euspen's 21st International Conference & Exhibition, Copenhagen, DK, June 2021
www.euspen.eu

# Guaranteeing nanometer positioning under manufacturing and modeling uncertainty

Matthias Fetzer[1]

[1]Carl Zeiss SMT GmbH, Oberkochen, Germany

*matthias.fetzer@zeiss.com*

**Abstract**
In this paper, we discuss the application of the µ-analysis or structured singular value framework to the problem of high precision motion control under manufacturing and material tolerances as well as modeling uncertainties. After clarifying the problem at hand, we first introduce the µ-analysis framework and set the stage by discussing the so-called generalized plant setting. In the following, we apply this framework to a control loop including an optical or mechanical payload controlled in six degrees of freedom, idealized actuator models as well as several dampers. Finally, we discuss the achieved results and compare them to other approaches.

Robust control, robustness analysis, structured singular value, uncertainty

## 1. Introduction

High precision motion control of mechanical structures such as optical payloads and stages is a key enabler for lithography systems. The increasing requirements on overlay and achievable critical dimension impose highly challenging specifications on the optically relevant structures themselves as well as on other subcomponents, here in particular the actuators. In addition, it also necessitates the introduction of various dedicated parts, such as tuned mass dampers (TMDs), that crucially affect the overall performance.

In this paper we focus on the problem of keeping an optical or mechanical element stable at a defined position in six degrees of freedom (DoF). The current position is measured and varied with respect to a certain reference frame using a set of sensors and actuators. The plant, consisting of a payload, actuators and dampers, is modelled using Finite Elements (FE) and subsequently integrated as a state space system into the control loop simulation.

In this process we accumulate several errors that have to be taken into account if we aim at the same time for pushing the overall performance to the limit and providing a robust design to our customer: On a fundamental level, we always have a mismatch between model and reality. This can be due to changing or uncertain material data, manufacturing tolerances or environmental conditions. Second, the final FE model will always be an approximation as we, for example, reduce complexity to reduce computational time or, intrinsically, assume a linear behavior.

Thus, the nature of the uncertainties in our model can be time-independent, e.g., stemming from modeling inaccuracies or manufacturing tolerances, but can also be time-varying, e.g., changing due to environmental properties like temperature and humidity or simply aging.

Consequently, robustly guaranteeing a desired performance under manufacturing and modeling uncertainties is the major challenge in position control with extremely high accuracy requirements. However, even if only considering key contributors, the number of varying parameters quickly ranges between 50 to 100, often even surpassing that. This sheer quantity of uncertainties already renders grid-based approaches such as Monte Carlo or its derivatives futile, as they rely on a sufficiently dense grid – a feat impossible to achieve: with just ten grid points per dimension and 100 parameters, we end up with $10^{100}$ points, a quantity larger than the number of atoms in the known universe.

## 2. Problem statement

Current lithography systems have overlay requirements in the order of nanometers which leads to error budget breakdowns of only picometers for individual components. In contrast to the extremely high accuracy requirements, manufacturing tolerances range in the order of micrometers. Therefore, dynamically relevant parameters such as stiffness, damping and moving masses may deviate significantly from the data in our perfect FE simulation.

In this complex interplay, small deviations from the nominal behavior lead to critical performance degradation or even loss of control stability. However, robustness analysis of a given design is always a trade-off between too much conservatism, i.e., performance degradation, and too optimistic forecasts, leading to failures. Lithography specific requirements such as cost of goods ranging in the order of several millions, in combination with low production numbers further add to the complexity of this problem.

In conclusion, what we need is an analysis tool that allows us to compute the worst-case performance of a given linear time-invariant (LTI) system representation under a range of pre-defined tolerances for dozens of parameters in a short amount of time. Additionally, we also want to learn from these worst-case scenarios, e.g., by improving the robustness of our system already in the design phase or reducing the tolerances by changing materials or adapting manufacturing processes.

### 3. The µ-Analysis framework

As robust control is still a very highly researched field, many techniques are available to investigate the robustness of an uncertain LTI system. However, the number of relevant states in a state space model compiled from a complex FE model can easily range between 100 and 1000. Thus, many classical Lyapunov approaches based on linear matrix inequalities (LMIs) are rendered useless simply due to the number of unknowns in the Lyapunov certificate (see, e.g., [1, 2]). For this reason, we apply frequency domain techniques that have the advantage to not scale with the number of states (see, e.g., [9, 10, 11] and [6, 7] for some background). A readily available (i.e., implemented in the MATLAB Robust Control Toolbox [3]) approach is the so-called Structured Singular Value (SSV) or µ-Analysis framework [9]. As discussed subsequently, this framework perfectly addresses the requirements stated at the end of Section 2.

