Corner loading and its influence on the tilt sensitivity of precision weighing cells

Maximilian Darnieder¹, Markus Pabst², Thomas Fröhlich³, René Theska¹

Technische Universität Ilmenau, Department of Mechanical Engineering
¹Institute for Design and Precision Engineering, Precision Engineering Group
²Institute for Process Measurement and Sensor Technology, Process Measurement Technology Group

maximilian.darnieder@tu-ilmenau.de

Abstract

Corner loading and ground tilt are relevant error sources for precision weighing cells. The sensitive compliant mechanism of the weighing cell is subject to large dead weights with the aim to resolve small mass differences. Tilting of the base or an eccentric load application relative to the centre of the weighing pan result in lateral force components, apart from those in measurement direction. This changes the indication of the balance resulting in measurement errors. In this paper, a finite element model of the weighing cell was used to evaluate the corner load sensitivity in three directions and the tilt sensitivity in two directions. Based on a parametric study, an optimal attachment point for the weighing cell was found where all considered mechanical errors can be reduced to second order effects.

precision weighing, corner load error, compliant mechanism, finite element modelling

1. Introduction

Precision balances with electromagnetic force compensation (EMFC) are applied as measurement instruments in many scientific and industrial fields. Special weighing cells are used in mass comparators for the most precise mass determination tasks [1]. The most demanding requirement for the mechanical system of precision weighing cells is expressed by the load to resolution ratio. For precision weighing cells, the ratio ranges from about \(10^5\) to up to \(10^{10}\) for applications in mass comparators [2]. This implies a high sensitivity of the weighing instrument in measurement direction, despite of high dead loads (e.g. 1 kg). At the same time, the errors due to lateral force components need to be minimized. These are induced by tilt angles between the mechanism and the g-vector (ground tilt) and eccentric placement of the weights on the weighing pan (corner load).

![Figure 1](image.png)

**Figure 1.** Schematic of the EMFC weighing cell with its parameters for the mechanical model (1-base, 2-lower lever, 3-upper lever, 4-load carrier, 5/6-weighing pan, 7-coupling element, 8-transmission lever).

It was shown that the sensitivity to ground tilt can be adjusted to zero for a certain configuration of the weighing cell [3]. However, both errors were found to be interdependent. An optimal configuration for a specific weighing cell would require both sensitivities, tilt sensitivity and corner load sensitivity, to be zero.

Many solutions are available to make the weighing system largely independent from eccentric load placings on the weighing pan: Transferring the lateral force components directly to the frame via a parallelogram guide is the most common solution. This is known as Roberval mechanism and is frequently used in the mechanism of EMFC weighing cells [4] (see Fig. 1, parts 1-4). The efficiency of this solution is highly dependent on the alignment of the flexures in the mechanism. Typical deviations in the manufacturing process are already problematic. Consequently, tedious adjustment of this part of the mechanism is required [5].

Additional devices or mechanisms can be attached to the weighing pan [6,7]. These mechanisms effectively place the centre of rotation over the weighing pan to create a pendulum type hanging weighing pan. An eccentrically placed weight is mechanically centred on the weighing pan, removing the error to a large extend. A similar solution is the gimbal mounted hanging weighing pan. Assuming a gimbal with zero stiffness, the hanging mass is perfectly centred [8].

From the literature, no investigation concerning the location of the weighing pan itself or the reference point for the eccentricities is known. For small eccentricities in precision applications, this can provide a highly effective and less cumbersome solution. The sensitivities of the weighing cell for tilt and corner load were evaluated for varying positions of the point mass \(m_S\), based on a mechanical model of the EMFC weighing cell, see Fig. 1. In a second step, the location of the weighing pan itself is varied to find its optimal attachment point. As a conclusion, recommendations for an improved weighing cell design are deduced.

2. Mechanical model of the weighing cell

For the determination of the mechanical properties of interest, corner load sensitivity \(E_c\) and tilt sensitivity \(D\), the non-structural components were simplified, excluding additional
effects on the indication of the balance. Due to the complexity of the load application and the complex stress within the mechanism, a three-dimensional finite element (FE) model was found to be most appropriate for the computation task. The compliant mechanism with thin flexures was loaded by point masses and a displacement constraint.

This displacement constraint with its resulting reaction force $F_{EMFC}$ replaces and simplifies the electromagnetic force compensation (EMFC), which can be described as precise position controller with a voice coil actuator and an optical position sensor. The indicated mass $m_{ind}$ was calculated using the reaction force $F_{EMFC}$.

