

Combined partitioning and approximation for optimized gear inspection

Axel von Freyberg¹, Andreas Fischer¹

¹University of Bremen, Bremen Institute for Metrology, Automation and Quality Science (BIMAQ), Linzer Str. 13, 28359 Bremen, Germany

a.freyberg@bimaq.de

Abstract

Quality inspection of gears is a standardized process, but it is based on only a few random surface points. In order to perform a more comprehensive quality inspection, precise optical measurement systems are being developed to acquire the complete geometrical data with high speed. The resulting point clouds are in particular convenient for the estimation of unknown gear parameters, but require a partitioning prior to the evaluation of geometrical features. However, the standard procedure will only lead to reliable results if the evaluation range is well defined. In case of unknown gear parameters, it is not able to automatically partition the integral geometric elements.

In order to automate and optimize the assessment of geometric gear parameters, e. g., for the estimation of unknown geometric gear parameters, a holistic approximation is presented, which combines the partitioning and the approximation of geometrical features in a single optimization routine. The holistic approximation is extended by a root point iteration that allows the calculation of the orthogonal distance of the measurement point to the surface of the geometric model for any object geometry, e. g. involutes modified by crownings. Simulations and experiments demonstrate that the holistic approximation is capable of precisely determining the geometric parameters with an implicit optimal partitioning of the measurement points.

Automation; Gear; Geometric modelling; Measurement

1. Introduction

Gear metrology is challenging due to the narrow tolerances and the boundary conditions of dimensional data acquisition. Often, the golden rule of metrology is not fulfilled. Instead of a measurement uncertainty of less than 10 % of the tolerance, only 20-30 % are reached [1]. Both the data acquisition and evaluation contribute to the total measurement uncertainty.

The standard data processing is tailored to the evaluation of line- and pointwise data to calculate deviations to the nominal, non-modified involute. Actual gear software does not provide area-oriented evaluation of dimensional data, but there are approaches to calculate areal deviation parameters [2, 3, 4]. Anyway, neither the standard evaluation nor other approaches offer a partitioning capability and are able to automatically estimate unknown gear parameters. Therefore, this article analyses the potential of gear data evaluation by applying a model-based holistic approximation in combination with an automated partitioning method. This method was validated for different applications already [5, 6]. Extended by a root point iteration [7], it now offers the potential for a precise evaluation of gear measurements for modified profiles.

2. Method

The standard inspection of involute gears consists of evaluating two lines on both flanks of 4 teeth (profile and slope deviations) and a single point on each flank (pitch and runout deviations). In the following, the methods and analysis are limited to the evaluation of the profile slope deviation $f_{H\alpha}$ as an exemplary quality parameter. Here, after approximating the nominal, non-modified involute to the measuring points, the residual distances of the measurement points to the nominal involute are evaluated in a second step by a regression [8]. If the

involute is designed with a crowning, the approximation with an ideal involute will induce errors in the calculation of the orthogonal distances, as non-correct root points were used.

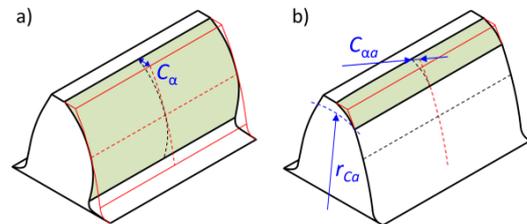


Figure 1: Involute flank with modifications (the red lines indicate the non-modified involute geometry): a) crowning (C_α) in profile direction; b) tip relief ($C_{\alpha r}$) starting at radius $r_{b,r}$.

The holistic approximation (HA) is a model-based approximation, which minimizes the least squares sum

$$\min_{\mathbf{a}_p, \mathbf{a}_g} \left[\sum_{i=1}^{N_i} d_i(\mathbf{a}_p, \mathbf{a}_g)^2 + \sum_{k=1}^{N_r} d_k(\mathbf{a}_p, \mathbf{a}_g)^2 \right] \quad (1)$$

of the orthogonal distances d_i , d_k of the $N = N_i + N_r$ measuring points to the approximating elements. These elements are a crowned involute with shape parameters base circle radius $r_{b,i}$ and crowning C_α (cf. **Error! Reference source not found.** a) and the tip relief described by its base circle radius $r_{b,r}$. The shape parameters are summarized by the vector $\mathbf{a}_g = [r_{b,i}, C_\alpha, r_{b,r}]$, and the position parameter vector $\mathbf{a}_p = [\varphi_b, r_{Ca}]$ contains only the rotation angle φ_b around the z-axis and the start radius of the tip relief r_{Ca} (see **Error! Reference source not found.** b), because the origin of the workpiece coordinates system (WCS) is known from the measurement already. In addition, the HA is capable of partitioning the measurement data into integral geometric elements by optimizing the associations between the measuring points and the approximating elements. Therefore, the point numbers N_i and N_k are varying during the iteration. Whereas the

orthogonal distances of the measuring points to the tip relief d_k can be directly calculated [2], a root point iteration [7] enables the calculation of the orthogonal point distances d_i between the measurement data and the crowned involute.