***3.1 Some background***

It is well-known that the problem of determining the worst-case performance of an LTI system with real uncertainty is NP complete and thus impossible to solve efficiently [11]. This inspired a different approach, namely the µ-Analysis that tackles this challenge by solving two substitute problems, thus giving upper and lower bounds on the original one. The key advantage is that both problems are convex, hence, efficient computation as well as global optimality are guaranteed.

***3.2 Prerequisites***

In order to apply the µ-framework to our problem, we need to transform the system under investigation into a standard form, the so-called generalized plant framework [1]. As this is an important step to apply the whole framework, we dedicate this section to discuss it in some detail. For further background we recommend the comprehensive and insightful lecture notes [1].

Let us explain the process using a standard tracking interconnection depicted in Figure 1.

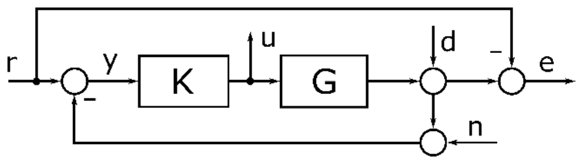


**Figure 1.** Standard tracking interconnection

In the first step, we disconnect the controller K and note all external inputs (d,n,r,u) and outputs (e,u,y) to the remaining interconnection (see Fig. 2). Then, we combine the external inputs that cannot be influenced by the controller in the so-called generalized disturbance $w \coloneqq \mathrm{col}(d; n; r) \coloneqq (d^T, n^T, r^T)^T$ and the signals that allow us to measure whether the controller shows a desired characteristic using the controlled variable $z \coloneqq \mathrm{col}(e; u)$.

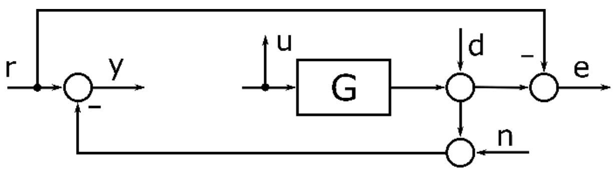


**Figure 2:** Tracking interconnection with decoupled controller

Subsequently, we stack these with the control input u and the measurement output y, respectively, in order to arrive at the following two system equations:

$$\begin{pmatrix} e \\ u \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} I & 0 & -I & G \\ 0 & 0 & 0 & I \\ -I & -I & I & -G \end{pmatrix}}_{=:P} \begin{pmatrix} d \\ n \\ r \\ u \end{pmatrix} \quad \text{and} \quad u = Ky.$$

With P as above, we can equivalently depict the two interconnections in Figure 1 and 2, respectively, as follows.

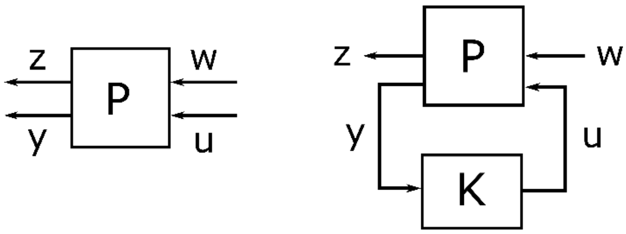


**Figure 3:** Generalized open and close loop interconnection

This very flexible framework allows us to merge rather different settings and develop as well as apply algorithms to this standard form. Furthermore, it is important to note that the algorithms implemented in MATLAB also require this specific structure.

In a next step, we now introduce uncertain systems, and demonstrate how the generalized plant setting may be easily extended to incorporate uncertain dynamics. The µ-framework allows for three different types of uncertainties: real or complex (repeated) and complex full-block uncertainties. Using the first option, we may take real parameter variations into account, typically physical parameters as stiffness, damping or mass. The latter ones may be used to introduce dynamic uncertainties that often stem from modeling inaccuracies or flawed identification (see, e.g., [10]). For all uncertainties, we are required to define bounds, i.e., for the real case we can simply attribute a certain percentage of variation often derived from manufacturing tolerances.

The key idea in applying this framework is to pull out the uncertainties as independent dynamical systems by defining additional inputs and outputs to the original system, and thus incorporating them into a feedback loop (see Fig. 4). As this process is rather standard, we refer the reader to the literature ([3], [1] and [8]).

