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Logarithmic spiral for referencing a camera-screen micro positioning system

O. de Francisco Ortiz[1], G. Azzopardi[2], I. Ortiz Sánchez[1]

[1]*Engineering and Applied Techniques Department, University Center of Defense, San Javier Air Force Base, MDE-UPCT, Spain*
[2]*L'École de l'air, French Air Force Academy, Salon-de-Provence, France*

oscar.defrancisco@cud.upct.es

Abstract
In any precision manufacturing process, positioning systems play a very important role in achieving a quality product. As a new approach to current systems, camera-LCD positioning systems are a new technology that can provide substantial improvements enabling better accuracy and repeatability. However, in order to provide stability to the system a global positioning system is required. This paper presents an improvement of a positioning system based on the treatment of images on an LCD in which a new algorithm with absolute reference has been implemented. The method is based on basic geometry and linear algebra applied to computer vision. The algorithm determines the spiral centre using an image taken at any point. Consequently, the system constantly knows its position and does not lose its reference. The simulation and test of the algorithm provide an important improvement in the reliability and stability of the positioning system providing errors of microns for the calculation of the global position used by the algorithm.

Keywords: Accuracy; Pattern recognition; Positioning; Artificial vision; Precision

1. Introduction

Identifying the same points in two images to control the position is essential in the analysis of the movement controlled by vision. The determination of the movement of an object from a sequence of two-dimensional images has been extensively analysed and discussed by Roach [1] and Yachida [2] using two frames of a sequence to solve this problem. Later Sethi [3] used a sequence of frames to explore the smoothness of the motion by proposing an iterative algorithm to find trajectories of points in a monocular image sequence.

The use of geometric and symmetric patterns in a vision system provides a good local positioning system, which requires additional external information to achieve absolute positioning successfully [4] or depend on the previous relative positions assuming the cumulative errors generated. To improve this perspective, this work presents an asymmetric and non-periodic pattern so that each image taken is unique. A spiral pattern, including the conditions mentioned, is presented as a solution. Pattern analysis using the artificial vision algorithm will be more difficult but will provide better results. Consequently, the algorithm used for the analysis was initially created using Matlab. The solution presented in this work consists in the use of a logarithmic spiral as a geometric base to calculate the global positioning. Mathematical aspects of the logarithmic spiral have been studied by Catrakis [5], who clarifies and summarizes years of mathematical studies on spirals. Logarithmic spirals have also been analysed and used in image processing, but with different approaches in Weiman [6] or Rojer [7] and, more recently, in Palacios [8] and Zhao [9].

The method presented consists in the calculation of the centre position of the spiral determining the consistency and accuracy during position calculation.

2. Methodology

Since a periodic and symmetrical pattern presents a weakness in the vision positioning system, it has been proposed to use an asymmetric and non-periodic pattern so each image taken by the camera, regardless of the area of the image seen, is unique. Therefore, a logarithmic spiral pattern has been implemented. Pattern analysis using the artificial vision algorithm will be more complex but will lead to better results.

The calculation method uses the properties of the logarithmic spiral such as the angle α formed by the radius and tangent vectors at a point of the curve, although it is not perpendicular, remains constant for all points of the curve.

2.1. Algorithm

The method studied is based on the well-known Newton-Raphson method, which is a root search algorithm that uses tangents although the determination of the central point cannot be performed perfectly. Therefore, the error of the calculated coordinates of the centre point must be evaluated. The method presented does not require to know the logarithmic spiral parameters beforehand.

As a first step, the tangent lines at 3 points of the logarithmic spiral are calculated, using an image taken by the camera. At this point the angle of rotation α is unknown. The objective is to find the coordinates of the central point O of the spiral (figure 1), which in this case will be designated by O_3 which includes an error estimation (figure 2).

Three points P_1, P_2, P_3 are taken over the spiral of figure 1 and the coordinates of each of these points are, respectively, x_1, y_1, x_2, y_2, x_3, y_3. The coordinates of the central point O will be x_c, y_c and the angle between the tangent and the radius at each point is designated as α. The radius direction vectors for each of the three points are r_1, r_2, r_3 and the tangents t_1, t_2, t_3.

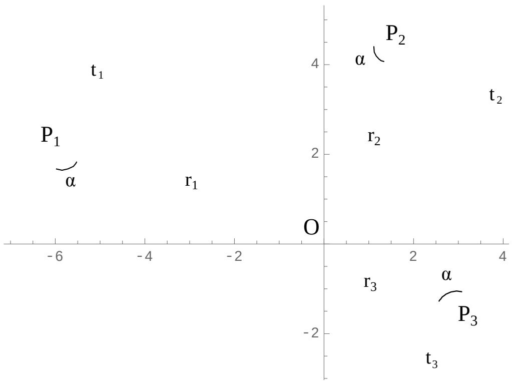

Figure 1. Scheme, elements and nomenclature used in the method

To relate the angle α with the 3 points chosen, the formal definition of the scalar product is used to calculate the cosine of the angle between two vectors, so that the equation 1 is fulfilled, where vectors t_1, t_2, t_3 are fully known because the tangent lines are known and calculated previously using discrete points over the spiral.

