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Reduction of noise bias in 2.5D surface measurements

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Abstract
Surface roughness amplitude parameters tend to be biased towards positive deviations due to the presence of noise in the measurement. This is due to the nature of the roughness parameters themselves, e.g. *Sq* is essentially the standard deviation of the measurement. Generally this effect is counteracted by averaging the measurement before calculating the roughness parameter. This increases the measurement time, requiring quite a lot of measurements to sufficiently reduce the noise. This paper extends an earlier developed method for noise bias reduction of profile measurements to 2.5D measurements. The noise bias effect is reduced by calculating the limit value of the *Sq* parameter. A noise reduced topography is reconstructed by considering the decomposition of the measurement into Legendre polynomials, using the unbiased estimation of *Sq* as a limiting value for the *Sq* of the surface. Due to the orthogonality of the Legendre polynomials, the Legendre coefficients are reduced in a similar way as the *Sq* parameter. This method is applied to both simulated and experimental data. The experimental data are obtained by 2.5D areal measurements using focus variation microscope. The method proved to be well suited for noise reduction of flat surfaces, e.g. for measuring flatness deviation, but could have its value for other applications with relatively smooth shapes.

Keywords: Dimensional Metrology, Noise, Roughness, Microscopy, Focus Variation

1. Introduction

In surface measurements the reported surface parameters are essentially deviations of the measured surface. The presence of noise in these measurements causes a systematic error where the reported parameter is positively biased. This paper focusses mainly on the RMS roughness parameter, *Sq,* as defined in ISO 25178 [1], which is the standard deviation of the measurement (1).

$$Sq = \sqrt{\frac{1}{A}\iint_A z^2(x,y)\,dx\,dy} \qquad (1)$$

Generally the influence of noise in the measurement is reduced by averaging multiple measurements. Arithmetic averaging does not completely remove the bias however. Haitjema [2] proposed a method for estimating an unbiased roughness and roundness parameter for 1.5D profile measurements which is based on a paper by Wyant and Creath [3]. The reduction of the bias is also proposed by Davies and Levinson [4] for RMS flatness.

Since noise added to the measurement is in theory independent of the measurement itself, the *Sq* parameter contains contributions of both the measured object and the noise:

$$Sq^2_{measurement} = Sq^2_{object} + Sq^2_{noise} \qquad (2)$$

By considering the mean of a number of measurements *N*, the noise component is reduced and the mean is given by:

$$Sq^2_{mean} = Sq^2_{object} + \frac{Sq^2_{noise}}{N} \qquad (3)$$

As the number of measurements increases, the influence of the noise diminishes. Note that for an increasing amount of measurements, Sq^2_{mean} approaches Sq^2_{object}.

Combining (2) and (3) gives an unbiased estimate for the *Sq* parameter of the object.

$$Sq^2_{object} = \frac{N * Sq^2_{mean} - \overline{Sq^2_{measurements}}}{N-1} \qquad (4)$$

Where $\overline{Sq^2_{measurements}}$ is the mean of the Sq^2 of the individual measurements. Similarly, the *Sq* of the noise can be calculated by eliminating Sq_{object} from (2) and (3):

$$Sq^2_{noise} = \frac{\overline{Sq^2_{measurements}} - Sq^2_{mean}}{1 - N^{-1}} \qquad (5)$$

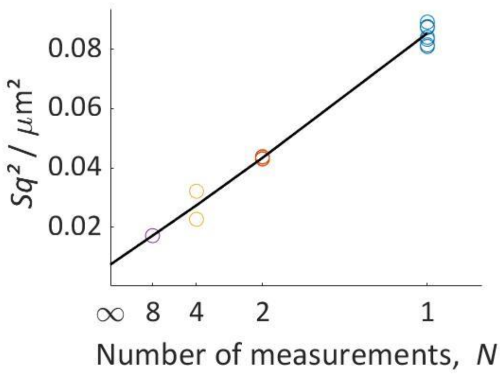

***Figure 1**. Sq² of a roughened optical flat as a function of the number of measurements taken for arithmetic averaging. The extrapolated value corresponding to an infinite amount of measurements is the unbiased estimate as obtained by (4).*

Using a Fourier decomposition, Haitjema [2] reconstructs a bias reduced topography with *Sq* equal to the value obtained by (4). This method used on 2 profile measurements is reported to obtain an even better results than arithmetic averaging of 16 measurements.

In this paper the Fourier decomposition method is extended to 2.5D areal surface measurements. However since elementary forms such as sphericity and torque and edge effects in surface measurements challenge the independence of Fourier terms, the surface is decomposed in Legendre polynomials.