3. Results

The HA was verified by Monte-Carlo simulations. 100 repetitions with simulated data with uniformly distributed noise in normal direction of the involute within the range $[-0.75 \dots +0.75] \mu\text{m}$, for example, resulted in a standard error of $0.13 \mu\text{m}$ for the base circle radius and a mean deviation of $0.03 \mu\text{m}$ to the given value of a crowned involute with tip relief.

3.1. Validation with experimental data

For validating the HA, profile lines on a helical gear (number of teeth $z = 40$, module $m_n = 2 \text{ mm}$, helix angle $\beta = -30.75^\circ$, crowning $C_\alpha = 5 \mu\text{m}$) were measured as reference on a coordinate measuring machine (CMM) with ruby balls of 1.0 mm diameter. The acquired data points were evaluated with the CMM software as reference and with the HA within the same boundary conditions (automatic partitioning disabled). The results are presented in Table 1. The differences to the reference values are $0.2 \mu\text{m}$ and $0.1 \mu\text{m}$, respectively. As these differences are a fraction of the probing error $P_{\text{MPE}} = 0.9 \mu\text{m}$, the HA algorithm is validated.

Table 1: Validation results for the involute form parameters r_b and C_α of tooth #1.

	Reference	HA
Base circle radius r_b in mm	43.5337	43.5339
Crowning C_α in μm	5.0	5.1

3.2. Reduction of point density

The standard evaluation of a modified profile does not use the correct root point for calculating the orthogonal distance. It is analysed whether the root point iteration of the HA might lead to more precise results. For this analysis, real measurement points are evaluated with both methods in the same evaluation range, and the point density was stepwise reduced by a factor f_r . The results are presented in Figure 2. A reduction of the point density by a factor of 8 leads to deviations of the base circle radius up to $13 \mu\text{m}$ for the HA. The more accurate distance calculation within the HA requires additional degrees of freedom, and this multidimensional optimization is more sensitive to a reduced signal to noise ratio (SNR) than the standard evaluation.

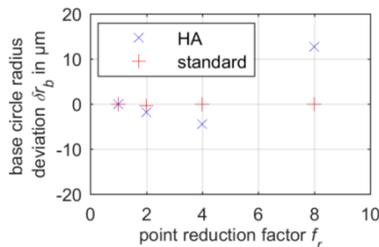


Figure 2: Influence of a reduced point density on the deviation of the base circle radius with respect to the standard evaluation and HA. The dataset without point reduction ($f_r = 1$) contains 111 measuring points.

3.3. Automated partitioning

The calculation of gear deviation parameters is sensitive to the evaluation boundaries. Figure 3 a) shows that varying the evaluation range only fractions of a millimeter causes deviations in the calculated parameters. The first line is the reference evaluation of the right flank of tooth 1.

Rows 2-4 of Figure 3 a) contain variations of the evaluation range ($d_{\text{cf}} - d_{\text{fa}}$) in an order of less than a millimeter leading to differences of almost $2 \mu\text{m}$ for the profile slope deviation, which corresponds to more than $5 \mu\text{m}$ difference for the base circle radius. Note that the tolerance for the profile slope deviation is $\pm 4.9 \mu\text{m}$ for gears in this dimension. The C_α values of the crowning vary within a range of $0.7 \mu\text{m}$.

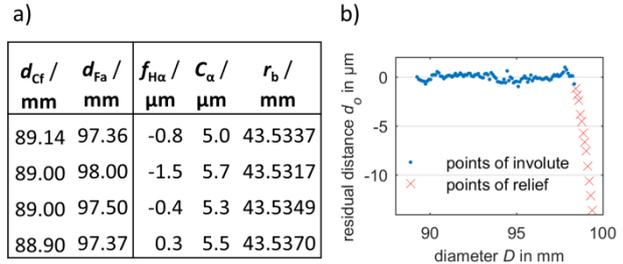


Figure 3: Partitioning of involute data: a) influence of the evaluation borders d_{cf} , d_{fa} on the deviation parameters calculated for the right flank of tooth #1; b) residual distances of measuring points to approximating involute, the red crosses are defined by the model-based partitioning to points belonging to the tip relief.

The HA provides an automated partitioning. One result is exemplary shown in Figure 3 b). The associations of the measuring points to both the involute and the tip relief are automatically selected in an optimal way based on the least squares sum according to Equation (1). Thus, the HA delivers reproducible results in cases where the evaluation ranges are not specified or the gear parameters are completely unknown.

4. Summary and conclusion

The extension of the HA by a root point iteration is validated for the application of gear inspection. A comparison between the standard two step approximation and the approximation of the measuring points directly by a crowned involute revealed that the two step approximation is more robust against a decreasing point density of noisy data. However, the implicit optimal partitioning of the HA enables automated evaluations and delivers robust results for an unspecified evaluation range. Small variations of the evaluation borders lead to significant differences in the calculated parameters for the standard evaluation. Here, the HA delivers repeatable results based on an optimal evaluation range due to the automatic partitioning.

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