The finite element computations were based on the software ANSYS®. The weight on the weighing pan was modelled as point mass firmly connected to the load carrier (4). The most relevant part of the mechanism, the thinnest sections of the flexure hinges, were finely meshed with SOLID186 elements following the mesh sensitivity study in [9]. The resulting mesh is shown in Fig 2:

Figure 2. Geometry and mesh of the flexure hinges used in the FE model ($h = 50 \mu m$, $r = 1.5 \ mm$, $d = 6 \ mm$). The depth of the structure is 10 mm.

It was sufficient to refine a central zone of 1.5 mm to cover the zone with relevant stress gradients, see Fig. 2. The non-matching meshes of the two mesh zones were connected via multi-point constraints (MPC).

Slight offsets of the weight of the weighing pan as well as different locations of the weighing pan itself can be considered. This way, the corner load sensitivity was calculated $E_{c}= \Delta m_{ind}/(m_{c} \Delta \phi)$. The tilt sensitivity $D$ was evaluated for two directions of tilt. It is defined as the apparent mass change on the weighing pan over a small tilt angle around the zero position $(D_{\theta} = \Delta m_{ind}/\Delta \theta$ and $D_{\phi} = \Delta m_{ind}/\Delta \phi)$. The angle $\theta$ is the nick rotation of the weighing cell about the $y$-axis, whereas $\phi$ is the roll rotation about the $x$-axis, see Fig. 1.

3. Corner loading and tilt sensitivity

In the model, the mass point ($m_{c} = 0.433 \ kg$, see Fig. 1) was intentionally displaced in $xyz$-direction on the weighing pan to evaluate the corner load sensitivity $E_{c}$ and the influence of the corner load on the tilt sensitivity $D_{\theta}$ and $D_{\phi}$. For the determination of the tilt reaction, the $g$-vector was inclined relative to the base of the weighing cell in two directions. The resulting relative reading of the mass value is shown in Figs. 3 to 5 for different configurations of the weighing cell.

Figure 3 shows the calculated curves for a nick rotation of the weighing cell for different configurations and different eccentricities of the point mass representing the weight on the weighing pan.

The curves are dominated by a linear trend that results from vertical distance between the centre of mass and the effective centre of rotation on parts of the mechanism undergoing a rotational motion. This linear trend can be adjusted by trim masses on the respective parts to shift the centre of mass in the centre of rotation [3]. The behaviour, with the linear trend removed, can be observed in Fig. 4. A pronounced change of the nonlinear behaviour can only be observed for a variation of the mass on the weighing pan $\Delta m_{s}$. This results in an additional component of second order, while all other changes lead to components of third order or higher. The mass imbalance that leads to the second order effects has already been described in [10]. Weighing cells with small electric weighing ranges, e.g. weighing cells in mass comparators, exhibit only a small second order effect for nick motions; thus, the tilt sensitivity can be strongly minimized by the vertical adjustment of trim masses.

Figure 4. Relative mass change for pronounced eccentricities of the weight on the weighing pan for nick angle.

The second order effect is also visible for the roll motion of the weighing cell, as Fig. 5 shows. Most relevant for the roll motion is the $y$-eccentricity of $m_{c}$ on the weighing pan since it leads to a first order effect, see Fig. 5.

To minimize the roll sensitivity, the position of the weighing pan and the allocated weight on the pan needs to be as close as possible to the $xz$-plane.
The corner load error without tilt ($\theta = \Phi = 0$) and eccentricities in all three spatial directions is displayed in Fig. 6. The comparison revealed a pronounced change of the indicated mass for eccentricities in $x$-direction with a local corner load sensitivity of $E_{Lx} = 411.2 \mu g \cdot kg^{-1} \cdot mm^{-1}$, hereby exceeding the aspired maximum value of $E_l = 100 \mu g \cdot kg^{-1} \cdot mm^{-1}$.

Displacements of the mass in $y$-direction result in a parabolic curve symmetric to the origin, where the sensitivity or pitch of the curve around the neutral position is zero $E_{Ly} = 0.00 \mu g \cdot kg^{-1} \cdot mm^{-1}$. The first derivative of $m_{ind}(y)$ has its zero crossing at $e_{Sy} = 0$. As for the roll sensitivity, the weighing pan should be attached in the $xz$-plane to minimize the corner load sensitivity.

A displacement of the point mass in $z$-direction by 10 mm results in an indicated mass differences of merely $0.05 \mu g$ corresponding to a corner load sensitivity of $E_{Lz} = 0.01 \mu g \cdot kg^{-1} \cdot mm^{-1}$, thus the $z$-direction can be neglected for small eccentricities.

It became evident that the $x$-direction eccentricity of $m_3$ has a major effect on the indicated mass value of the weighing cell. This raised the question, whether a position for $m_3$ relative to the weighing cell exists where the sensitivity to corner loads $E_{Lx}$ has a minimum.

