equivalent plants and deriving Nyquist based stability methods, allowing to obtain structural stability system information in addition to pure stability information. The methods are evaluated comparatively using a modelled example system.

Stability analysis, Multivariable control, Multivariable analysis, Multivariable systems

1. Introduction

Stability is one of the most important system parameter in the analysis of feedback control systems and must be guaranteed before other system parameters such as performance or robustness are considered. For the stability analysis of Single-Input Single-Output (SISO) systems, a variety of methods are available to the control engineer [1]. An important method is the Nyquist Criterion, which provides information about the robustness of the system in addition to the pure binary stability information, stable or unstable. Therefore, it can be determined directly how, for example, the gain of the control loop has to be adjusted in order to achieve a stable controller behavior.

An extension to Multiple-Input Multiple-Output (MIMO) systems exists for the Nyquist criterion. However, when switching from SISO to MIMO systems, the additional information about the robustness of the system is lost, or can no longer be assigned to individual physical axes. Thus, the MIMO Nyquist Criterion provides binary stability information and no conclusions can be drawn about individual axes of the system. Therefore, in this paper, the MIMO Nyquist Criterion is extended in order to derive advanced stability analysis methods. This allows the investigation of individual transmission paths considering cross-couplings in the MIMO system. For this purpose, stability of feedback control systems is defined. Subsequently, the Nyquist Criterion is first derived for SISO and MIMO systems and then extended in different ways. To compare the methods, a representative system of a position-controlled payload with flexible eigenmodes is modelled.

A widely used approach in high-performance position control of mechanical and optical payloads in up to six Degrees of Freedom (DoFs) is the use of static decoupling by scheduling matrices [2]. By using lightweight design and maximizing controller bandwidths to optimize performance, static decoupling decreases in effective range and decoupling is not trivial. It is assumed that the results from static decoupling will not be able to fulfill ever increasing requirements in high-performance position control. Consequently, MIMO modelling, control and analysis techniques need to be considered to ensure a more accurate and advanced mechatronic system design.

This paper extends and adapts the performance analysis discussed in [3]. For a more detailed description of MIMO system analysis, see e.g. [4].

2. Multivariable stability analysis

Several methods are available to the control engineer for stability analysis of feedback multivariable systems [5]. Most of these methods provide a binary statement about stability, stable or unstable. If more structural stability system information is to be obtained, more advanced methods must be used. For example, it is often not possible to make a statement about which of the individual Transfer Functions (TFs) contributes to instability. In the following, we present classical and commonly known methods for MIMO stability analysis and then extend them to provide methods for obtaining structural system stability information besides the binary stability information.

2.1. Definition of stability for feedback control systems

In this section we define the stability of feedback systems and prepare the basis for the stability analysis, which is discussed in the following sections. In this paper scalar quantities are indicated by thin formula symbols, vector quantities by vector arrows and matrices by capital bold formula symbols.

Consider the standard feedback control loop depicted in Figure 1. The open-loop Transfer Function Matrix (TFM) of the $m \times m$ MIMO system is defined as $\boldsymbol{L} = \boldsymbol{GC}$ with the input controller $\boldsymbol{C}$ and the plant $\boldsymbol{G}$. By this definition, the sensitivity matrix $\boldsymbol{S} = (\boldsymbol{I} + \boldsymbol{L})^{-1}$, the complementary sensitivity matrix $\boldsymbol{T} = (\boldsymbol{I} + \boldsymbol{L})^{-1}\boldsymbol{L}$ and $\boldsymbol{I}$ as the identity matrix of appropriate dimension are defined. To define stability, we apply the general valid MIMO definitions to the SISO case ($\boldsymbol{S} \rightarrow S$ and $\boldsymbol{T} \rightarrow T$). Thereby it is important to notice, that the poles of S and the poles of T in $\mathbb{C} \cup \{\infty\}$ are exactly given by the zeros of $1 + L$ in $\mathbb{C} \cup \{\infty\}$, assuming no unstable pole-zero cancellation occurs in L [6]. With this, $(1 + L)^{-1}$ is stable if

- it is proper and
- has only poles in the open left-half plane (real part < 0).

