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Ultra-precision direct diamond shaping of composite polygonal Fresnel lenses

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Abstract
Fresnel lenses have gained much attention in recent years due to their weight savings, large spatial coverage and small device footprint. However, production of these lenses has been limited to mostly circular contours as conventional diamond turning is used. While Fresnel applications continue to see increasing uptake in industrial applications, purely circular lenses may prove inadequate in serving their purpose. Enabled by the Virtual Rotating Tool Shaping (VRTS) algorithm on a two-dimensional point cloud with a depth component, various geometrically contoured three-dimensional functional surfaces can now be fabricated, while achieving maximum fill-factor compared to their radial counterparts. As such, Fresnel lenses can now be directly manufactured with various geometrical profiles without being limited to radial contours. The centre of the lenses was also designed to remain radial to reduce dispersive losses, while the lens periphery takes the form of the chosen polygon. In this study, a composite polygonal square Fresnel lens was successfully machined on a brass workpiece using a 5-axis ultra-precision machine.

Ultra-precision, Diamond, Micromachining

1. Introduction

Fresnel lenses are created by segmenting the curvature of a conventional spherical lens radially into various zones, each of which is known as a Fresnel Zone Plate (FZP), and aligning them in plane to significantly reduce the thickness of the lens [1-3]. As such, these lightweight, thin and highly efficiently lenses have greatly benefited optoelectronics, photonics and various solar applications, significantly enhancing their performance [3-5].

To increase the spatial coverage of such optoelectronic components, it is quite prevalent to observe the use of multiple instances of these lenses, often deployed in an array. As typical Fresnel lenses inherit their circular nature from their conventional counterparts, these lens arrays usually lack the ability to achieve maximum fill-factor [2,6,7].

The conventional approach to address this issue would require the trimming the round profile of these Fresnel lenses to obtain straight edges. This allows for assembly of these lens arrays by arranging the edges together. However, this often leads to geometric inaccuracies due to the need for alignment of an asymmetrical geometry to the rotational centre of the machine. The assembly process for these types of lens arrays is also extremely time-consuming and tedious as the tolerances between each lens are usually kept to ± 10 µm. Furthermore, the optical performance around the fringes of the inscribed circle is usually compromised as incident light may not be well received due to the differing focal points for each facet [7].

Alternatively, the groove of the facets can be directly generated into geometrical shapes to allow for ease of tessellations [2,6,7]. This will allow the Fresnel arrays to achieve maximum fill-factor without any gaps in between the lenses. Although the cross-sectional profile of these lenses is akin to that of their radial counterparts, the facets remain perpendicular the apothem of the polygon.

Polygonal lenses which have been produced till date have also experience losses in optical performance due to a degradation in focusing efficiency [2]. Also, the corners of these polygonal lenses possess sharp edges between the facets, further affecting optical performance.

The composite polygonal Fresnel lens solves these issues by firstly removing the internal corners of the polygon, rounding the radial feature into an elliptical facet. To increase the focusing efficiency, the middle of the lens possesses a conventional radial Fresnel lens. As these composite Fresnel lens possess both radial and linear features, the Virtual Rotating Tool Shaping (VRTS) algorithm must be employed as conventional diamond turning cannot produce these features [8].

In this paper, the surface generation of the composite polygonal square Fresnel lens is discussed, followed by a modified VRTS algorithm to enable three-dimensional surface generation on the UPM. The experimental setup and geometrical considerations were then examined.

2. Direct Diamond Shaping

Direct Diamond Shaping of the composite Fresnel lenses requires the generation and design of the three-dimensional surface profile, presented in section (*2.1*), and the corresponding VRTS algorithm that enables the UPM to directly shape the workpiece (*2.2*).

2.1. Composite Fresnel lenses

Composite Fresnel lenses consist of two distinct profiles when observed from the XY plane view (Fig 1), namely the rotary symmetric focal centre and the polygonal periphery. This allows the Fresnel lens to possess regular edges while maintaining an optimal focal centre, achieving maximum fill-factor with tessellated patterning. As such, two separate cutting strategies are required to develop the functional surface.

