euspen's 21st International Conference & Exhibition, Copenhagen, DK, June 2021
www.euspen.eu

Rocker oscillator time bases

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Abstract

Since the invention of the pendulum clock by Christiaan Huygens in 1657, precision timekeepers have been regulated by oscillators. Although the pendulum is not an isochronous oscillator, its introduction as a time basis for clock regulation led to a significant increase in accuracy. Indeed, numerous clocks were subsequently retrofitted with pendulums to replace the foliot time base. Oscillators have defined revolutions in accuracy, as seen by the terms *quartz watch* and *atomic clock*. Observation of a rocking chair and its mathematical modelling shows that it provides oscillatory motion and can therefore theoretically be used as a clock time base. This led to the study, design, and fabrication of a rocker oscillator for the regulation of a table clock. The demonstrator consists of a rocker oscillator designed to replace the original pendulum of a commercial precision table clock. Two identical clocks were used to compare the operation of the modified clock with that of the original one. The rocker oscillator was dimensioned so that its natural frequency as well as its quality factor fit those of the original pendulum. The performance of both clocks was then the subject of an experimental study. This project serves as a demonstration of how new oscillators can be retrofitted to existing timekeepers.

Rocking oscillator, clock oscillator, time bases, rocking motion

1. Introduction

The retrofitting of timekeepers with new oscillators is a procedure that has been widely used in the history of timekeeping. In previous work [1, 2], novel two degree of freedom oscillators were installed in a traditional table clock. The balance-spring of a mechanical watch has also been replaced by a quartz oscillator, the first time in [3]. These results concentrate on the oscillator and not on the replacement procedure. This paper gives simple outline of how to replace a traditional oscillator with a novel oscillator.

The new time base is a *rocker oscillator* inspired by the observation that a rocking chair has oscillatory motion. By rocker oscillator, we mean a solid object subject to gravity, rolling without slipping on a surface and having a unique stable state about which it oscillates. The paper describes how the classical pendulum time base of a commercial clock was successfully replaced by this type of oscillator and compares the performance of the clock with each of these time bases.

This paper summarizes the results of an EPFL semester project by the first author, in which the methodology allowed him to gain a deep understanding of the theoretical and practical aspects of mechanical timekeeping.

2. Timekeeper theory

Mechanical timekeepers measure time by means of an energy source, usually a spring or a weight, whose energy release is regulated by a time base, most commonly an oscillator. The interface between the driving torque and the oscillator is the *escapement* which has a dual purpose. The first is to transfer maintaining torque to the oscillator by replacing the energy lost due to friction. The second is to regulate the clock's gear train so it can be connected to the hands so as to display civil time.

To replace the oscillator of an existing timekeeper without modifying the movement and the escapement means that the new oscillator has to match the natural frequency of the old oscillator and that the frictional losses of each oscillator have to be matched so that amplitude remains similar in each case.

2.1. Isochronism

Isochronism is the property of an oscillator where its frequency is independent of its amplitude. This is the basis of precise timekeeping as the measure of time is liberated from the maintaining energy. This principle was discovered by Galileo ca. 1600 and first applied to a timekeeper by Huygens in 1657, see [4]. The pendulum is not theoretically isochronous, but Huygens found an elegant theoretical modification, the *cycloidal pendulum,* which unfortunately does not work in practice [4]. Pendulum clocks owe their accuracy to careful constructions and keeping amplitude small.

2.2. Quality factor

The standard theory of harmonic oscillators, see [5], shows that E_l the energy lost to friction during one cycle, can be expressed as

$$E_l = 2\pi\, E/Q, \qquad (2.2.1)$$

where E is the total mechanical energy of the oscillator and the dimensionless quantity Q its *quality factor*. The energy loss is thus fully determined by the quality factor and the total mechanical energy of the oscillator.

3. New oscillator and base clock

3.1. Rocker oscillator concept

The rocker oscillator studied here is a homogeneous solid having the shape of a right circular cylinder cut by a plane parallel to its axis. Its curved face is placed on a horizontal plane surface. It is assumed that the cylinder rolls without slipping.

