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Foucault pendulum properties of spherical oscillators

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Abstract

In 1851 Léon Foucault created a sensation with his pendulum providing a direct demonstration of the turning of the Earth. This simple device consists of a pendulum which is launched in a purely planar orbit. Following Mach's principle of inertia, the mass will continue to oscillate in the same planar orbit with respect to absolute space. For an observer on Earth, however, the plane of oscillation will turn. Conceptually speaking, Foucault constructed a very precise demonstrator showing that, when put on a rotating table, planar oscillations of an isotropic two degree of freedom oscillator remain planar with respect to an inertial frame of reference.
These oscillators have currently been under study in order to construct new horological time bases. A novel concept was a spherical isotropic two degree of freedom oscillator. Theoretical computations indicate that when put on a rotating table, planar oscillations of the spherical oscillator neither remain planar in the inertial frame nor in the rotating frame of reference, but in a frame of reference rotating at exactly half the rotational speed of the rotating table.
This intriguing result led to the design, construction and experimental validation of a proof of concept demonstrator placed on a motorized rotating table. The demonstrator consists of a spherical isotropic oscillator, a launcher to place the oscillator on planar orbits, a motorized rotating table and a measurement setup. The experimental data recorded by the lasers validates the physical phenomenon.

Foucault pendulum, two degree of freedom oscillator, spherical oscillator, mechanics

1. Introduction

In previous work [1,2], it was shown that that two degree of freedom (2-DOF) harmonic oscillators can be used advantageously as time bases for mechanical timekeepers since they bypass escapement mechanisms. More specifically, attention was brought to spherical oscillators which are relatively insensitive to the effect of tilting the mechanism in the presence of gravity [2]. By spherical oscillator, we mean a spherical mass having purely rotational kinematics and subject to restoring torque.
These 2-DOF oscillators behave to some extent like a spherical pendulum, the Foucault pendulum being a physical constrution of the latter. This paper shows that spherical oscillators are also subject to a precession when undergoing planar oscillations, but this precession is only at half the rate of the standard Foucault pendulum. This paper first introduces the required theoretical background including the description of spherical oscillators. We then present the design and construction of a proof-of-concept demonstrator (Fig. 1) intended to make this Foucault effect visible to the naked eye. As the Foucault effect induced by the Earth's rotation is extremely small (one revolution per 32 hours at our 47 degree latitude), the prototype was placed on a motorized rotation stage to increase the effect. The experimental method will be described and the results of quantitative measurements will be given and discussed.

2. Theoretical Background

2.1. The Foucault pendulum

The Foucault pendulum is a physics experiment that enables the rotation of the Earth to be visible to the naked eye and was first imagined and constructed in 1851 by the famous French physicist Léon Foucault. This simple device consists of a spherical pendulum launched in a straight line towards the center of attraction, which defines a plane of oscillation. Following the principle of inertia, the mass will try to follow a straight line with respect to absolute space. Due to the Earth's rotation, for a terrestrial observer, the plane of oscillations will seem to turn. See [3] for a historical summary.

2.2. Spherical oscillators

In his *Principia Mathematica*, Isaac Newton showed that if gravitation's restoring force were to be linear, then the orbits of a planet around the sun would be an ellipse with the sun in the center.

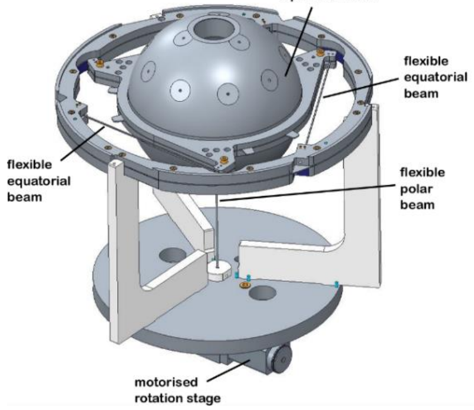

Figure 1: General CAD view of the demonstrator prototype

Moreover, Newton showed that all orbits of a planet would have the same period, which is the basic requirement for an oscillator to be a precise time base for a timekeeper.

In order to have portable timekeepers, the time base must be insensitive to gravity and in [2] we considered spherical oscillators, since they can be rotated without displacing their center of gravity.

Since a sphere has 3 degrees of freedom and Newton's result applies only to 2-DOF oscillators, we must restrict a degree of freedom. This is achieved by having the sphere follow *Listing's Law* known to described human eye movement, see [2].

Listing's law:

There is a direction called the *primary position* so that any admissible position is obtained from this position by a rotation whose axis is perpendicular to the direction of the primary position.

Mechanical Realisation

A flexure mechanism following Listing's law can be obtained with four flexible beams as shown in Fig 1. The three equatorial beams prevent rotation around polar axis and the polar beam acts as a bearing to counteract gravity.