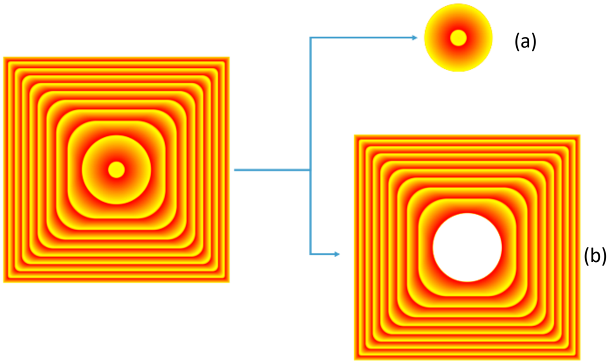

Figure 1. Constituent profiles of composite polygonal Fresnel lenses, with the (a) rotary symmetric focal centre and (b) the polygonal periphery.

The first feature developed was the rotary symmetric focal centre (Fig 1a). A conventional constant angle Archimedean spiral was used to generate the point cloud on the XY plane, given by:

$$\rho = r - f_r\theta \tag{1}$$

The radial feed, fr, was set at 2 µm for the roughing and 1 µm for the finishing pass, while the spiral starting position, r, was set to 0.89661 mm. Once the XY plane profile has been developed, it can be mapped onto the XZ profile. This is mathematically described by a piecewise function, given as follows:

$$z_r = \pm\sqrt{c^2 - (x-h)^2} + k \qquad (\text{if } x = x_p\,, z_r = z_p) \tag{2}$$

$$z_f = \left(z_{p+1} - z_p / x_{p+1} - x_p\right)\left(x - x_p\right) + z_p \tag{3}$$

Where h and k signify the *x* and *z* coordinates for the centre of the arcs respectively, while *c* represents the radius of circle. The subscripts *r, p* and *f* signify the rounded facets, the piece-wise boundaries and the facets respectively. This provides the third dimension to the point cloud to generate the functional surface. The cross-sectional geometry and the associated tool path can be observed in Figure 3.

After determining the Fresnel profile, the point cloud for the polygonal periphery was developed. Using the maximum radius of the focal centre as reference, the fillets of the polygonal profile linearly decrease to a point, depending on the final size of the feature. Given the number of sides of the polygon, n, the polygonal profiles can be developed as follows (*ref.* Fig 2):

For $i = 1 \rightarrow n$, where $n \geq 3$

$$\alpha = (2\pi/n) \tag{4}$$

$$L = (R - r_f)/tan(\tfrac{\pi}{2} - \tfrac{\pi}{n}) \tag{5}$$

$$r_c = \sqrt{L^2 + (R - r_f)^2} \tag{6}$$

$$h_i = r_c \cos\left(\tfrac{\pi}{n} + \alpha(i-1)\right) \tag{7}$$

$$k_i = r_c \sin\left(\tfrac{\pi}{n} + \alpha(i-1)\right) \tag{8}$$

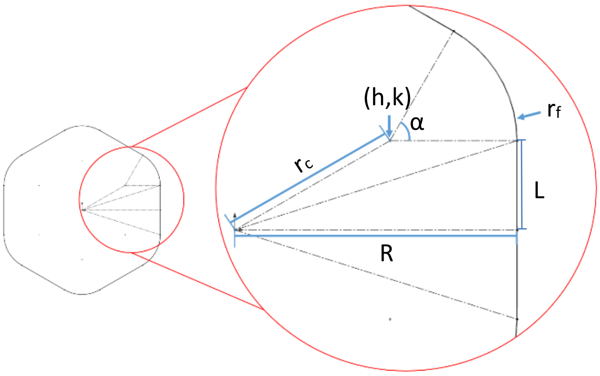

Figure 2. Schematic of the various geometrical relations for a single polygonal profile, using a hexagonal example. The radius of the fillet linearly reduces to a point as the profile tends towards the maximum perimeter to achieve maximum tessellation fill-factor

Where α represents the opening angle of each fillet. L is the straight-line distance connecting each fillet. R signifies the apothem of the feature from the focal centre, while r_f and r_c represent the radius of the fillet and the radius of the fillet centre respectively. The planar position of the fillet centres is represented by h_i and k_i for their X and Y displacement respectively. The entire point distribution of the profile is created with the same point spacing as the maximum arc distance of the Archimedean spiral to maintain a constant cutting speed across the feature. By including Z profile for each facet, the two-dimensional point cloud is transformed into a three-dimensional one, ready for VRTS conversion.

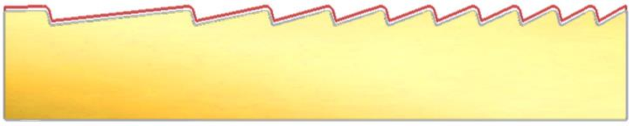

Figure 3. Cross-sectional profile of Fresnel lens with tool path compensated with the tool nose radius offset, as shown in red.