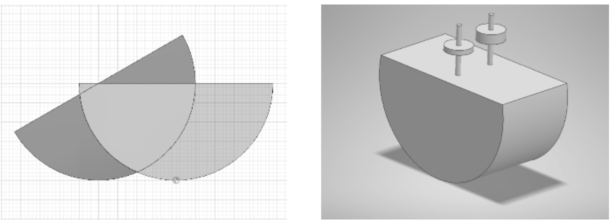

Figure 1. Shape and motion concept of the novel rocker oscillator. On the right with the discs to adjust the frequency.

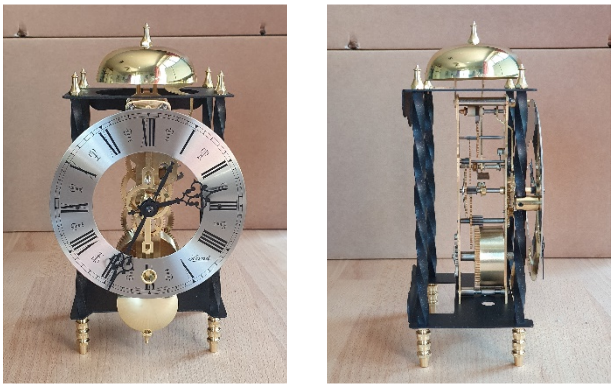

Figure 2. Base clock from Hermle (model Tischuhr 22734-000701). Its pendulum is to be replaced by the novel rocker oscillator while keeping its original movement and escapement.

When the cut is nontrivial, there is a unique stable state around which it oscillates (for small amplitudes), see Fig. 1.

In addition to this object, a system has to be designed to allow the coupling of the oscillator to the base clock's escapement. In order to correct inevitable practical inaccuracies, the final device requires empirical adjustment of the natural frequency and energy, i.e, amplitude, of the oscillator.

3.2. Base clock

The chosen clock is a pendulum clock powered by barrel spring made by Hermle (model Tischuhr 22734-000701), see Fig. 2. Two copies were used, one to retrofit the new rocker oscillator while the other serves as a reference.

4. Frequency matching

The first issue is to match the new oscillator's frequency to match that of the original oscillator, since the movement is designed to display civil time according to this frequency.

4.1. Base clock frequency

The nominal frequency of the base clock can be directly determined by counting the number of teeth of the geartrain and computing the transmission-ratio from the minutes wheel to the escapement wheel. This process applied to the base clock gives a nominal frequency $f_N = 1.3975\ \mathrm{Hz}$.

4.2. Base clock setting resolution

It is also useful to estimate the original clock's setting resolution. This setting is achieved by turning a nut which modifies the pendulum's length and therefore its natural frequency. The setting resolution can be estimated by measuring the change in the clock's daily rate produced by a single turn of the nut. The obtained result is that a single turn of the nuts leads to a variation of $270\ \mathrm{s/day}$. Considering a minimum setting step of the nut of 20 degrees, the clock setting resolution is approximately $15\ \mathrm{s/day}$.

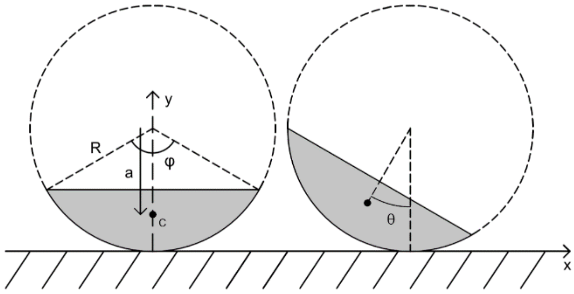

Figure 3. Parameters for the derivation of the equation of motion of the rocker oscillator. The point c indicates the center of mass.

The methodology used to measure the actual pendulum frequency is to count, by means of a laser barrier, the number of periods performed by the pendulum over a certain time interval.