### 4. Location of the weighing pan

The weighing pan of EMFC weighing cells is commonly located in proximity to the $yz$-plane including the ideal rotation axes of flexures C and D ($x = 0$), see Fig. 1. The corner load sensitivity was evaluated in the local coordinate system $(xyz)$. The position of the point mass $m_3$ was varied in the local coordinate system and the indicated mass value $m_{ind}$ was observed. An optimal location for the weighing pan is found where $\frac{d}{dx}m_{ind} = 0$ holds. If the weighing pan is placed in this position, the weighing cell is innocent to corner loads (remaining errors of second order or higher).

The $m_{ind}$-curve in Fig. 7, for an eccentricity in $x$-direction, is of second order. The interesting discovery was the location of the zero-crossing of the first derivative in Fig. 7.

In $x$-direction, this zero crossing is far away from the common weighing pan location. This is remarkable, since none of the existing weighing cells is designed accordingly. The results suggest that the centre of the weighing pan needs to be designed in this special $x$-position to make the weighing cell innocent towards corner loads in $x$-direction.

The cause for the behaviour was found in the variation of parallelism between the two levers of the parallelogram linkage (A-D), as indicated in [5]. By slight deformations of the assumed rigid parts (e.g. load carrier), the positions of the flexures are slightly shifted. Figure 8 shows the scaled deflection states of the parallelogram linkage for the initial weighing pan position and the determined optimum. It is evident that the remote weighing pan position leads to a larger bending deflection of the load carrier, bringing flexures C and D closer together. This seems to compensate the initial deflection on the left side which is reducing the distance between flexure A and B. Finally, the angle $\angle \overline{AD} \overline{BC}$ approaches zero.
The tilt insensitive optimum is not achieved for perfect parallelism of the levers. A slight angle of a few arcseconds seems to be required for the compensation of other effects.

Since the determined optimal weighing pan location is far from the original position, it was checked whether the corner load sensitivities in the other direction are influenced by the major shift in x-direction.

5. Conclusion and Outlook

Corner loads or small eccentricities of the weight on the weighing pan change the roll sensitivity of the weighing cell. A first order error component is added to the second order roll angle error. Corner loads in y-direction result in a second order error symmetric to the origin. Both errors stress the need to minimize any eccentricities of the weighing pan and the weights placed upon it in y-direction.

A location of the weighing pan relative to the mechanism was determined where the both tilt and corner load sensitivity can be minimized to negligible values with remaining small errors of second order. Surprisingly, this location is far off the common location in commercially available EMFC-weighing cells. The reason for the discovered phenomena is the small deflections of the assumed rigid parts of the weighing cell mechanism. These are changing the positions of the flexures in the parallelogram linkage of the weighing cell. The discovered relationship can be used for the adjustment of the corner load sensitivity. Instead of designing the load carrier with a large bending stiffness, the bending stiffness can be designed to fit the required adjustment resolution and to limit the required eccentricity of the mass on the weighing pan.

This method provides the possibility to keep the design of the mechanism as simple as possible and to achieve the adjustment of the corner load sensitivity by a simple variation of the position mass points attached to the mechanism. This finding enables the reduction of the overall measurement uncertainty with minimal effort.

In theory, the discovered phenomena should also apply for gimbal mounted weighing pans, where the gimbal has a finite stiffness. Corner loads on the hanging weighing pan, result in angular deflections of the gimbal and small torques on the load carrier (4). Mechanically the small additional torque is equal to a slightly displaced mass. Hence the optimal weighing pan position should equally apply for the position of the gimbal of the hanging weighing pan.

For future weighing cell designs, the results of this study suggest:

- The centre of the weighing pan or the gimbal of the hanging weighing pan should be allocated at the x-position where \( \frac{\partial}{\partial Lx} m_{\text{ind}} = 0 \) holds.
- The second order tilt effect for both directions is proportional to the mass imbalance on the weighing cell or \( F_{\text{EMFC}} \) respectively.
- Weighing pan and weights need to be carefully aligned to the xz-plane of the mechanism to minimize tilt- and corner load errors.

Before applying this concept to other weighing cells, the parametric dependence of the optimal location for the weighing pan needs to be checked thoroughly. An experimental verification with existing weighing cells is required: In operation of the weighing cell, the corner load errors are likely to be higher than the pure mechanical values in this paper, since position sensor and actuator are influenced by lateral deformations and deflections of the structure.

The consideration in this paper omits any type of manufacturing deviations and changes to the flexure positions within the mechanism or the corner load sensitivity. Most likely, manufacturing deviations are influencing the optimal weighing pan position, which will be investigated as part of the future work.

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