Legendre polynomials $P(x)$ are solutions to the Legendre differential equation, which are normalised so that $P(1) = 1$. Any 2D function can be written as an infinite series of Legendre polynomials, also called a generalised Fourier series:

$$f(x,y) = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty} a_{km} P_{km}(x,y) \tag{6}$$

Where P_{km} are the two dimensional Legendre polynomials and a_{km} are the corresponding Legendre coefficients.

Legendre polynomials are defined as an orthogonal system, with the corresponding orthogonality relation:

$$\int_{-1}^{1}\int_{-1}^{1} P_{kl}(x,y)\, P_{nm}(x,y) dxdy = \frac{\delta_{nm}}{m+0.5}\frac{\delta_{kl}}{k+0.5} \tag{7}$$

Where δ_{nm} is the Kronecker delta symbol.

The orthogonality of Legendre polynomials is important, because due to the orthogonality the Legendre coefficients are linear independent. Therefore $S_q^2 \sim a_{nm}^2$:

$$S_q^2 = \frac{1}{4}\int_{-1}^{1}\int_{-1}^{1} f^2(x,y)\, dx\, dy = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty} \frac{a_{km}^2}{\left(k+\frac{1}{2}\right)\left(m+\frac{1}{2}\right)}$$

This allows the coefficients to be reduced similar to (3):

$$a_{nm,object}^2 = \frac{N * a_{nm,mean}^2 - \overline{a_{nm,measurements}^2}}{N-1} \tag{9}$$

Where N is the number of measurements. The surface is reconstructed based on the reduced Legendre coefficients given by (9).

This paper aims to compare arithmetic averaging (3), to the method which uses a Legendre polynomial decomposition (9). In the next section, the methodology for the noise bias reduction method and the experiments performed are discussed in more detail. In section 3, both methods are compared on simulated data. Finally, the algorithms are applied to real measurement data.

2. Methodology

Noise reduction algorithms are implemented in Matlab®, and are applied on both simulated and experimental data.

For arithmetic averaging, the point clouds of *N* measurements are first levelled subtracting the least-squares plane (this effectively zeros the a_{00}, a_{10} and a_{01} components), after which the point clouds are averaged pointwise. The rms roughness parameter *Sq* is calculated according to (1).

The *Sq* of the average profile, Sq_{mean}^2, is also used in the averaging noise bias reduction method, along with the *Sq* parameters of each of the individual measurements. All measurements are first levelled by subtracting the least-squares plane, then the *Sq* is calculated according to (1), and their average is calculated to obtain $\overline{Sq_{measurements}^2}$. Now, the unbiased estimation is calculated using (4).

The Legendre method first approximates the averaged profile, calculating the coefficients of the mean profile, $a_{km,mean}^2$. Then, similar to the averaging method, the decomposition of each individual measurement is calculated. The corresponding coefficients are averaged to obtain $\overline{a_{km,measurements}^2}$. Now, the noise reduced components can be calculated using (9). However during this calculation the sign of the coefficients is lost and should be restored, which is done based on the signs of $a_{km,mean}$:

$$a_{km,object} = sign(a_{km,mean})\sqrt{a_{km,object}^2} \tag{10}$$

The profile is then reconstructed using (6). It is important to note that due to numerical instability of higher-order polynomials, as well as calculation time constraints, only a limited order of polynomials can be calculated. As a result, high frequency components (*K*,*M* > 20) of the measurement cannot be approximated by the Legendre method. A residual component, $g(x,y)$, remains:

$$f(x,y) = \sum_{k=0}^{K}\sum_{m=0}^{M} a_{km} P_{km}(x,y) + g(x,y) \tag{11}$$

Where *K* = *M* is the highest order polynomial which is still stable.

This residual component of the surface is again reduced in a similar way as the averaging noise reduction method. Introducing this reduction in (11), the profile is approximated as:

$$f(x,y) = \sum_{k=0}^{K}\sum_{m=0}^{M} a_{km} P_{km}(x,y) + \sqrt{\frac{N * Sq^2(g_{mean}) - Sq^2(\overline{g_{measurement}})}{(N-1)\, Sq^2(g_{mean})}}\, g(x,y) \tag{12}$$

Where *g(x,y)* is scaled such that (4) holds. Finally, from this noise reduced profile, *Sq* is calculated according to (1).