$$\cos\alpha = \frac{<\vec{r_1}|\vec{t_1}>}{||\vec{r_1}||\ ||\vec{t_1}||} = \frac{<\vec{r_2}|\vec{t_2}>}{||\vec{r_2}||\ ||\vec{t_2}||} = \frac{<\vec{r_3}|\vec{t_3}>}{||\vec{r_3}||\ ||\vec{t_3}||} \quad (1)$$

r_1, r_2, r_3 can be expressed based on the coordinates x,y, the central point O and the 3 selected points (P_1, P_2, P_3) obtaining the equations 2. A system of 3 equations with 3 unknowns (x_c, x_c, α) is provided

$$\cos\alpha = \frac{<(x_c-x_1,y_c-y_1)|\vec{t_1}>}{||(x_c-x_1,y_c-y_1)||\ ||\vec{t_1}||} = \frac{<(x_c-x_2,y_c-y_2)|\vec{t_2}>}{||(x_c-x_2,y_c-y_2)||\ ||\vec{t_2}||} = \frac{<(x_c-x_3,y_c-y_3)|\vec{t_3}>}{||(x_c-x_3,y_c-y_3)||\ ||\vec{t_3}||} \quad (2)$$

When solving and calculating these values, the angle α found will not correspond to the real value, since the tangents used are approximations that contain a small error. Figure 2 shows a general graphic scheme of the method showing how the three lines whose vectors are r_1, r_2, r_3 and passing across, respectively, through the points P_1, P_2, P_3, intersecting at the same point O_3 which is the centre of the spiral calculated with this method.

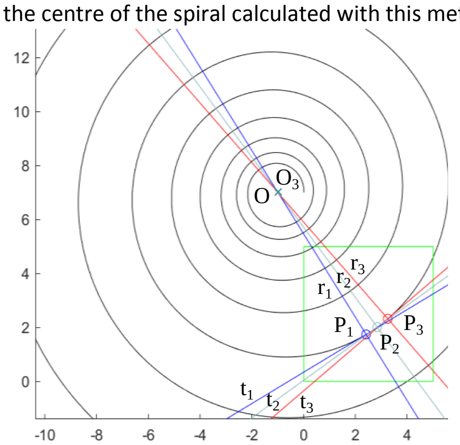

Figure 2. Graphical representation of method with the tangent and radial lines that pass through the points P_1, P_2, P_3.

4. Simulation and Results

Taking an amplitude or scale factor of the spiral of 1, for a screen with a width of 640 pixels, it corresponds to 4.20 units on the Matlab scale. Although the pixel size is a limiting factor, considering a screen size of 640 pixels and width of 51.70 mm, it is obtained that each Matlab unit corresponds to 12.30mm.

The consistency of the method is demonstrated by performing more than 100 simulations with each construction of the spiral, where d is the distance between two consecutive discretized points when the spiral is made. The method has been tested with 11 different distance d values. Errors in the calculation of the position of the centre of the spiral O vs distance d in mm provided by the method are shown in figure 3.

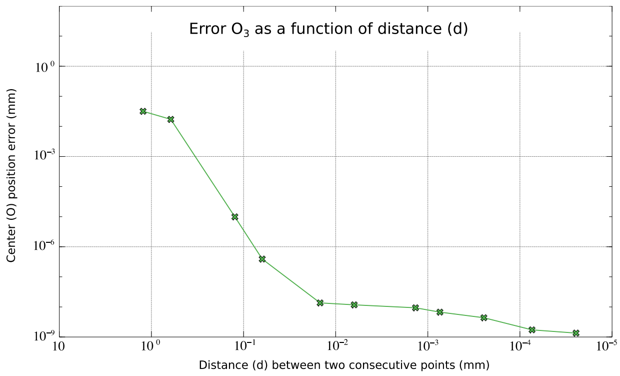

Figure 3. Errors in the calculation of the position of the center of the spiral in mm.

5. Conclusions

A method has been presented to achieve an absolute positioning system based on a vision system in which an asymmetric logarithmic spiral pattern has been used as the basis. This pattern is different from the one used for the development of the camera-screen system presented in previous studies [10] based on a pattern of repetitive LEDs lighten on the screen whose position calculation was relative to the previous calculated point, which may cause loss of reference.

The method is based on the property that the tangent line at any point of the spiral and the line that passes through the centre of the curve drawn from that point form a constant angle. The algorithm has been tested by performing different simulations in Matlab, taking random photos of an area of the logarithmic spiral represented and calculating the position of the centre of the spiral. With the position of the centre, the system is able to position itself with respect to the rest of the system, so that the desired absolute positioning system is achieved. In addition, the method, does not need to know in advance any of the spiral parameters. Since the errors depend on the discretization of the curve in its simulation that is defined by the distance between points of the spiral used, different simulations have been made finding that with a distance of 0.05 mm between points on the curve, the method It is able to position the centre of the spiral with an error of less than $1x10^{-6}$ mm.

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