4.3. New oscillator frequency

The frequency of the rocker oscillator is derived from its equation of motion. The derivation of the equation of motion of such an object is similar to the known case of the rolling motion of non-axisymmetric cylinders, see [6]. After computing the position of the center of mass and the inertia of the homogeneous solid presented in Fig. 1, the equation of motion can be derived using Lagrangian mechanics. The final result is of the form:

$$(R^2 - 2Ra\,\cos\theta + a^2 + I/m)\ddot{\theta} + Ra\dot{\theta}^2 \sin\theta + ga\,\sin\theta = 0,$$

where the parameters are as in Fig. 3, m is the mass of the cylinder and I its moment of inertia along its axis at its center of mass. A first order approximation of this equation gives this result:

$$\ddot{\theta} + \frac{ga}{(R-a)^2 + I/m}\theta = 0.$$

This is the equation of a harmonic oscillator and the corresponding frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{ga}{(R-a)^2 + I/m}}.$$

The eccentricity of the center of mass a and the moment of inertia I are parametrized only by the geometric angle φ and the cylinder radius R. Thus, the natural frequency of the rocker oscillator is fully determined by m, R and φ according to the equation above.

This result shows that the rocker oscillator as defined in this paper is isochronous with a second order error. This allows us to consider the frequency as independent of the amplitude and to separate the frequency matching from the energy matching.

Note that this result is valid as long as the curve of the cylinder is circular. A computation of the curve yielding a theoretically perfect isochronous oscillator, in analogy with Huygens' cycloidal pendulum, was announced in [7].

4.4. New oscillator setting resolution

To allow empirical adjustments of the oscillator natural frequency, two solid discs, one heavy and one light, were mounted on threaded rods on top of the cylinder as shown on the right of Fig. 1. The setting of the distance between the discs and the cylinder changes the inertia of the system as well as the position of the center of mass and thus modifies the natural frequency. The lightweight disc is designed to allow for fine adjustment of the frequency (about $5\ \mathrm{s/day}$ per revolution), and the heavier one for coarse adjustment (about $120\ \mathrm{s/day}$ per revolution).

5. Quality factor matching

The second condition that the new oscillator must comply with in order to correctly replace the pendulum of the base clock is that its energy loss per cycle corresponds to the energy supplied by the escapement impulse. This is required as the amplitude of the oscillator at the escapement must be similar. If the base clock is properly constructed, this energy should be equal to the energy loss per cycle of the original pendulum.

5.1. Base clock oscillator quality factor

The quality factor of the pendulum can be evaluated by measuring the decrease in the amplitude of the oscillations of the free pendulum (decoupled from the clock) as a function of the number of periods completed.

According to [5], the evolution of an oscillator damped by a friction force proportional to the speed of the movement (viscous friction) is given by:

$$x(t) = Ae^{-\pi f t/Q}\cos(2\pi f_a t + \phi), \text{ with } f_a = f\sqrt{1 - 1/(4Q^2)}.$$

Therefore, starting with an amplitude A_0, after k periods the amplitude is $A_k = e^{-\pi k/Q} A_0$, yielding the linear relation

$$\log A_k = -\frac{\pi k}{Q} + \log A_0.$$

Using this equation, it is straightforward to derive Q from a series of k and A_k measurements. Using a camera to record the evolution of the amplitude of the original pendulum, the quality factor is $Q = 2700$.

5.2. Base clock impulse energy

According to formula (2.2.1), the energy E_i given by the escapement to the pendulum of the base clock at each cycle is

$$E_i = 2\pi\, mgL(1 - \cos\theta_0)/Q,$$

where m is the mass of the pendulum's bob, L the length of its rod and θ_0 the amplitude of oscillations. Measuring the amplitude of the pendulum of the base clock provides the estimate $E_i \approx 1.24 \times 10^{-6}$ J.

5.3. New oscillator quality factor

The theoretical derivation of the quality factor of the rocker oscillator is a difficult task as it requires a quantitative identification of the frictional losses inherent to the system. The chosen approach is therefore empirical and based on experiment.

The frictional losses are rolling friction and air friction. The second is considered negligible as compared to the first. Rolling friction has two main causes: the deformation of the rolling object or the surface and the slippage between the rolling object and the surface.

The rolling resistance caused by material deformation can be expressed as

$$F = bN/R,$$

where b is the rolling resistance coefficient, N is the normal force and R the rolling object radius. This indicates that to maximize the quality factor of the rocker oscillator, the rolling resistance coefficient should be minimized which is typically the same as maximizing the hardness of the involved materials. According to this equation, the ratio of the oscillator mass to its radius should be minimized.