2.1 Simulated data

The function $z = (3x^2 - 5x^5 + 4\sin x + 5\cos 7x)(-2y^2 - 3y^4 + 4\sin 2y - 2\cos 5y)$ is evaluated on a 200 x 200 pixel grid. The *Sq* of this surface is about 15.34 µm. Gaussian random noise is added to the surface with an *Sq* of about 19.43 µm to obtain simulated measurements.

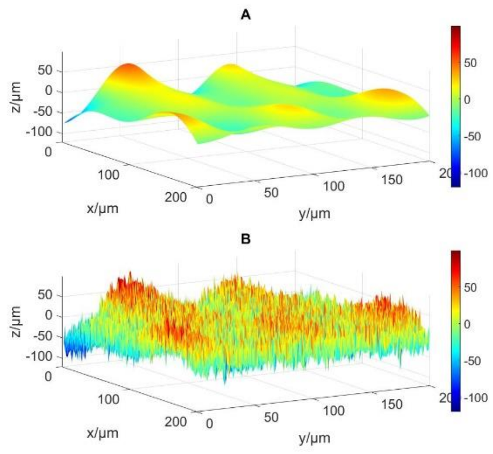

Figure 2*. a) Simulated topography, Sq = 15.34 µm. b) Simulated topography with added noise, Sq = 24.76 µm.*

2.2 Experimental data

In order to evaluate the noise reduction algorithms on real measurements, both a roughened optical flat and a realistic roughness sample are measured using a Sensofar Neox® in focus variation mode. Because the focus variation method needs some roughness in order to operate, necessarily a somewhat roughened flat surface must be used to evaluate the instruments flatness deviation. To average out the remaining roughness the roughened optical flat sample was measured at 8 random positions on the surface. The roughness sample is measured 8 times at the same location. The specification of the objective and measurement conditions are shown in Table 1.

For the optical flat measurements, a Mitutoyo type 158-118 roughened optical flat with a reported flatness deviation of < 0.1 µm was used. The realistic roughness sample used is a Rubert Microsurf 315 with a reported *Ra* of 0.4 µm when filtered using a Gaussian filter with λ_s= 2.5 µm and λ_c=0.25 mm.

Table 1. Optical parameters for the microscope objective

Optical parameter	Value
Magnification	20x
NA	0.45
FoV / µm	877 x 660
Pixel size / µm	0.69 x 0.69

3. Results and discussion

Figure 3 shows the topographies obtained by the noise reduction methods on simulated data. From Fig. 3 (a) and Fig. 3 (b) it is obvious that the noise added to the measurement is reduced when arithmetically averaging 3 and 8 simulated topographies respectively. However when compared to Fig. 2 (a) it is obvious that some of the noise remains even after averaging 8 topographies. Fig. 3 (c) shows the noise reduced topography obtained by the Legendre decomposition method, which is very similar to the original simulated profile. This shows that Legendre polynomials can be used to reconstruct a noise bias reduced topography.

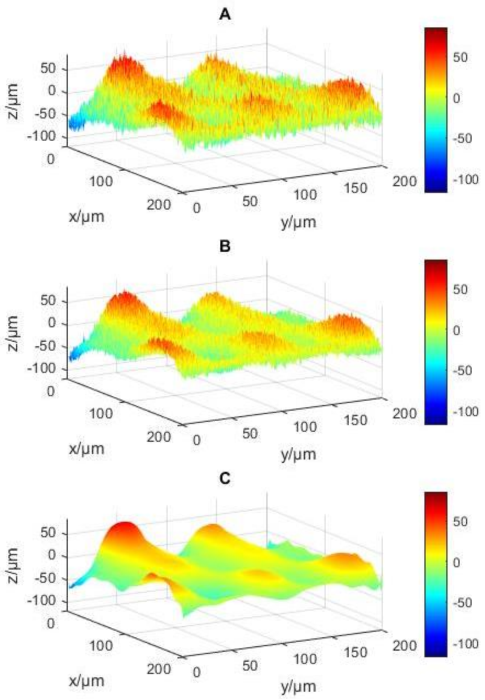

Figure 3*. Noise reduction algorithms applied to simulated topographies. a) Arithmetic averaging of 3 topographies as per ISO standard requirements [5]. b) Arithmetic averaging of 8 topographies. c) Legendre noise bias reduction of 3 topographies.*

Table 2 shows the *Sq* values corresponding to the noise reduction methods applied on simulated topographies. When comparing the *Sq* parameter of the arithmetic averaging method, it is clear there are diminishing returns for increasing the number of measurements. When calculating the unbiased estimate using (4), it is clear that increasing the number of topographies does not influence the unbiased estimate of *Sq. However it can he expected that the actual topography is approximeted better with multiple measurement*. From these simulations it is also clear that the *Sq* obtained by arithmetic averaging follow the expected trend similar to Figure 1, but in order to approach the unbiased estimate, a lot of measurements are required.

