Effect of oil film thickness on motion errors for closed hydrostatic guideway with four pads

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Abstract
Hydrostatic guideways are widely applied as the vital functional unit in precision and ultra-precision machine tools owing to their low running friction, high stiffness, high motion accuracy, etc. Since the motion errors of hydrostatic guideways directly affect the accuracy of the machined parts, there have been many researches concerning the motion accuracy of hydrostatic guideways. However, it seems that the influence of oil film thickness (OFT) on motion errors in closed hydrostatic guideways has not been thoroughly studied yet. Different from those previous approaches, the quasi-static analysis model with less simplification is directly developed by incorporating the concept of pose in this paper, which is then employed to explore the effect of OFT on motion accuracy in the field of hydrostatic guideways. The typical closed hydrostatic guideway with four pads is still adopted as the sample here. The numerical results demonstrate that, OFT hardly affects the motion errors if the external force acting on the hydrostatic table is not considered, whereas the effect does occur if the external force is taken into account. Further, the mechanism of motion errors variation is attributed to the fluctuation of the difference between the average film thicknesses of the two adjacent pads, the greater the fluctuation, the larger the motion errors.

Oil film thickness; Motion error; Hydrostatic guideway; Guide rail; Pose; Oil pad

1. Introduction
Hydrostatic guideways have been widely applied as the vital functional unit in precision and ultra-precision machine tools for their low running friction, high stiffness, high motion accuracy, superior vibration-resistance, virtually no wear, long service life, etc [1-5]. Due to that the motion errors of hydrostatic guideways affect the accuracy of the machined parts directly, quite a few researches related to the motion accuracy of hydrostatic guideways were carried out. However, it seems that the influence of oil film thickness (OFT) on motion errors in closed hydrostatic guideways has not been thoroughly studied yet. In this paper, the typical closed hydrostatic guideway with four pads is taken as the sample, which also is commonly used in other researches [5-7]. Besides, different from the previous approaches including the transfer function method (TFM) [8], the simplification of oil film as the linear spring element [5, 9] and finite element analysis (FEA) [10], the quasi-static analysis model with less simplification is directly developed by incorporating the concept of pose, which is then employed to explore the effect of OFT on motion accuracy in the field of hydrostatic guideways. Our research is expected to be inspiring to the designers, and some theoretical references for precision design of hydrostatic guideways can be drawn.

2. Quasi-static analysis model
Figure 1 shows the typical closed hydrostatic guideway with four pads. The OFT proposed in our study does refer to the sum of $h_{01}$ and $h_{02}$, as seen in Figure 1 (a), it also can be considered as the designed total oil film clearance. Additionally, [A] is introduced as the reference coordinate system, and the distance between ideal rail 1 (2) and the plane $o_{3x_3}y_3a_3$ is $h_{0}$/2. Then, [8] is determined as the hydrostatic table coordinate system, as depicted in Figure 1 (b). Figure 1 (c) exhibits the structure of every rectangular pad. As observed from Figure 1 (a), the linear motion error $z_k$ and angular motion error $\phi$ do exist because of the real guide rails 1 and 2 with profile errors. They can be expressed as [5-6, 9]

$$Z_1 = E_1 \sin \left( \frac{2\pi}{L_1} \left( X - \frac{L_1}{2} \right) + \phi_1 \right) - \frac{h_0}{2}$$

(1)

$$Z_2 = E_2 \sin \left( \frac{2\pi}{L_2} \left( X - \frac{L_2}{2} \right) + \phi_2 \right) + \frac{h_0}{2}$$

(2)

Figure 1. Typical closed hydrostatic guideway with four pads (a) pose of hydrostatic table, (b) structure of hydrostatic table, and (c) rectangular pad
Where $E_i$, $E_o$, $A_i$, $A_o$, $\varphi_i$, $\varphi_o$ denote the profile error amplitude, wavelength and phase of guide rails 1 and 2, respectively. The quasi-static analysis model then is developed based on the static equilibrium. Both the resultant force and resultant moment of the hydrostatic table must equal to zero wherever it moves [4-5, 9]. As shown in Figure 2, the force and the moment equilibrium equations are expressed as

$$F_i + F_{z(u)} + F_{z(v)} = F_{z(i)} + F_{z(s)}$$

(3)

$$M_{o(b)} = F_{o} \cdot e + F_{v} \cdot \frac{l}{2} + F_{s} \cdot \frac{i}{2} = F_{i} \cdot \frac{i}{2} + F_{s} \cdot \frac{i}{2}$$