Instead of analyzing the poles of $(1 + L)^{-1}$, the zeros of $1 + L$ can be evaluated since they correspond to each other. Therefore, $(1 + L)^{-1}$ is stable if no zeros of $1 + L$ lie in the closed Right Half Plane (RHP) (i.e. real part ≥ 0).

This results in the conclusion, that for analyzing the stability of the feedback control system, we simply need to find zeros the TF and evaluate if any of them lies in the closed RHP.

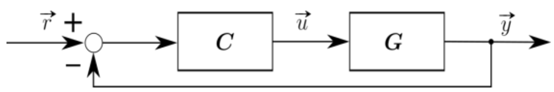

Figure 1. Standard feedback control loop

2.2. The Nyquist Criterion and extension to MIMO systems

In this section we derive the Nyquist criterion as a classical graphical test for checking the stability of the closed-loop system using the open-loop TF [7]. This method has the advantage that robustness information can be obtained additionally to the pure stability information.

As described in Section 2.1, the stability is evaluated by checking wether any zeros of $1 + L$ lie in the closed RHP. In order to evaluate all unstable poles of $(1 + L)^{-1}$ a contour that encircles all of them is needed. This contour is called Nyquist Contour $\mathcal{N}$, shown in Figure 2. It passes along the $j\omega$ axis from $-j\infty$ to $j\infty$ and closes by a significantly large semicircle $R \to \infty$ to encircle all poles of L. To enclose poles on the $j\omega$ axis, small semicircles r are inserted in the Left Half Plane (LHP) [6]. The plot defined as $s \in \mathbb{C}$ travesers $\mathcal{N}$ is called Nyquist plot $\Gamma_{\mathcal{N}}$.

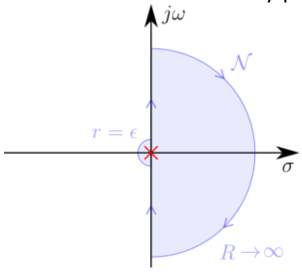

Figure 2. Definition of the Nyquist Contour $\mathcal{N}$

The mathematical background for the following theorems is given by the *Argument Principle* combined with *Cauchy's Theorem* and can be read in [8]. Here we use the definition of the Nyquist criterion and the Nyquist Contour $\mathcal{N}$ from [6].

Theorem 1 (Generalized Nyquist Criterion) *Let P denote the number of unstable poles in L. The closed-loop system with open-loop TF L and negative feedback is stable if and only if the Nyquist plot $\Gamma_{\mathcal{N}}$ of $L(s)$ does not pass through the critical point (-1,0) $\in \mathbb{C}$ and makes P counter-clockwise encirclements of (-1,0) $\in \mathbb{C}$ as s traverses $\mathcal{N}$ in clockwise direction, assuming no unstable zero pole cancellation takes place.*

The proof follows from the derivation of the Nyquist Criterion [6]. Theorem 1 is extended to MIMO-systems $(1 + L \ \rightarrow \ \boldsymbol{I} + \boldsymbol{L})$. Therefore, the closed-loop poles are now solutions to the Equation

$$\det(\boldsymbol{I} + \boldsymbol{L}(s)) = 0 \ \ \forall s \in \mathbb{C}. \qquad (1)$$

Note that in the SISO case, $\det(\boldsymbol{I} + \boldsymbol{L}(s))$ equals $1 + L(s)$. As a consequence of (1) the critical point shifts to (0,0). This leads to the Generalized MIMO Nyquist Criterion according to [5].