2.3. Foucault pendulum properties of spherical oscillators

We give an overview of the demonstration of the Foucault effect for spherical oscillators. The method is standard and calculates the oscillator's Lagrangian to derive its Euler-Lagrange equations. Computing the kinetic energy requires the calculation of the infinitesimal angular velocity ω.

In previous work [2], an expression for ω in the case of the spherical oscillator in an inertial frame was derived:

$$\omega = \dot{\varphi}(\boldsymbol{i} - \boldsymbol{v}) + \dot{\theta}\boldsymbol{n},$$

where $\boldsymbol{i}, \boldsymbol{v}, \boldsymbol{n}$ are the Euler angles rotation axes as shown in [2, Fig. 5]. As we are considering the case of a spherical oscillator whose primary position $\boldsymbol{i}$ is vertical and is placed in a frame rotating around this axis at rate ζ, the angular velocity ω has an additional term along $\boldsymbol{i}$:

$$\omega = \dot{\varphi}(\boldsymbol{i} - \boldsymbol{v}) + \dot{\theta}\boldsymbol{n} + \zeta\boldsymbol{i}$$

From this, one calculates the kinetic energy

$$K = \frac{I}{2}\left[\dot{\theta}^2 + 4\dot{\varphi}^2(\sin(\theta/2))^2 + 4\,\zeta\,\dot{\varphi}(\sin(\theta/2))^2 + \zeta^2\right]$$

and the Lagrangian is $L = K - V$, where V is a central isotropic potential. The Euler-Lagrange equation in φ simplifies to

$$(2\dot{\varphi} + \zeta)(\sin(\theta/2))^2 = \mathrm{C}$$

where C is a constant. The key observation is that this equation is satisfied when $\dot{\varphi} = -\zeta/2$. This states that planar oscillations precess at a rate one half of the constant sphere rotation. Note that this holds for any central isotropic restoring force.

3. Demonstrator design, construction and measurement

The designed demonstrator is based on the previous clock sphere time base of [2]. Its specifications are an inertia of $2{*}10^{-2}$[kg m^2] and a frequency of 1.75 [Hz]

To verify the ratio of ½ between the precession of the plane of oscillations and the rate of the rotating frame, the oscillator was set on a rotating table having a variety of possible speeds and launched on planar orbits. Following our theoretical analysis, when plotting the shift of the plane of oscillation described by φ (seen from the rotating frame) with respect to the shift undergone by the rotating frame, the angle φ should have a linear behaviour with slope ½. To represent the position of the sphere, the principal position corresponding to the north pole is projected onto the equatorial plane.

Due to the inevitable isotropy defect, the orbits will degenerate and no longer be planar. The angle φ was therefore measured at the orbits' apogees and only the first six periods were considered as they were deemed sufficiently linear. Fig.2a shows the orbits considered and Fig.2b shows the angle φ with respect to the shift of the table along with the slope of the linear regression.

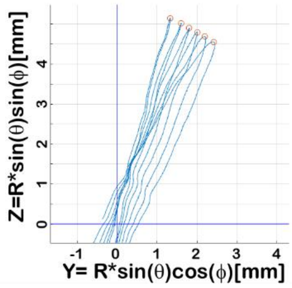

Figure 2a: Enlarged view on the first six periods with their apogees highlighted by the circles

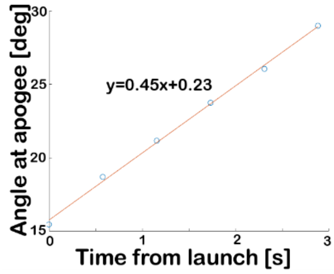

Figure 2b: Plot of the angle φ of the apogees in respect to the shift of the table with a linear regression

4. Results

The slopes of the linear regression for a variety of table speeds are shown in Table 1.

Table speed [deg/s]	Slope of linear regression
10	0.46
6	0.57
2	0.50
1	0.51
0.5	0.75

Table 1: Summary of the ratio of shift in the plane of oscillation versus the shift of the table for a selection of table speeds

These slopes are close to the theoretical value of 0.5, with exception of the lower table speed for which the slope is 0.75. An explanation is that the Foucault effect is small at this low speed and therefore the effects of the anisotropy dominate.

This experiment is a first validation of the predicted phenomenon: the frame of reference is rotating at half the rotational speed of the rotating table.

5. Future work

The natural question is whether it is possible to detect the rotation of the Earth. This would be a useful alternative to the standard Foucault pendulum which requires a high ceiling: they are usually over 5 [m] long in order to reduce parasitic precession [3]. The main result shows that these are more prominent for this mechanism, since the Foucault precession is reduced by half. Current work involves estimating the modifications required to detect the rotation of the Earth.

References

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