(4)

\[\begin{align*}
F_2 & = F_{o} \cdot e + F_{z(u)} + F_{z(v)} \\
F_3 & = F_{o} \cdot e + F_{z(u)} + F_{z(v)} + F_{z(s)} \\
F_4 & = F_{o} \cdot e + F_{z(v)} + F_{z(s)}
\end{align*}\]

Figure 2. Static equilibrium of hydrostatic table with four pads

By solving the Eqs. (3) and (4), the two motion errors can be determined. Hence, the oil film reaction force $F_i (i = 1-4)$ needs to be obtained first, the expression of $F_i$ is given as [11]

$$F_i (i = 1 - 4) = P_o \cdot A_i$$

(5)

Here, $P_o$ is the hydrostatic pressure in the recess. $A_i$ is the effective bearing area of the rectangle pad, it can be written as

$$A_i (i = 1 - 4) = (W - w) \cdot (M - m)$$

(6)

Note that, the progressive mengen (PM) flow controllers are adopted in the closed hydrostatic guideway shown in Figure 1. PM controller is the product of Hyprostatik Company in Germany and belongs to the membrane-type restrictor, which has been widely used as the key component in other precision machine tools [12-16]. Its structure and working principle can be easily found in [12, 15]. So $P_o$ can be calculated by

$$P_o = \frac{Q_0}{(i = 1-4)} \cdot \frac{R(h_o)}{1 - \frac{Q_0}{(i = 1-4)} \cdot R(h_o)}$$

(7)

where $Q_0$ is the initial flow rate for $P_o = 0$, $K_i$ is named as flow ratio, $P_o$ is the supply pressure. They are regarded as the three key characteristic parameters of PM controller. Besides, $R(h_o)$ is the flow resistance of the land and related to the average oil film thickness $h_o$ [4, 9]. $R(h_o)$ can be obtained as

$$R(h_o) = \frac{3\eta}{h_o^3} \left( \frac{M - m}{4w} + \frac{W - w}{4m} \right)$$

(8)

Lastly, the calculation of $h_o$ is essential. As seen in Figure 1 (a), CASE 2 is chosen to determine $h_o$ and it is given as

$$h_{o1} = \frac{1}{A_{o1}} \left[ \int F_{p,0}^x dY \int F_{p,0}^y dX (Z_{o1} - Z) dX \right]$$

(9)

$$h_{o2} = \frac{1}{A_{o2}} \left[ \int F_{p,1}^x dY \int F_{p,1}^y dX (Z_{o2} - Z) dX \right]$$

(10)

where $i$ denotes the number of pad, and $j$ denotes the number of point in one pad plane. Based on the example depicted in Figure 1, $i$ ranges from 1 to 4 and $j$ from 0 to 8. The descriptions of point $q$ on the hydrostatic table in [A] and [B] are different and denoted by $q_i$ and $q_j$, respectively. $q_{i1}$ and $q_{i2}$ are the coordinates of the $i$th point of the $j$th pad plane in [A] and [B], respectively. $A_{i1}$ denotes the projected area of land in the plane $o_{1i}A_{i1}$, it is

$$A_{i1} = 2w \left( y_{i2} - y_{i1} \right) + 2(W - 2w) \left( y_{i2} - y_{i1} \right)$$

(11)

Besides, pad 1 and pad 3 are in the same plane, and the corresponding mathematical equation can be denoted by $Z_{o1}$, so is $Z_{o2}$. Therefore, $Z_{o1}$ and $Z_{o2}$ can be expressed as

$$Z_{o1} = \frac{1}{2} \lambda \cdot X + \frac{1}{2} \lambda \cdot Y + \frac{1}{2} \lambda \cdot Y + \frac{1}{2} \lambda \cdot Y$$