Theorem 2 (Generalized Nyquist Criterion for MIMO systems) *Let P denote the number of unstable poles in $\boldsymbol{L}$. The closed-loop MIMO system with open-loop TFM $\boldsymbol{L}$ and negative feedback is stable if and only if the Nyquist plot $\Gamma_{\mathcal{N}}$ of $\det(\boldsymbol{I} + \boldsymbol{L}(s))$ does not pass through (0,0) and makes P counter-clockwise encirclements of (0,0) as s traverses $\mathcal{N}$ in clockwise direction, assuming no unstable zero pole cancellation takes place.*

2.3 Characteristic Loci

The stability analysis using Characteristic Loci is an extension of Theorem 2 and provides an if and only if statement on the stability of the $m \times m$ MIMO system using the eigenvalues of the open-loop TFM [9]. The main idea behind this technique is, that the determinant of a matrix is equal to the product of all its eigenvalues: $\det(\boldsymbol{I} + \boldsymbol{L}(s)) = \prod_{i=1}^{m}(1 + \lambda_{L_i}(s))$. Therefore, rather than checking $\det(\boldsymbol{I} + \boldsymbol{L}(s))$ with the MIMO Nyquist Criterion, it is also valid to check each of the eigenvalues $\lambda_{L_i}(s)$ with the SISO Nyquist Criterion as defined in Theorem 1. This poses already the following Theorem.

Theorem 3 (Nyquist Criterion with Characteristic Loci) *Let P denote the number of unstable poles in $\boldsymbol{L}$. The closed-loop MIMO system with open-loop TFM $\boldsymbol{L}$ and negative feedback is stable if and only if the Nyquist plots $\Gamma_{\mathcal{N}_i}$ of the Characteristic Loci λ_{L_i} do not pass through (-1,0) and drawn together make P counter-clockwise encirclements of (-1,0) as s traverses $\mathcal{N}$ clockwise, assuming no unstable zero pole cancellation takes place.*

2.4. Concept of equivalent plants

The Individual Channel Analysis and Design (ICAD) is an approach to handle multivariable control problems with SISO techniques by forming new equivalent SISO channels under consideration of the multivariable system behaviour, the so called equivalent plants (EPs). O'Reilly and Leithead presented the ICAD as a general analysis and design framework for multivariable control problems and especially considered 2 x 2 systems in [10]. In this paper we link the concept of equivalent plants and ICAD to the classical Nyquist stability analysis and thus define further Nyquist-like stability theorems.

Consider an open-loop TFM $\boldsymbol{L}$ for a 2×2 MIMO system with a diagonal control scheme. The closed control loop with negative feedback is represented in terms of a block diagram in Fig. 3.

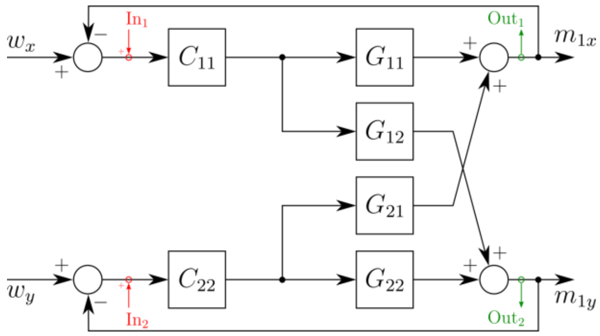

Figure 3. 2x2 feedback control structure

With the ICAD method, SISO TFs are derived for the two defined inputs In_1, In_2 and the two defined outputs Out_1, Out_2:

$$L_{11}^{EP} = \frac{\text{Out}_1}{\text{In}_1} = C_{11} \cdot {}_2\text{EP}_{11} = C_{11} \cdot G_{11}(1 - \xi \cdot h_2) \qquad (2)$$

$$L_{22}^{EP} = \frac{\text{Out}_2}{\text{In}_2} = C_{22} \cdot {}_2\text{EP}_{22} = C_{22} \cdot G_{22}(1 - \xi \cdot h_1) \qquad (3)$$