Table 2. *Sq* parameter of simulated topographies

Noise reduction method	*Sq*/µm
Arithmetic averaging (3x)	19.03
Arithmetic averaging (8x)	16.77
Arithmetic averaging (16x)	16.10
Legendre noise reduction (3x)	15.33
Legendre noise reduction (8x)	15.29
Original topography	15.34

Noise reduction algorithms find their practical use when assessing metrological characteristics such as flatness deviation, a systematic error introduced due to instrument limitations. The flatness deviation can be assessed by measuring an optical flat in at least 3 different locations on its surface [5]. When calculating the average topography of these measurements, the error introduced by the optical flat is reduced, better estimating the actual flatness deviation. When considering Legendre Polynomials it must be mentioned that the cylindricity components a_{20} and a_{02} and the torque a_{11} are not reduced by shifting the optical flat however the effect of the torque of the optical flat can be reduced by rotating the flat as well.

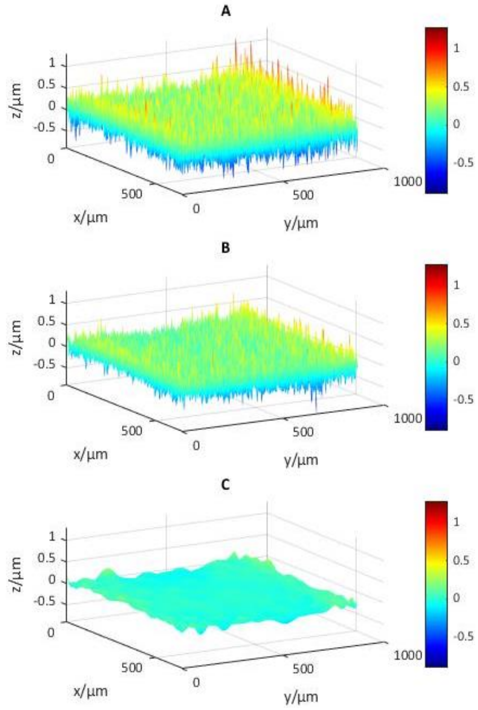

Figure 4*. Roughened Mitutoyo optical flat measured at 8 random locations. a) Arithmetic averaging of 3 measurements as per ISO standard requirements [5]. b) Arithmetic averaging of 8 measurements. c) Legendre noise reduction on 3 measurements.*

Figure 4 shows the noise reduced topographies obtained by such a measurement. Fig. 4 (a) is the topography obtained by arithmetic averaging of 3 measurements, which is the minimum as mentioned in ISO ISO/CD 25178-700:2019 [5]. Just like in the simulations, the noise influence of the optical flat decreases when more measurements are averaged.

However since the instrument flatness deviation, which is related to the curvature of the focal plane, has a smooth shape, Legendre polynomials are well suited for approximating this flatness deviation. Fig. 4 (c) shows the corresponding noise reduced topography, which is in accordance with the expected smooth shape.

Since the expected instrument flatness deviation has an expected smooth shape, 'noise' from both the instrument and the residual surface roughness is expected to have a large influence on the measured *Sq*. This is apparent when comparing the arithmetic averaging of 3 and 8 measurements with the noise reduction methods. The noise reduction compared to a single measurement can also be calculated using (5). The *Sq* of the noise is 0.28 µm, which is high when compared to the measured *Sq*. This method may reduce the need of taking many data, e.g. the 232 and even 2035 measurements reported in the literature [6].

Table 3. *Sq* parameter of roughened Mitutoyo optical flat measured at 8 different locations on the surface.

Noise reduction method	*Sq*/µm
Arithmetic averaging (3x)	0.173
Arithmetic averaging (8x)	0.129
Legendre noise reduction (3x)	0.039
Legendre noise reduction (8x)	0.037
Noise	0.282

When applying the noise bias reduction method to an actual realistic roughness sample, it appears that there is limited improvement due to the noise reduction algorithm. However when Sq_{noise} is considered, it is apparent that the noise already is quite low, so only minor improvements are expected. The lower noise Sq_{noise} of the roughness sample compared to the roughened optical flat is most likely due to the lower reflectivity of the roughnened optical flat and the randomization of its roughness

Note that the measurements are not filtered. Considering the limited presence of noise in the realistic measurements, the Legendre method still performs quite well in reconstructing the surface.