Now let us assume that we have identified the uncertainties, defined the associated tolerances and, thus, specified our uncertain system (see left hand side in Fig. 4). Note that the resulting uncertainty $\Delta$ will always be block-diagonal. Then, we can exploit the flexibility of the generalized plant framework that allows us to easily incorporate this additional feedback loop: we only have to exchange the system G with the uncertain system representation on the left in Figure 4 and reproduce the steps outlined above in order to arrive at the interconnection depicted on the right in Figure 4.

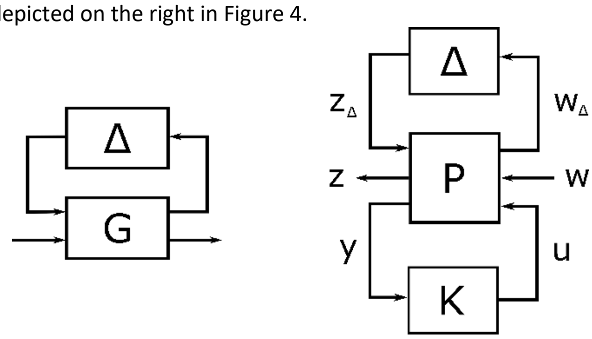


**Figure 4:** Uncertain system and uncertain general interconnection

Finally, as we will keep the controller fixed in our further analysis, we close the lower feedback loop containing the controller to arrive at the uncertain interconnection depicted in Figure 5, where we defined $N \coloneqq lft(P, K)$; with lft denoting the (lower) linear fractional transformation. Note that this command is actually readily implemented in Matlab.

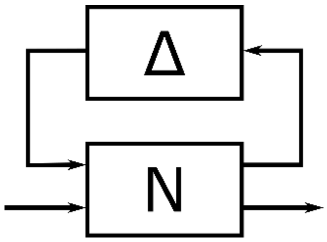


**Figure 5:** General robust performance interconnection

The uncertain feedback interconnection depicted in Figure 5 allows us to verify a certain performance under all possible uncertainties specified by Δ together with the corresponding tolerances. The last remaining step is to define the performance limit that we are interested in to verify. A common performances measure is the comparison of a system transfer function, for the tracking interconnection typically the sensitivity, with a specified bound. As the μ-analysis problem is evaluated frequency by frequency, also the performance bound may be frequency dependent. We will give a specific example in Section 4.

***3.3 Application of the μ- framework***

Now, we have everything in place to discuss robust stability and performance analysis based on the SSV. As already mentioned, the problem of assessing the worst-case performance of the uncertain interconnection depicted in Figure 5 is impossible to solve efficiently. The μ-framework, however, calculates upper and lower bounds on the original problem, where both are derived from convex problems, which are computationally relatively cheap.

As a control engineer, we are usually interested in two things: first a bound on the worst-case performance that allows us to deduce robust performance guarantees. In the μ-analysis this is given by the upper bound. Second, we want to understand what uncertainties lead to this performance deterioration in order to optimize our system – or, at least, understand its weaknesses better. This is provided by the lower bound. The underlying reasoning here is that the actual worst-case performance of the system cannot be worse than what the upper bound predicts; at the same time, it cannot be better than the performance achieved with the parameter combination derived from the lower bound. If both bounds are close to each other, we know very precisely what can happen in our system, as the original problems lies between both bounds. Furthermore, it is important to note, that the upper bound can in principle be calculated with arbitrary accuracy [4, 14]. Let us discuss this at the example of an optical or mechanical payload with about 60 uncertain parameters in Section 4.

**4. Application to a concrete example**

***4.1. Control loop***

In our example, we consider a payload that is position-controlled in six DoFs. The control scheme measures the position of the payload at six points and estimates the point to be controlled (e.g. the center of gravity) assuming rigid body behavior. The payload is actuated by six actuators with given internal dynamics, here approximated by a mass spring damper model. Furthermore, as flexible modes of the payload can have a negative impact on the achievable maximal bandwidth, TMDs are added to attenuate the impacts of these flexibilities on the control performance. A sketch of the control loop with only two actuators and one TMD is shown in Figure 6.

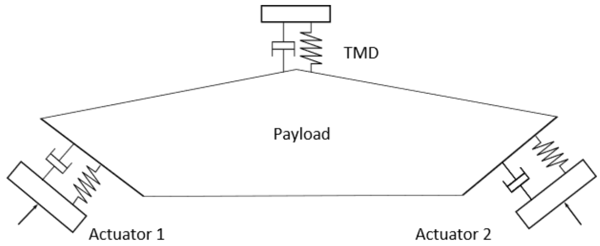


**Figure 6:** Sketch of the plant

From experience, we know that internal actuator dynamics as well as the flexibility of the payload have a significant impact on the control loop performance, in this case a bound on the transfer function from reference to error (see red dotted line in Fig. 8). This rather standard measure for tracking control performance allows us to balance the desired (low frequent) tracking behavior against the undesired (high frequent) control degradation due to the water-bed effect [12].