In (2, 3), the EPs for the 2×2 system are obtained as

$${}_2EP_{11} = G_{11} - G_{12} \cdot \frac{C_{22}}{1 + G_{22}C_{22}} \cdot G_{21} \qquad (4)$$

$${}_2EP_{22} = G_{22} - G_{21} \cdot \frac{C_{11}}{1 + G_{11}C_{11}} \cdot G_{12} \qquad (5)$$

and by rearranging (2, 3), the right hand side is defined

$$h_k = \frac{C_{kk} \cdot G_{kk}}{1 + C_{kk} \cdot G_{kk}}, \qquad \xi = \frac{G_{12} \cdot G_{21}}{G_{11} \cdot G_{22}} \qquad (6a, 6b)$$

with $k = \{1,2\}$. The TFs h_1 and h_2 form the closed-loop TF of channel 1 and 2 respectively. ξ is called the Multivariable Structure Function (MSF) and describes the multivariable nature of L_{11}^{EP} and L_{22}^{EP} and of the underlying system [10]. The product of ξ and h_k is a weighted product with diagonal and off-diagonal elements and therefore describes a measure of cross-coupling. If $\xi(s)$ is small in magnitude, loop signal interaction is low and the two loops act almost independently for that $s \in \mathbb{C}$. Vice versa, loop interaction is high if $\xi(s)$ is large in magnitude. Consequently, ξ can be used to define a frequency dependend measure of the coupling ratio of MIMO systems. Furthermore, $\max\limits_{\omega \in [0,\infty]} |\xi(j\omega)|$ can be evaluated to assess the maximum cross-coupling and the underlying frequency.

2.5. Stability analysis based on individual channel analysis and determinant decomposition

The SISO TFs defined in (2,3) can be used to evaluate the stability of the closed-loop system with open-loop TFM . For this investigation it is important to notice, that $\det(\boldsymbol{I} + \boldsymbol{L})$ can be rearranged in such a way, that the TFs h_1 and h_2 and the MSF ξ occur:

$$\det(\boldsymbol{I} + \boldsymbol{L}) = (1 + G_{11}C_{11})(1 + G_{22}C_{22})(1 - \xi h_1 h_2) \qquad (7)$$

Therefore, stability of the multivariable system depends on the stability of each SISO loop $(1+C_{11}G_{11})$ and $(1+C_{22}G_{22})$ and of the multivariable coupling described by $(1-\xi h_1 h_2)$. This can be rewritten to a generally valid case requiring

$$\det(\boldsymbol{I}+\boldsymbol{L})=\prod_{i=1}^{m}(1+l_i)\cdot\left(1-\xi\prod_{i=1}^{m}h_i\right) \tag{8}$$

not having zeros in the RHP for asymptotic stability of a $m\times m$ feedback control system, assuming no unstable pole zero cancellation takes place. In Eq. (8), $l_i=C_{ii}G_{ii}$ are the diagonal elements of $\boldsymbol{L}$. Consequently, we derive an alternative version of Theorem 2 with the advantage that instabilities can be attributed to one of the diagonal or off-diagonal elements.

Theorem 4 (Nyquist Criterion of determinant decomposition)

Let P denote the number of unstable poles in $\boldsymbol{L}$. *The closed-loop* $m\times m$ *MIMO system with loop TFM* $\boldsymbol{L}$ *and negative feedback is stable if and only if the m Nyquist plots* $\Gamma_{\mathcal{N}}(l_i(s)), i=\{1,\ldots,m\}$ *and the Nyquist plot of* $-\xi(s)\cdot\prod_{i=1}^{m}h_i(s)$ *do not pass through (-1,0) and the net sum of counter-clockwise encirclements of (-1,0) equals P as s traverses* $\mathcal{N}$ *in clockwise direction, assuming no unstable zero pole cancellation takes place.*

The proof is straightforward and can be done by calculating the determinant of a $m\times m$ matrix and applying the Nyquist criterion on each of the factors [4].

Despite the fact that similar functions occur in Eq. (8), no clear connection between the ICAD functions (2,3) can be determined as the individual channels L_{ii}^{EP} are not used. It can be shown, that $\det(\boldsymbol{I}+\boldsymbol{L})\neq\prod_{i=1}^{m}(1+L_{ii}^{EP})$ and therefore no if and only if statement on stability is possible by considering all L_{ii}^{EP}. However, a sufficient condition for stability of a multivariable system is that each of the functions L_{ii}^{EP} are stable. Nevertheless, there is an distinct connection between the individual channels and the determinant, which is derived in the following and extends the concept of EPs in [10] to $m\times m$ systems.