4. Conclusion

In this paper, an earlier developed method for noise reduction and reconstruction of a noise bias reduced topography is extended from 1.5D profile measurements to 2.5D surface measurements. In order to avoid challenges with the original Fourier decomposition method due to sphericity, cylindricity and torque components in 2.5D measurements, a Legendre polynomial approach was used. This noise reduction approach is compared to the commonly used method for noise reduction, where multiple measurements are arithmetically averaged in order to reduce the influence of noise.

The methodology was developed and tested on a relatively smooth topography, to which Gaussian distributed noise is added artificially to simulate a realistic measurement. The results obtained by arithmetic averaging follow the expected trend. The unbiased estimate is well obtained using only 3 simulations, and the original surface is reconstructed quite well using Legendre polynomials.

In order to test the methodology on realistic measurements, two use cases were examined. First, the method is applied to flatness deviation measurements, where an optical flat is measured on 8 random locations on its surface. The Legendre method proves very effective here in reconstructing a noise reduced topography, even though the measurement noise was quite high in these measurements, due to the low reflectivity of the measured optical flat and the randomization of the surface roughness. Secondly, a roughness standard was measured. Since the noise was already quite limited in this case, the reduction was not as apparent as in the previously mentioned example.

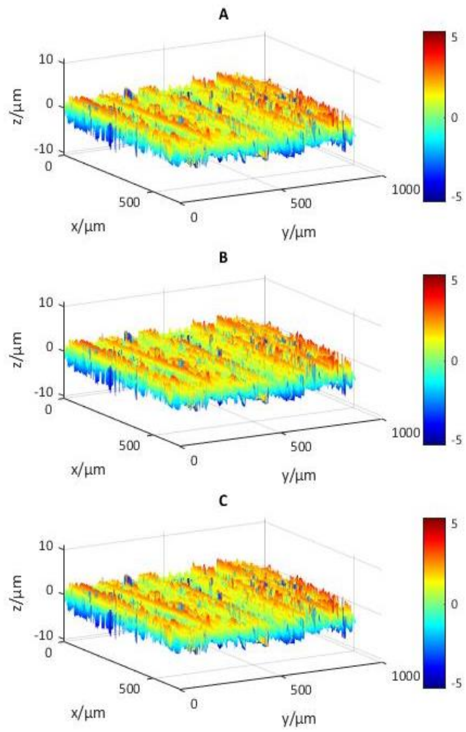

Figure 5*. Roughness sample measurements. a) Arithmetic averaging of 3 measurements. b) Arithmetic averaging of 8 measurements. c) Legendre noise reduction on 3 measurements.*

Table 4. *Sq* parameter of Rubert roughness sample

Noise reduction method	*Sq*/µm
Arithmetic averaging (3x)	1.526
Arithmetic averaging (8x)	1.529
Legendre noise reduction (3x)	1.517
Legendre noise reduction (8x)	1.515
Noise	0.139

However, the algorithm was still able to reconstruct the surface quite well.

The methodology proved to be most useful in situations where the measured topography is quite smooth without steep transitions.

Due to the nature of Legendre polynomials, it is well suited for measuring many kinds of form measurements, of which an example was given for flatness deviation.

References

[1] ISO 25178-2:2012 Geometrical product specification (GPS) – surface texture: Areal – Part 2 : Terms, definitions and surface texture parameters. *ISO*, Geneva

[2] Haitjema, H., & Morel, M. A. A. 2005. Noise bias removal in profile measurements. *Measurement*, **38**(1) 21-29.

[3] Creath, K., & Wyant, J. C. 1990. Absolute measurement of surface roughness. *Applied optics*, **29**(26) 3823-3827.

[4] Davies, A., Levenson, M.S., Estimating the root mean square of a wave front and its uncertainty (2001) *Applied Optics*, **40** (34) 6203-6209.

[5] ISO/CD 25178-700:2019 Geometrical product specification (GPS) – surface texture: Areal – Part 700: Calibration, adjustment and verification of Areal topography measuring instruments. *ISO*, Geneva

[6] Giusca, C.L., Claverley, J.D., Sun, W., Leach, R.K., Helmli, F., Chavigner, M.P.J. 2014 Practical estimation of measurement noise and flatness deviation on focus variation microscopes *CIRP Annals - Manufacturing Technology*, **63** (1), pp. 545-548.