Slippage between the rolling object and the surface occurs if $F_t > \mu_s N$. Therefore, to avoid this and maximize the quality

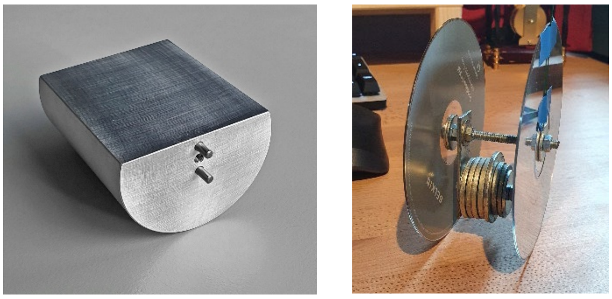

Figure 4. Test oscillators for quality factor estimation, on the left a homogeneous aluminium cylinder and on the right a test device to evaluate the influence of mass.

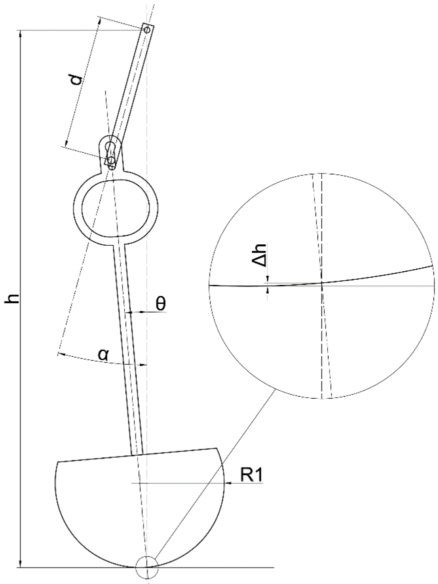

Figure 5. Coupling system and geometry of the rocker oscillator and the escapement of the base clock.

factor, the materials should be chosen to maximize the static friction coefficient μ_s between the cylinder and the rolling surface. The static friction coefficient is usually highest when the materials of the surfaces in contact are identical.

Moreover, in theory, the quality factor is related to the oscillator mass: if m is multiplied by a factor λ, then Q is multiplied by a factor $\sqrt{\lambda}$, see [5]. To verify this statement, the test oscillator presented on the right of Fig. 4 was constructed. Its mass can be modified as for a dumbbell while keeping the position of its center of mass and the radius unchanged. This experiment confirmed the relation above.

These theoretical results give qualitative insight but do not allow us to estimate the quality factor of the rocker oscillator. To this end, it is possible to build a prototype of the oscillator and measure its quality factor using the methods described above. The value obtained for the aluminium prototype shown on the left of Fig. 4 is $Q = 172$.

Subsequent experiments compared different types of rolling surfaces. The tests were carried out by counting the number of oscillations before stopping of the aluminium cylinder of Fig. 4 for different types of supports. As predicted by theory, the results are best when the materials of the cylinder and the rolling surface are identical. These experiments also show that reducing the contact patch increases quality factor.

5.4. New oscillator operating amplitude

By introducing the total mechanical energy of the rocker oscillator in the formula (2.2.1), it is possible to express its energy loss per cycle E_l as a function of its amplitude θ_0:

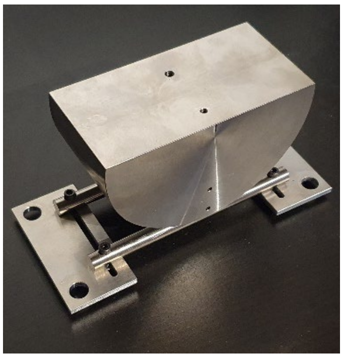

Figure 6. On the left, the constructed cylinder and support rods. On the right, the complete demonstrator with the base clock.

$$E_l = 2\pi\, mga(1 - \cos\theta_0)/Q,$$

where m is the mass of the oscillator, a the eccentricity of its center of mass and Q its quality factor. This equation can be used to find what oscillation amplitude will make the oscillator quality factor match the impulse energy of the escapement.