Let $\boldsymbol{L}$ be the open-loop TFM of a $m\times m$ MIMO system with a diagonal controller $\boldsymbol{C}\coloneqq\operatorname{diag}(C_{11},\ldots,C_{mm})$ and a fully populated plant matrix $\boldsymbol{G}:=\left(G_{ij}\right)_{ij}$.

Consider the square submatrix $\boldsymbol{G}_{(1:n;1:n)}$ defined by deleting each row and column of $\boldsymbol{G}$ greater than n with $n\in\{1,2,\ldots,m\}$. Let ${}_nEP_{nn}$ be the EP TF of the submatrix $\boldsymbol{G}_{(1:n;1:n)}$ defined in a similar way as in (4) and (5) and suppose that no pole zero cancellations take place within the multiplications. With that, the equivalent open-loop TFs L_n are defined for each EP of the submatrices with the corresponding controller according to [11]:

$$L_n=C_{nn}\cdot{}_nEP_{nn}\quad\forall\, n\in\{1,2,\ldots,m\}. \tag{9}$$

Theorem 5 (Stability of MIMO Systems using ICAD)

With the definition of L_n *in Equation (12), the following holds:*

$$\det(I+L)=\prod_{n=1}^{m}(1+L_n). \tag{10}$$

Thus the stability of the MIMO system can be assessed by considering the open-loop TFs L_n *using SISO Nyquist criterion. An equivalent statement about the number of unstable poles can be made as in Theorem 2 by counting the net sum of encirclements of the critical point (-1,0).*

Proof: A lower-upper decomposition $\boldsymbol{\mathcal{L}\mathcal{U}}$ of the matrix $\boldsymbol{A}\coloneqq\boldsymbol{I}+\boldsymbol{GC}$ is done [12]. The result of the $\boldsymbol{\mathcal{L}\mathcal{U}}$ decomposition are two matrices, $\boldsymbol{\mathcal{L}}$ as a lower triangular matrix with only ones on the diagonal elements and $\boldsymbol{\mathcal{U}}$ as an upper triangular matrix. Because $\det(\boldsymbol{\mathcal{L}})=1$ and $\boldsymbol{\mathcal{U}}$ is an upper triangular matrix, $\det(\boldsymbol{A})$ is equal to the product of all diagonal elements of $\boldsymbol{\mathcal{U}}$. The diagonal elements of $\boldsymbol{\mathcal{U}}$ are given by u_{ii} and therefore

$$\det(\boldsymbol{A})=\det(\boldsymbol{\mathcal{L}\mathcal{U}})=\det(\boldsymbol{\mathcal{U}})=\prod_{i=1}^{m}u_{ii}\,. \tag{11}$$

By applying the $\boldsymbol{\mathcal{L}\mathcal{U}}$ decomposition on $\boldsymbol{A}$, $\boldsymbol{\mathcal{U}}$ is given by

$$\boldsymbol{\mathcal{U}}=\begin{bmatrix}1+G_{11}C_{11} & {}_1G_{12}C_{22} & \cdots & {}_1G_{1m}C_{mm}\\ 0 & 1+{}_2G_{22}C_{22} & \cdots & {}_2G_{2m}C_{mm}\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 1+{}_mG_{mm}C_{mm}\end{bmatrix} \tag{12}$$

$${}_nG_{nj}={}_1G_{nj}-\sum_{i=1}^{n-1}\frac{{}_iG_{ij}\,{}_iG_{ni}C_{ii}}{1+{}_iG_{ii}C_{ii}}\ \forall n,j\in\{1,\ldots,m\} \tag{13}$$

Interestingly, for $j=n$ Equation (13) equals the EP of the submatrix $\boldsymbol{G}_{(1:n;1:n)}$ and therefore

$$L_n=C_{nn}\left({}_1G_{nn}-\sum_{i=1}^{n-1}\frac{{}_iG_{in}\,{}_iG_{ni}C_{ii}}{1+{}_iG_{ii}C_{ii}}\right) \tag{14}$$

As the $\boldsymbol{\mathcal{L}\mathcal{U}}$ decomposition is unique, Theorem 5 is proven for the general $m\times m$ case since

$$\det(\boldsymbol{I}+\boldsymbol{L})=\prod_{i=1}^{m}u_{ii}=\prod_{n=1}^{m}(1+L_n). \tag{15}$$

■

Note that with Equation (13), a closed expression for the EPs of a ICAD system can be defined.