In order to ensure robust performance of our control loop under manufacturing tolerances as well as modeling inaccuracies we introduce uncertainties for these parameters. Since the actuator mechanism as well as the TMD are modelled ideally as mass-spring-damper systems, this is easily done using position and velocity feedback at the respective nodes in the FE model. The manipulation of the flexible eigenmodes of the payload is similarly straightforward and discussed in some detail in [8]. In our example, this leads to about 50-60 uncertain parameters that have a key impact on the overall system performance. For each of these parameters we estimate the required tolerances in order to guarantee robustness against these variations.

***4.2. Worst case analysis***

In Figure 7, we show a plot of the resulting upper and lower bound of the μ-problem over the critical frequency range. For simplicity, we focus here on just one DoF. Both bounds are normalized to one (the horizontal blue line), i.e.,

- If the upper bound is below one, the system achieves the desired performance specification under all considered uncertainties; robust performance is guaranteed.
- If the lower bound is above one, there exists a parameter combination that violates the robust performance specification.
- If the upper bound is above one and the lower bound below, robust performance cannot be guaranteed, yet, the algorithm was unable to find a parameter combination that actually leads to a violation.

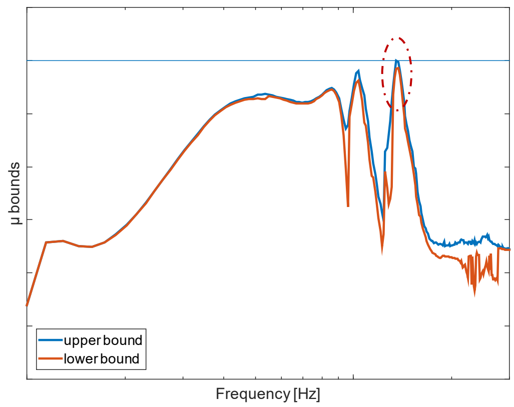


**Figure 7:** Upper and lower bound obtained from μ-analysis

As can be seen, upper and lower bound are in general very close to each other, thus the results can easily be interpreted: first, the upper bound is always below the blue line, hence, robust performance is guaranteed. Second, the lower bound is for the critical frequencies close to the blue line. Hence, there exist parameter combinations that lead to a performance close to the specification.

Let us now discuss how we can use the lower bound to analyze the system in its worst-case state. In Figure 8, we plotted the nominal transfer function in blue. As can be seen, especially in the high frequent area there is significant margin to the specification plotted as a dashed red line. Using the lower bound we can immediately recover the worst-case parameters that lead to the most critical performance. If we plug them back into our model, we obtain the sensitivity transfer function plotted in red in Figure 8. In the critical frequency range, the sensitivity changes dramatically, nearly touching the specification, as was to be expected from the lower bound in Figure 7.

Note that worst case parameters of the μ-analysis do not necessarily correspond to the worst-case performance of the system. This is indicated by the gap between upper and lower bound. The μ-analysis tells us that the performance can even be worse than the one depicted in red in Figure 8. However, we know that it will never be violating the specification, as the upper bound in Figure 7 is below one.

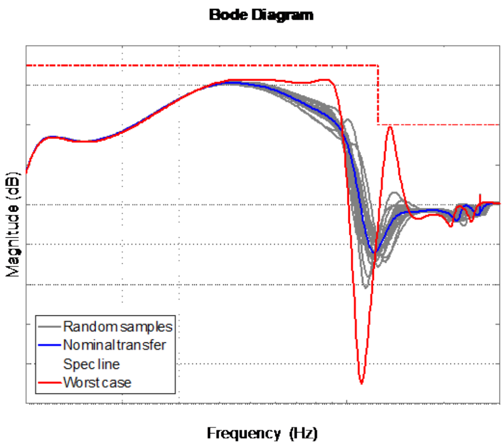


**Figure 8:** Sensitivity transfer functions

### *4.3. Discussion of results*

In this section, we want to point out that the μ-analysis can actually be used in different ways. First, and more obviously, one can verify that, given a certain set of uncertain parameters with associated tolerances, a system robustly achieves a desired performance. Second, and this might even be a more important aspect, it can be used to assess the impact of certain tolerances and to specify their range. This allows the engineer on the one hand to robustly optimize the control system already in the design phase. While, on the other hand, it enables specification of sub-components not only regarding their nominal performance but also concerning the allowed tolerances.