2.2. 3D Virtual Rotating Tool Shaping

As the composite Fresnel lens possesses both radial and linear features around the focal centre, conventional turning techniques cannot be used to develop the functional surface. By modifying the existing VRTS algorithm, the rake face of the tool will always be kept perpendicular to the cutting direction, as well as provide the dimension of varying depth to develop the Fresnel facets [8]. This is done by synchronizing the rotational and linear axes to compensate for translational displacements, while guiding the tool along the intended path.

The modified VRTS algorithm was applied to the XYZ point clouds developed in section *2.1*, using the cartesian coordinates around the focal centre. Depending on the machine used, the orientation of the tools and the axial direction of the translation between the workpiece and tool, the algorithm is modified as follows:

$$\varphi_j = \begin{cases} \frac{3\pi}{2} + atan2\left((y_{(j+1)} - y_j)/(\,x_{(j+1)} - x_j)\right) & \text{if x<0 and y<0} \\ atan2\left((y_{(j+1)} - y_j)/(\,x_{(j+1)} - x_j)\,\right) - \frac{\pi}{2} & \text{Otherwise} \end{cases} \tag{9}$$

$$\begin{bmatrix} x'_j \\ -y'_j \\ z'_j \end{bmatrix} = \begin{bmatrix} cos(\varphi_j) & -sin(\varphi_j) & 0 \\ sin(\varphi_j) & cos(\varphi_j) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} \tag{10}$$

$$c_i = \sum_{i=1}^{n} \varphi_i \tag{11}$$

Where φ represents the trajectory of the point to the next, while the subscript j indicates the iteration. The initial location of the points is indicated with x, y and z, while the transformed points are given by x', y' and z'. The rotational position of the spindle is given by c, from 0 to 360 degrees. Hence, the XYZ input coordinates of the functional surface can be translated to obtain the XYZC positions to develop the functional surface.

3. Experimental verification

To verify the VRTS technique, the process was conducted on a 5-axis ultra-precision machine. The physical setup of the machine is presented in Figure 4. A cylindrical brass workpiece measuring 10 mm in diameter was used. A conventional single crystal natural diamond tool was used, with a nose radius of 14 µm, 20° included angle, 120° opening angle and a 15° cylindrical front clearance angle.

To avoid collision with the facets, the tool was set at a 15° negative rake angle, as observed in Figure 5. This provided sufficient clearance for the front clearance of the tool to avoid any interaction with the formed facets. The clearance required due to the inclination of the facets can be determined by the following equations:

For $n \geq 3$

$$\beta = 90 - 180/n \tag{12}$$

$$\gamma = atan\,[tan(\varepsilon)tan\,(\beta)] \tag{13}$$

Where β is half of the geometrical plane shift angle and ε is the actual facet angle, taken from the top surface of the profile towards the facet. γ is the projected facet angle, which also determines the minimum clearance required for the tool so as not to damage the corresponding facet at the corners. This equation for the projected feature angle still holds for the intersection of any two facets if the plane shift angle, 2β, is determined.

The point cloud generated the rotary symmetric centre had approximately 316,000 points with a feed rate of 2 mm/s. While the point cloud generated for the periphery amounted to approximately 7,879,000 points with a 0.002 mm spacing, keeping the feed rate to consistent with the rotary symmetric centre. Each pass took approximately 4 hours to complete with a total of 4 roughing passes and 1 finishing pass at 10um and 5um respectively.

4. Results and Discussion

Using the VRTS algorithm, a polygonal square composite Fresnel lens was successfully machined on the surface of the brass workpiece, as shown in Figure 6 and 7. A stylus profilometer was used to investigate the quality of the machined surface.

For the rotationally symmetric centre, the surface roughness was measured with an average Ra of 8.3 nm, while the periphery features averaged a surface roughness of Ra of 13.6 nm. This corroborates well with the exceptional surface finishing that diamond turning produces, which is reflected by the mirror finishing as seen in Figure 7. Figure 6 provides a close-up view on the composite polygonal Fresnel lens.