Fig. 5 shows a drawing of the coupling system between the rocker oscillator and the escapement of the base clock. Some geometrical considerations along with a first order approximation allowed us to derive formula

$$h = d(\theta + \alpha)/\theta \Rightarrow \theta = d\alpha/(h - d)\,,$$

where h is the distance between the rolling surface and the rotation axis of the clock's escapement, d the length of the escapement arm, α the amplitude of motion of the escapement and θ the amplitude of the rocker oscillator. The required value of h was computed using these equations.

6. Demonstrator design and fabrication

The final determination of the parameters of the demonstrator was done using the above methods. In particular, the material of the cylinder was chosen to be steel so as to increase mass (as opposed to using aluminium or titanium). The radius of the cylinder is $R = 50$ mm and the horizontal cut is realized at a distance of 7.19 mm from the cylinder centre ($\varphi = 196.53$ degree). The rolling support is made of two steel rods to maximize friction and minimize the contact patch. Their radius is 8 mm. The cylinder and the support rods were machined in EN 1.4305 stainless steel using standard processes. A picture of the fabricated cylinder and rolling support is presented in Fig. 6 on the left.

The results of the section 5.3 predict a quality factor of the rocker oscillator of order $Q = 400$. A stand was designed to support both the rocker oscillator and the clock and allow adjustment of the vertical distance between them, h in Fig. 5. The amplitude of the oscillator will then be adjusted in order for the energy of the oscillator to fit the impulse energy of the clock's escapement. The complete constructed demonstrator is pictured in Fig. 6 on the right.

7. Results

The quality factor of the oscillator was measured at $Q \approx 430$, validating the analysis of section 5.3. Fig. 7 shows the periods of the free rocker oscillator. After the sixtieth period, the amplitude is too small to observe pure rolling and oscillatory motion is altered. Averaging on the stable part, the natural frequency of the oscillator can be derived and is $f_0 = 1.3935$ Hz.

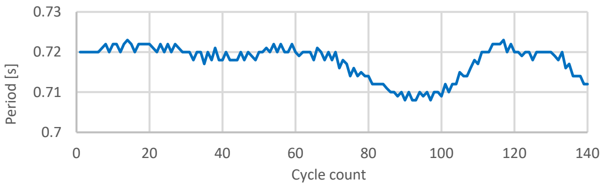

Figure 7. Evolution of the period of the free rocker oscillator.

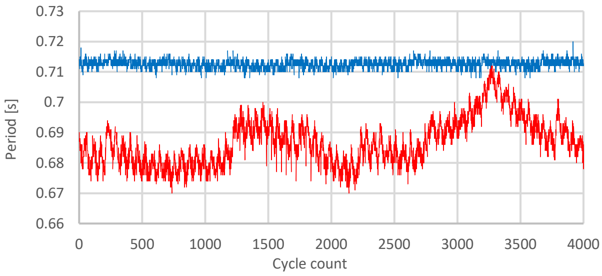

Figure 8. Comparison between the evolution of the period of the clock regulated by the rocker oscillator (in red) and the one of the unmodified pendulum clock (in blue).

This corresponds to an error of 0.29% with respect to the expected frequency. The effects of the frequency setting disks on the natural frequency were evaluated and the results validates the predicted theoretical values.

The oscillator was then installed in the clock and its operation recorded for several values of the parameter h. The resulting frequency was always superior to the natural frequency. Moreover, for small h, the oscillator stops after a number of periods since the maintaining torque does not compensate for oscillator losses. For large h, the periods exhibits significant instability. This seems to be due to the fact that for small amplitudes, the rolling distance is too short to allow pure rolling and the rocker no longer oscillates smoothly. Fig. 8 compares the periods of the rocker oscillator (in red) and the pendulum of the unmodified clock (in blue). Measurements were done over 4000 cycles corresponding to approximately 46 minutes. In this experiment the period of the free rocker oscillator was about: $T = 0.718\ s$. The rocker oscillator rate fluctuates significantly more than the original pendulum.

8. Conclusion and perspectives

We described the successful retrofit of a new oscillator to a standard commercial clock. Our current research is to identify the causes of the rocker frequency instability when coupled to a clock and to design and build a prototype clock whose accuracy is comparable to a standard pendulum clock.

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