

Measurement and identification of translational stiffness matrix for static loads in machine tools

Nikolas Theissen^{1*}, Theodoros Laspas¹, Károly Szipka¹, Andreas Archenti¹

¹ KTH Royal Institute of Technology, Department of Production Engineering, Brinellvägen 68, 10044 Stockholm, Sweden

*Corresponding Author: theissen@kth.se

Abstract

Stiffness is an important characteristic of production machinery, as it contributes to its ability to precisely maintain the pose between a tool centre point with respect to a workpiece under loads. For machine tools, it directly affects the geometric dimensions and surface properties of the parts, i.e. how closely the parts match their design drawings.

This work presents an efficient measurement procedure to measure and identify the full translational stiffness matrices of machine tools. The measurement procedure consists of inducing static loads, which vary in magnitude and direction, at the tool centre point of the machine tool using the Loaded Double Ball Bar and measures the displacement with three Linear Variable Differential Transformers. The main components of the uncertainty budget related to the measurement of the cross compliance are also summarized.

The measurement procedure is implemented in a case study on a 5-axis machining centre. Finally, the manuscript concludes with a discussion on the utility value of the translational stiffness matrix for the design and operation of machine tools as well as the possibility to expand the measurement procedure to capture the quasi-static and dynamic compliance.

Keywords: Machining, Measurement, Performance, Stiffness

1. Introduction

Machine tools are essential for technologically developed societies. Their accuracy enables machining complex products of tight Geometric Dimensions and Tolerances (GD&T). One of the main design criteria of machine tools is static stiffness which affects their performance under load. Several works exist which focus on the stiffness evaluation and identification at the Tool Centre Point (TCP) of a machine tool. There are *a priori* modelling approaches employing FE analysis [1] and *a posteriori* experimental approaches [2]. The experimental approaches can be further divided into machining tests [3] and experiments [4]. ISO230-1:2012 provides guidelines for the measurement and evaluation of machine tool static compliance (inverse of static stiffness) and hysteresis. The measurement consists by applying a static load between the spindle and the workpiece or any machine component (e.g. table and spindle) and measure the displacements for each axis. The limitation of the process is the measurement of deflections solely in the direction of the applied force ignoring possible cross stiffness effects.

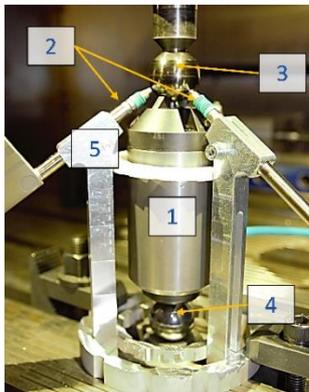


Figure 1. Experimental setup for the measurement and identification of the translational stiffness matrix.

This research work is a first step towards expanding the capabilities of the Loaded Double Ball Bar (LDBB) proposed by Archenti and Nicolescu [5] to identify the Cartesian translational stiffness matrix K_t .

2. Methodology

2.1 Experimental setup and procedure

The experimental setup is displayed in Figure 1. It consists of the LDBB (1), three wireless LVDT probes *TESA GTL21-W* (2) fixed on a mounting adaptor (5), a Tool Adaptor (TA) (3), which consists of a sphere of radius 15 mm attached to a shaft of diameter 25 mm, and the table adaptor (4) which is a steel sphere of radius 15 mm mounted on the table with a washer and a nut. For data capturing the wireless probes are connected to a *TESA TWIN-STATION* data acquisition system. The design of the mounting adaptor for the probes is inspired by Trapet et al. [6].

The setup in Figure 1 is used for the z-axis loading and measurement. For the x- and y-axis, the same probe configuration and placement is used (invariant probe mounting) but the LDBB is re-aligned along the x- and y-axis of the Machine Coordinate System (MCS) using a higher table adaptor with a similar sphere.

First the table adaptor is probed using the machine tool touch trigger probe to identify the center point in MCS. This way the alignment of the table and tool spheres will orient the force vector along the z-axis in MCS. The orientations of all force vectors are defined through the relative position of the table and tool spheres and identified using the touch trigger probe. Then the LVDTs' output is mapped to the MCS. The process consists of measuring controlled motions of the unloaded TCP (tool adapter) of 20 μm , 50 μm and 500 μm along x-, y- and z-axes of the MCS. The transformation matrix ($T \in R^{3 \times 3}$) that maps the LVDT's coordinate system into the MCS is calculated by solving the linear least-squares problem:

$$\left\| T \times \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{LVDT} - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{MCS} \right\|_2^2 \quad (1)$$

The uncertainty associated with this transformation has been assessed by repeatedly measuring controlled motions approaching the corner points of a cube with side length 40 μm , 100 μm , and 1000 μm .

Once the transformation matrix is identified, the LDBB is placed between the tool and table adaptor. A predefined set of loads of 150 N, 300 N, 450 N, 600 N, 750 N are applied incrementally. The TCP displacement is recorded by the LVDTs for each load level. The process is repeated three times for each axis of principal loading. The measurement procedure and the data analysis is visualised in Figure 2.