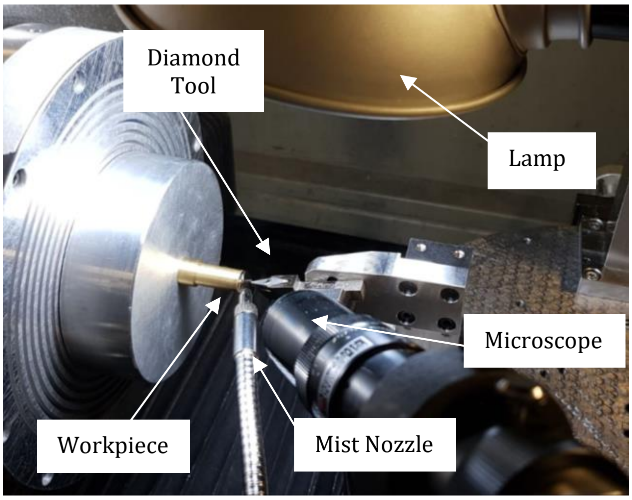

Figure 4. Physical set-up for VRTS on the 5-axis Ultra-Precision Machine with the tool set upon the B-stage. The tool is set with a negative 15 deg rake angle to manoeuvre the tool without clashing into the succeeding facets of the lens.

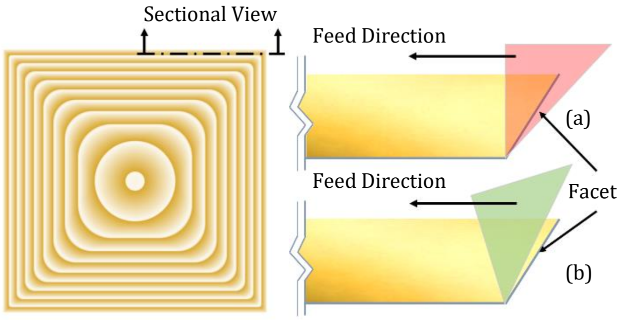

Figure 5. Geometrical considerations must be considered to avoid tool collision with the Fresnel facets. (a) Tool with zero rake angle, with the front clearance colliding with the corresponding facet. (b) tool with negative rake angle to avoid collision with the corresponding facet. With the B-axis of the machine still available, full 5-axis machining can be used not only for collision avoidance, but also for optimal rake angle adjustments.

Figure 6. A close-up microscope shot of the composite polygonal Fresnel lens.

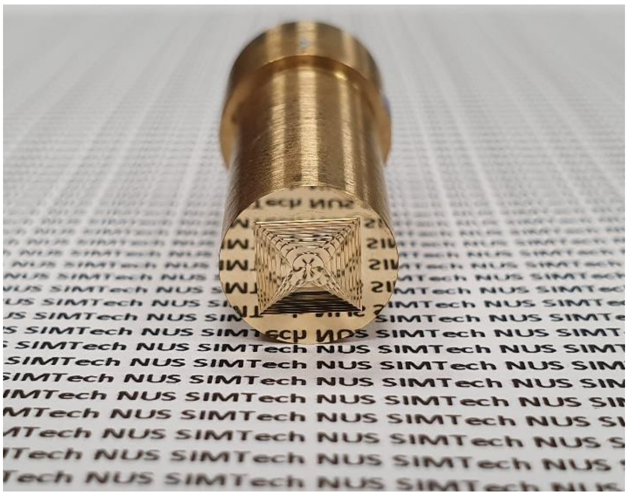

Figure 7. The composite polygonal Fresnel lens was successfully machined on the surface of a 10mm cylindrical brass workpiece by employing the VRTS algorithm.

With the successful shaping of the composite polygonal Fresnel lens, the VRTS algorithm has proven its viability and flexibility in producing high quality three-dimensional functional features, not limited to rotary symmetric features. This opens up the possibility of optical designs which have yet to be realised due to current manufacturing constraints.

5. Conclusion

The composite polygonal Fresnel lens was successfully fabricated by employing a modified VRTS algorithm on a three-dimensional point cloud. As the functional surface possesses both linear and radial features, the VRTS algorithm was required to guide the tool along the intended tool path continuously, enabling orthogonal cutting despite having off-axis radial features.

Two separate approaches were taken to develop the distinct features of the functional surface. For the rotary symmetric centre, a conventional constant angle Archimedean spiral was used. The lens periphery was developed using a geometric algorithm to cater for the rounded edges of the lens, after which a depth component was added to the profile to realise the three-dimensional surface. The surface finish of the rotary symmetric centre was measured with an Ra of 8.3 nm, while the periphery was measured at an Ra of 13.6 nm. This corroborates well with the excellent surface finish of diamond turned products.

Hence, the design, development and production of Fresnel lenses are now not just limited to radial or linear lenses, but a combination of these features can also be realised on the UPM. By producing lenses with brilliant finishing new optical designs which have not been realised till date can now be brought to fruition to advance optoelectronics and photonics applications.

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