3. Modelling of the controlled mechanical system

After having discussed different methods for multivariable stability analysis let us now turn to the comparison of these methods. A model of a dynamically coupled MIMO system is required for a comparative analysis. For this purpose, the dynamic behavior of actuated and controlled payloads with flexible eigenmodes is analysed using existing simulations and literature. On this basis, the dynamic behavior is duplicated by mechanical elements with concentrated parameters, i.e. masses, dampers and springs.

The example system, Figure 2, consists of two basic parts.

- The payload mechanical element.
- The actuators to manipulate the mechanical element.

The payload mechanical element consists of four masses M_1-M_4. The position of mass M_1 is controlled in two DoFs, x and y. In order to model the eigendynamics of the payload and thus flexible eigenmodes, a three mass osciallator with masses M_2-M_4 is placed on mass M_1. By adjusting the parameters of this masses and the interconnecting springs and dampers, the dynamic behavior of a payload with flexible eigenmodes is duplicated in two DoFs. The cross coupling between x and y depends on the angles ψ and φ and can thus be modified.

Two actuators M_{A1} and M_{A2} modelled as masses at which the actuation forces f_{A1} and f_{A2} apply are connected to mass M_1 with a spring under the angles α and β to actuate mass M_1 in x and y. To attenuate the resulting resonance, a Tuned Mass Damper (TMD) is attached to each of the actuator masses. Similar to the payload mechanical system, the cross coupling of x and y depend on α and β and can therefore be adjusted.

Based on this modelling architecture the system is determined by the second order Ordinary Differential Equation (ODE) system

$$\boldsymbol{M}\cdot\ddot{\vec{q_i}}(t)+\boldsymbol{C}\cdot\dot{\vec{q_i}}(t)+\boldsymbol{K}\cdot\vec{q_i}(t)=\vec{f_i}(t) \tag{16}$$

with the mass matrix $\boldsymbol{M}$, the damping matrix $\boldsymbol{C}$, the stiffness matrix $\boldsymbol{K}$, the actuation force vector $\vec{f_i}(t)$ as input vector and the displacement vector $\vec{q_i}(t)$ as output vector.

By applying the Laplace transform to (16) and tuning a variant of a classical PID controller with a diagonal control scheme as reported in [2], the open loop TFM of the example system is defined to

$$\boldsymbol{L}=\boldsymbol{GC}=\begin{bmatrix}G_{11}C_{11} & G_{12}C_{22}\\ G_{21}C_{11} & G_{22}C_{22}\end{bmatrix}. \tag{17}$$

As described, the cross-coupling of the system depends on the angles $\alpha,\beta,\psi,\varphi$. Therefore the MSF ξ as introduced in Section 2.4 of the example system is a function of these angles $\xi=\xi(\alpha,\beta,\psi,\varphi)$ and the multivariable behaviour and cross coupling can be influenced and adapted by adjusting these angles. This makes it possible to create different system configurations by modifying $\alpha,\beta,\psi,\varphi$. Thereby, the methods presented in Chapter 2 can be investigated and comparatively evaluated with different system characteristics, for example strongly or weakly coupled systems.