(12)

$$Z_{o2} = \frac{1}{2} \lambda \cdot X + \frac{1}{2} \lambda \cdot X + \frac{1}{2} \lambda \cdot X + \frac{1}{2} \lambda \cdot X$$

(13)

The relationship between the four oil film reaction forces and the two motion errors can be summarized as follow: Oil film reaction force $F_i \rightarrow$ Hydrostatic pressure in the recess $P_o \rightarrow$ Flow resistance of the land $R(h_o) \rightarrow$ Average oil film thickness $h_o \rightarrow$ Motion position of the hydrostatic table and the motion errors ($x_o, z_o, \theta$). Up to now, the establishment of the quasi-static analysis model has been completed.

3. Results and discussion

Three case studies are selected to investigate the effect of OFT on motion errors for closed hydrostatic guideway with four pads. The simulation condition has been listed in Table 1.

<table>
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<tr>
<th>Parameter</th>
<th>Hydrostatic table [6]</th>
<th>Rail 1</th>
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<th>Parameter</th>
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<td>$Q_o$</td>
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<table>
<thead>
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<th>Parameter</th>
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<th>Rail 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_2$</td>
<td>$\lambda_2$</td>
<td>$\varphi_2$</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Hydrostatic oil dynamic viscosity [6]</th>
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<td>Value</td>
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3.1. Case study one

Case study one means that both the external force $F_0$ and its arm $e$ shown in Figure 2 are set as zero. As can be observed in Figure 3 (a) and (b), the influence of OFT on the two motion errors are not significant at all, and the PV values of the two motion errors are also unchanged with the variation of OFT, as depicted in Figure 3 (c). However, it can be seen that the angular
and linear motion errors fluctuate periodically with the moving of hydrostatic table along the positive direction of $x_A$ axis. Here, the horizontal axis named travel appears in Figures 3, 4 and 5, and it is just the $x_A$ axis seen in Figure 1.

3.2. Case study two
Case study two refers to that the external force $F_0$ is set as 1500 N, but its arm $e$ is still zero. Figure 4 shows the influence of OFT on motion accuracy in case study two. As seen in Figure 4 (a), with the increase of OFT, the peaks of the angular motion error curve move along the negative direction of $x_A$ axis, but the valleys move inversely. In Figure 4 (b), the peaks and valleys of these linear motion error curves are correspondent to each other, whereas the curve moves down with the growth of OFT, in other words, the hydrostatic table shown in Figure 1 (a) gets closer to the guide rail 1 along the positive direction of $F_0$. The two motion errors still fluctuate periodically. Additionally, the PV values of the two motion errors do rise with the increase of OFT, as depicted in Figure 4 (c), and they two change similarly.

3.3. Case study three
Case study three implies 1500 N of $F_0$ and 12.10 mm (0.2*$l$) of $e$. The two motion errors change periodically with the moving of hydrostatic table, as shown in Figure 5 (a) and (b). Moreover, with the increase of OFT, the peaks of the angular motion error curve move along the negative direction of $x_A$ axis, but the valleys move inversely, which is similar to the phenomenon exhibited in Figure 4 (a), but the difference also is noticeable, the angular motion error curve moves up. The information presented from Figure 5 (b) and (c) is similar to that from Figure 4 (b) and (c). Accordingly, the arm $e$ can affect the variation of the angular motion error significantly by comparing to case study two.
larger the motion errors. Just as shown in Figure 6 (a), no matter how the OFT changes, $p$ and its PV value are unaffected, which can interpret the phenomena shown in Figure 3. Does the explanation still work to the case studies two and three? As depicted in Figure 6 (b) and (c), the PV value of $p$ rises with the increase of OFT, which corresponds to the results shown in Figure 4 (c) and Figure 5 (c), respectively. So the variation of the motion errors can almost be figured out now. The research work [16] also is suggested to be referred due to the similarity with this study, but the themes are different with each other.

4. Conclusions

The influence of oil film thickness on motion errors for closed hydrostatic guideway with four pads does exist, but the effect can be ignored if the external force is not taken in account. Besides, the parameter $p = (h_{32} - h_{44})$ is found to interpret the mechanisms of the motion errors.

References