### *4.4. Comparison to grid-based algorithms*

In this section, we want to briefly compare the μ-framework applied in this paper to the typically used grid-based approaches. The most important problem in applying grid-based algorithms is the so-called curse of dimensionality: with 60 parameters and only two grid points per dimension, i.e., just the extreme points, we end up with more than $10^{18}$ grid points that need to be checked. Using a reasonably fine grid renders even medium sized problems impossible to solve efficiently. In contrast, the μ-analysis problem takes two to ten minutes to compute, depending on the choice of the underlying algorithm.

In Figure 8, we indicate what can happen if the sample size is too small. Here we randomly picked ten parameter configurations and plotted the resulting transfer functions in grey. While this already gives us an indication where we can expect a certain amount of deviation from the nominal transfer function, it is certainly useless for a quantification of the worst possible performance, as indicated by the large gap between all grey plots and the worst case plot derived using μ-analysis. However, even if we drastically increase the sample size, grid-based algorithms will never give us a robustness guarantee. We never know whether a finer grid would lead to different results.

On the other hand, it is important to note that also the μ-approach relies on a frequency grid. Nevertheless, this is only one dimensional, hence, grid refinement is not so costly in terms of computational time. And furthermore, the μ-algorithm gives us a certain kind of continuity [9] that guarantees a certain smoothness of the bounds.

## 5. Conclusions and future work

So far, our analysis is restricted to so-called parametric uncertainties. However, the μ-framework also allows to introduce dynamic ones to take complete model uncertainty into account. From this we expect a more accurate modeling of system uncertainty and thus a less conservative robustness estimate. In addition, even nonlinear behavior (see, e.g., [5, 13]) as well as specifically time-varying parameters [14] or even can be tackled within the generalized plant framework, which allows to add an even richer class of uncertainties.

## References

[1] Scherer, C. W., Theory of Robust Control. *Lecture notes, Delft University of Technology*, 2001. URL: https://www.imng.uni-stuttgart.de/mst/files/RC.pdf.

[2] Scherer, C. W., Weiland, S., Linear Matrix Inequalities in Control. *Lecture Notes, Dutch Institute for Systems and Control*, Delft, The Netherlands.

[3] Balas, G. J. , Doyle, J. C., Glover, K., Packard A., Smith R., μ-Analysis and Synthesis Toolbox, The MathWorks Inc. and MYSYNC Inc., 1995.

[4] Scherer C. W., Relaxations for robust linear matrix inequality problems with verifications for exactness. *SIAM Journal on Matrix Analysis and Applications* 2005; 27(2):365–395.

[5] Fetzer, M., Scherer, C. W., Full-block multipliers for repeated, slope-restricted scalar nonlinearities. *International Journal of Robust and Nonlinear Control* 27 (17), 3376-3411, 2017.

[6] Veenman, J., Scherer, C. W., Köroğlu, H., Robust stability and performance analysis based on integral quadratic constraints. *European Journal of Control* 31, 1-32, 2016.

[7] Fetzer, M., From classical absolute stability tests towards a comprehensive robustness analysis. PhD thesis, Department of Mathematics, University of Stuttgart, 2017.

[8] Schoenhoff, U., Robustness Analysis of Dynamics and Control: A New Method to Support High Performance Optical Manipulator Design. *Proceedings of the ASPE 2012 Summer Topical Meeting*, Berkeley, 2012.

[9] Packard, A., Doyle, J. C., The complex structured singular value. *Automatica*, vol. 29, no. 1, pp. 71-109, 1993.

[10] Balas, G. J. , Doyle, J. C., Identification of flexible structures for robust control. *IEEE Control Systems Magazine*, vol. 10, no. 4, pp. 51-58, 1990.

[11] Young, P. M. , Newlin, M. P. and Doyle J. C., mu analysis with real parametric uncertainty. *Proceedings of the 30th IEEE Conference on Decision and Control*, Brighton, UK, 1991, pp. 1251-1256 vol.2.

[12] Megretski, A., The Waterbed Effect. *MIT OCW*, 2004.

[13] Fetzer, M., Scherer, C. W., A general integral quadratic constraints theorem with applications to a class of sampled-data systems, *SIAM Journal on Control and Optimization,* 54 (3), 1105-1125

[14] Fetzer, M., Scherer, C. W., Stability and performance analysis on Sobolev spaces, *IEEE 55th Conference on Decision and Control (CDC),* 7264-7269, 2016.