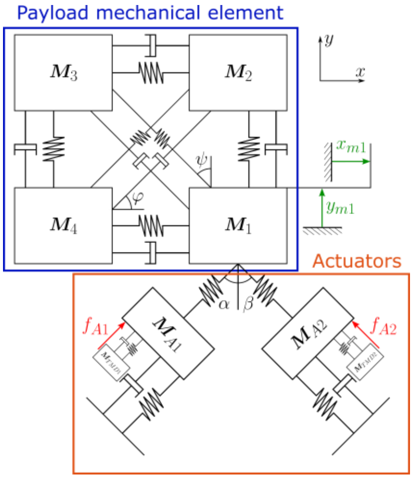

Figure 2. An actuated payload with flexible eigenmodes

4. Comparison results and recommendations

In this Chapter we present the comparison results of the methods investigated in Chapter 2 with the example system introduced in Chapter 3. To compare the stability methods, different system configurations and thus different system behaviors are modeled. Thus, for example, instabilities can be forced into specific individual TFs of the system TFM and the advantages and disadvantages of the methods become obvious.

The MIMO Nyquist as introduced in Theorem 2 provides a necessary and sufficient condition for stability through evaluating $\det(\boldsymbol{I}+\boldsymbol{L})$ with the advantage that only one Nyquist plot needs to be evaluated to assess stability. However, this method cannot be used to draw conclusions about the individual physical axes and does therefore not deliver structural stability system information. Consequently, the use of this method is recommended, if only pure stability information of the system needs to be evaluated.

The m Characteristic Loci as described in Theorem 3 deliver valid stability information in all cases as the eigenvalues of $\boldsymbol{I}+\boldsymbol{L}$ are directly connected to the determinant of this matrix. Similar to the MIMO Nyquist, the Characteristic Loci do not provide information about the individual, original physical axes and no structural system information is obtained. The robustness margins of the Characteristic Locis correspond to SISO margings for a simultaneously change in all channels [9].

The Nyquist of determinant decomposition as introduced in Theorem 4 provides valid stability information in all cases as seen in Equation (9). The determinant of $\boldsymbol{I}+\boldsymbol{L}$ is split into m SISO loops $1+l_i$ and one additional loop $1-\xi(s)\cdot\prod_{i=1}^{m}h_i$ considering the cross coupling of the system with the MSF ξ. This enables to find out whether there are instabilities on one of the diagonal SISO loops or in the coupling of these loops. Therefore the use of this method is recommended if stability and structural stability information needs to be obtained.

Similarly, the SISO Nyquist using ICAD functions as presented in Theorem 5 is an individual interpretation of the MIMO Nyquist and exactly matches the determinant of the matrix as shown in the proof of Theorem 5. Consequently, the stability information obtained with this method is valid in all cases. An advantage of this method is that information about the individual physical axes is obtained, e.g. occurrence of instabilities or robustness information. To gain full information about the individual axes, each permutation of the individual channel combination can be evaluated. The use of this method is recommended, if not only pure stability information needs to be obtained, but also structural information about the system behavior is of interest.

Additionally, with Equation (13) a closed expression for the EP as introduced in the ICAD is defined.

Table 1 summarizes the comparison results for the individual stability analysis methods.

Table 1 Comparison results

Method	Stability statement	Structural information	Number of functions for m x m system
MIMO Nyq.	yes	no	1
Char. Loci	yes	no	m
Det. Deco.	yes	yes	$m+1$
ICAD Nyq.	yes	yes	m

5. Conclusions

In this paper Nyquist based methods of multivariable stability analysis are evaluated comparatively. For this purpose stability is defined and the different methods are derived. By combining the concept of EPs and ICAD, individual interpretations of the Nyquist Criterion are derived and stated. Subsequently, an example system is modeled, representing a position controlled payload to enable a comparative analysis of the methods. Afterwards the stability analysis methods are applied to this system and the results of each method are compared.

All the investigated methods allow valid stability assessments. However, they differ in terms of the information content. While with MIMO Nyquist a pure stability statement can be made, additional robustness information is obtained with characteristic loci. In order to obtain structural stability information for MIMO systems, we extend the concept of EPs from [10] and combine it with the Nyquist criterion to derive two alternative Nyquist-like stability theorems. These provide structural stability information for MIMO systems.

With these alternative interpretations of the Nyquist criterion, multivariable stability analysis can be significantly improved and additional structural system information can be obtained.

For future work, the robustness analysis of multivariable systems can be examined, for which a variety of sophisticated methods is available, see e.g. [13].

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