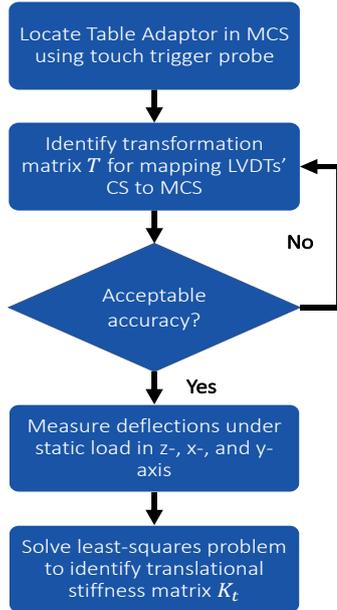


Figure 2. Flow chart of the measurement procedure and data processing

2.2 Static stiffness identification

Models of the static deflections of machine tools describe the spatial deflections (Δx) of the loaded TCP compared to the unloaded TCP due to the finite stiffness (K) of the structural members which are in the flow of forces of static loads (F). The stiffness of a mechanical system can be defined as its capacity to sustain loads, which result in a change of its geometry. This relationship is modelled using Hooke's law in tensor form:

$$F = K_t \times \Delta x \quad (2)$$

The translational stiffness matrix K_t is identified using the static loads f_x , f_y , and f_z of the force vector F from the LDBB and the translational deflections δx , δy , and δz of vector Δx , measured by the LVDT's using a linear least-squares solution according to :

$$\left\| \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \times \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} - \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} \right\|_2^2 \quad (3)$$

3. Results

A case-study has been executed on a Hermle C50U 5-axis milling machine. The deflections of the TCP as a result of a loading along the principal axis of the MCS can be seen in Figure 3. The shades on the plots highlight the combined expanded uncertainty of the measurands. The components of the uncertainty budget have been estimated by considering the following factors: Transformation of the measured deflections into the MCS, the eccentricity of the TA as well as of the LDBB,

the non-linearity of the LVDTs, the load variation of the LDBB, and the Type A uncertainty associated with the measurement. The identified translational stiffness matrix K_t equals:

$$K_t = \begin{bmatrix} 12.1 & 0.3 & 6.3 \\ -0.1 & 12.6 & -2.2 \\ 6.8 & 5.1 & 102.6 \end{bmatrix} N \mu\text{m}^{-1} \quad (4)$$

The magnitude of the components of the matrix K_t is in accordance with the design requirements of the type of machine tools. The x- and y-axis are similar in magnitude while the z-axis is significantly stiffer. However, in theory the terms k_{yx} and k_{xy} as well as k_{yz} and k_{zy} are considered to be equal in magnitude and sign, i.e. the stiffness matrix is symmetric. This is due to the fact that K_t in Equation (4) has been obtained according to an unconstrained linear least-squares formulation and the loading of the z-axis. Due to the comparably higher stiffness in the z-axis the uncertainty for the x and y component is significantly higher than the measurand, cf. Figure 3 bottom.

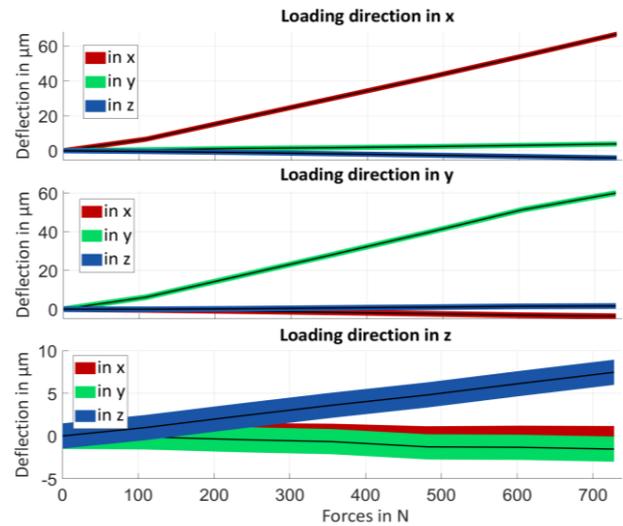


Figure 3: Measured translational deflections under load and their combined uncertainty.

4. Conclusion

In this paper a measurement and identification procedure was presented for measuring the full translational static stiffness matrix of a machine tool using the LDBB instrument. As can be observed from the measurement results in Figure 3, the procedure is able to identify translational stiffness values in an efficient way. The immediate next steps is reducing the uncertainties associated with the measurement system. Furthermore, the method should be improved in order to identify static stiffness for a subset of the workspace and include one rotary axis contribution in the measurement and identification procedure.

References

- [1] Neugebauer R, Scheffler C, Wabner M, Zwingerberger C., *IJMMS*, 2011,4(3/4),370-384. doi:10.1504/IJMMS.2011.041478
- [2] Szipka K, Laspas Tand Archenti A. *Precision Engineering*, 2018, **51**, 59-67, doi:10.1016/j.precisioneng.2017.07.011
- [3] Salgado M, López de Lacalle L.N., Lamikiz A., Muñoa J., Sánchez J.A. *IJMT*, 2005, **45**(6), 727-739, doi:10.1016/j.ijmachtools.2004.08.023
- [4] Brecher C., Weck M., *Werkzeugmaschinen Fertigungssysteme*. 2017. ISBN 978-3-662-46566-0.
- [5] Archenti A., Nicolescu M., *CIRP Annals*, 2013, **62**(1), 503-506, doi:10.1016/j.cirp.2013.03.100
- [6] Trapet E, Aguilar Martín J-J, Yagüe J-A, Spaan H., Zelený V. *Precision Engineering*, 2006, **30**(2), 165-179, doi:10.1016/j.precisioneng.2005.07.002